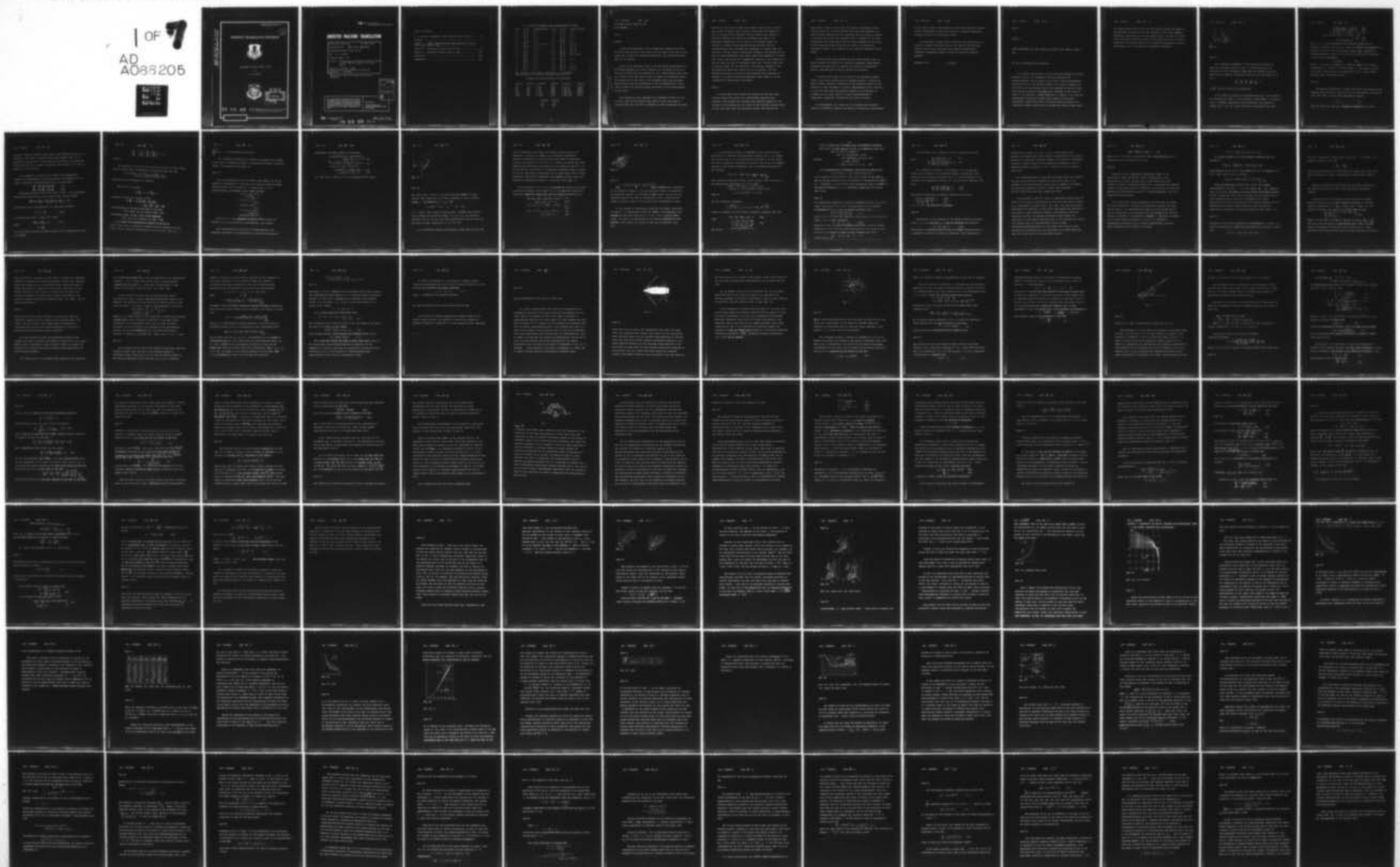
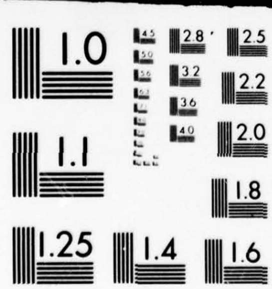
AD-A066 205 FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO F/G 20/4
NONLINEAR CONICAL FLOWS OF GAS,(U)
MAR 78 B M BULAKH
UNCLASSIFIED FTD-ID(RS)T-0266-78 NL

1 OF 7
AD
A066205





AD-A066205

FTD-ID(RS)T-0266-78

1

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

NONLINEAR CONICAL FLOWS OF GAS

by

B. N. Bulakh

FOREIGN TECHNOLOGY DIVISION

DDC
RECEIVED
22 MAR 1979
E

78 09 08 053

Approved for public release; distribution unlimited.

FTD-ID(RS)T-0266-78

UNEDITED MACHINE TRANSLATION

FTD-ID(RS)T-0266-78 27 March 1978

MICROFICHE NR: FTD-78-C-000420L

NONLINEAR CONICAL FLOWS OF GAS

By: B. N. Bulakh

English pages: 572

Source: Nelineynyye Konicheskiye Techeniya Gaza, Izd-vo, "Nauka", Moscow, 1970, pp. 1-343.

Country of origin: USSR
This document is a machine translation.
Requester: FTD/TQTA

Approved for public release; distribution unlimited.

ACCESSION for		
NTIS	White Section	X
DDC	Buff Section	☐
UNANNOUNCED		☐
JUSTIFICATION		
BY		
DISTRIBUTION/AVAILABILITY CODES		
Dist.	AVAIL. and/or SPECIAL	
A		

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.

PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

Table of Contents

U. S. Board on Geographic Names Transliteration System..... ii

Preface.. 1

Chapter 1. Common Properitties and Some Particular Forms of the Conical Flows of Gas.................................. 5

Chapter 2. Supersonic Conical Flows of Gas................ 197

Chapter 3. Hypersonic Conical Flows of Gas................ 435

Appendices... 531

References... 535

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ë in Russian, transliterate as yë or ë.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

NONLINEAR CONICAL FLOWS OF GAS.

B. M. Bulakh.

Page 7.

PREFACE

During the axisymmetric flow of supersonic uniform flow about the round cone of gas the bow shock near the apex of the cone is also round cone (if flow for abruptly supersonic), but flow behind shock wave is not uniform.

Since in the supersonic flow of gas the slight disturbances are not spread upstream, the which interests us flow about the cone of the finite length will be concealed as for a semi-infinite cone. Then as a result of the fact that in task is absent the reference length, the Parameters of flow prove to be constants along each ray/beam, which emerges from the apex/vertex of the streamlined cone. Such a field of flow he is called conical and serves as the object/subject of this book.

The problem of cone, examined by A. Busemann in 1929 [1], was the first task of the conical flow theory of gas. Long time it remained only. Only in 1943 A. Busemann in work [2] derived the basic

formulas of the linear conical flow theory, with the aid of which were solved at present many important tasks (see, for example [3, 4]). The theory of the nonlinear conical flows cf gas (based on "precise" equations of motion of nonviscous gas) began to be developed, essentially, in the 50's. In last/latter decade the interest in conical flows considerably was enforced. This is understandable, since although the parameters of conical flows are functions from two angular independent variables, they belong all the same to three-dimensional type flows. Within the framework of conical flow theory, are solved such fundamental problems of gas dynamics as task cf cone, the task cf triangular plate, etc. Conical flows serve as starting point for the solution to the spatial problems of the flow about the nonconical bodies and are of interest also for mathematical physics, since here many boundary-value problems are related to the mixed elliptical-hyperbolic type, almost in no way illuminated in mathematical literature.

Page 8.

In recent years were created the circuits cf the flow about conical bodies, were placed the corresponding boundary-value problems, were created the analytical and numerical methods of the solution of the problems of the theory of the nonlinear conical flows of gas, both with super and hypersonic speeds, were carried out

experimental studies. As a result the thecry of nonlinear conical flows acquired, to a certain extent, the final form. However, the special books, dedicated to this question, thus far there is neither in the USSR nor abroad. The proposed book is intended to fill this gap/spacing. The author hopes that all basic problems of the theory of the nonlinear conical flcws of gas fcund in it reflection, although the book is not handbook on the theory of nonlinear conical flows.

For this reason many theories are set forth concise. (For in detail research on question is indicated literature). Experimental materials are drawn only to evaluate the quality of theoretical results. Semi-empirical theories barely are set forth.

At present many tasks of the theory of the nonlinear conical flows of gas can be solved with the assigned degree of accuracy by ETsVM [digital computer]; therefore by the analytical method of the solution of such problems it is given comparatively little attention, in the same cases when the numerical methods of the solution of problems are not still created or their application/use is inexpedient, analytical theories are set forth in detail.

In bibliography, as a rule, are not indicated the technical reports of different companies and special institutions, unattainable

to the Soviet reader. Unfortunately, in the book could not preserve single designations and the same letters in different paragraphs designate now and then different values.

In conclusion we would like to note that this book arose as a result of author's many-year work in the range of the nonlinear conical flows of gas, initiated under Saveli Vladimirovich Falkovich's management/manual to whom the author is sincerely grateful.

December 1967. B. Bulakh.

Page 9.

Chapter 1.

COMMON PROPERTIES AND SOME PARTICULAR FORMS OF THE CONICAL FLOWS OF GAS.

§1. Basic assumptions and equations.

1.1. Lead-in observations. In this book are examined the steady motions of gas on the assumption that gas inviscid, that nonheat-conducts, and is found in the state of local thermodynamic equilibrium (or in the "frozen" state), i.e., there is equation of state of gas. In the general case of the equation of motion of this gas, contain as unknown functions three component of the velocity vector of the particle of gas $\boldsymbol{V}$, pressure p, density ρ (or other two thermodynamic functions of gas), which depend on three space coordinates. During flow those, limited by conical surfaces, the supersonic uniform flows of gas are formed the conical flows, which

are characterized by the facts that the velocity vector of V and the thermodynamic functions of gas are constant on half-lines, passing through the apex/vertex of the streamlined body - the pole of conical flow. (Incipient yielding of such type is caused by the fact that in supersonic flow conical body can be considered unlimitedly continued downstream, i.e., not having the significant dimension).

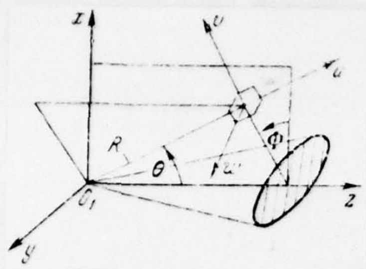


Fig. 1.

Page 10.

1.2. Spherical coordinates. If we introduce the system of spherical coordinates R, Φ, θ (Fig. 1) with beginning in the apex/vertex of the streamlined body, then the unknown values will depend only on angular coordinates Φ, θ, but will not depend on R, i.e.,

$$\frac{\partial V}{\partial R}=\frac{\partial p}{\partial R}=\frac{\partial \rho}{\partial R}=\frac{\partial S}{\partial R}=\frac{\partial i}{\partial R}=0$$

(S, i - specific enthropy and enthalpy).

In the selected system of coordinates (see Fig. 1) the equation of continuity div $(\rho\overline{V})=0$, the equation of Euler $d\mathbf{V}/dt = 1/\rho$ grad p (d/dt - complete, substantial time derivative), the equation of energy d/dt S = 0, for conical flow will be written in the form

$$(\rho v \sin\theta)_\theta + (\rho w)_\Phi = -2u\rho\sin\theta, \tag{1.1}$$
$$vu_\theta + w \operatorname{cosec}\theta u_\Phi - v^2 - w^2 = 0, \tag{1.2}$$
$$vv_\theta + w \operatorname{cosec}\theta v_\Phi + uv - w^2 \operatorname{ctg}\theta = -\frac{1}{\rho}\cdot p_\theta, \tag{1.3}$$
$$(vw_\theta + w \operatorname{cosec}\theta\, w_\Phi)\sin\theta + w\,(u\sin\theta + v\cos\theta) = -\frac{1}{\rho}p_\Phi, \tag{1.4}$$
$$vS_\theta + w \operatorname{cosec}\theta S_\Phi = 0. \tag{1.5}$$

Here u, v, w - the components of the velocity vector of V in direction increase with respect to R, θ, Φ; derived are designated by subscript about the sign of function, i.e. $f_\theta = \frac{\partial f}{\partial\theta}, f_\Phi = \frac{\partial f}{\partial\Phi}$. For the closing/shorting of system of equations (1.1)-(1.5) it is necessary to indicate the dependence

$$S = S(p, \rho) \tag{1.6}$$

[Also T = T(s, i), e = e(S, i), a = a(S, i), where T are absolute temperature, e - specific internal energy, a - the speed of sound. The graphic representation of dependences (1.7) is called Molier diagrams (see, for example [5])].

Page 11.

Multiplying the equation of Euler along flow line scalarly on **V**, replacing dp/ρ by di(di = dp/ρ with dS = 0) and producing integration along flow line, we will obtain Bernoulli's integral

$$\frac{V^2}{2} + i = i_0. \tag{1.8}$$

$V^2 = |V|^2 = u^2 + v^2 + w^2$, i_0 - stagnation enthalpy or the total

enthalpy. Since the conical flows appear from uniform flow and i_0 is constant in all field of conical flow. With integral (1.8) it is possible to replace one of the equations (1.2) is(1.4). Is simplified with the aid of (1.8) system of equations (1.1)-(1.5), (1.7) (see in regard to this also [6]).

It is temporary in order not to disrupt the designations, accepted in thermodynamics, the designations of derivatives let us extract completely. Differentiating (1.7), we will obtain

$$\frac{\partial\rho}{\partial\theta}=\left(\frac{\partial\rho}{\partial S}\right)_i\frac{\partial S}{\partial\theta}+\left(\frac{\partial\rho}{\partial i}\right)_S\frac{\partial i}{\partial\theta}, \tag{1.9}$$

$$\frac{\partial\rho}{\partial\Phi}=\left(\frac{\partial\rho}{\partial S}\right)_i\frac{\partial S}{\partial\Phi}+\left(\frac{\partial\rho}{\partial i}\right)_S\frac{\partial i}{\partial\Phi}. \tag{1.10}$$

Multiplying equation (1.9) on v sin θ, (1.10) - on w, piecemeal store/adding up and taking into account (1.5), we will obtain

$$\frac{\partial\rho}{\partial\theta}v\sin\theta+\frac{\partial\rho}{\partial\Phi}w=\left(\frac{\partial\rho}{\partial i}\right)_S\left(\frac{\partial i}{\partial\theta}v\sin\theta+\frac{\partial i}{\partial\Phi}w\right). \tag{1.11}$$

From second law of thermodynamics for reversible processes

$$T\,dS=di-\frac{dp}{\rho} \tag{1.12}$$

it follows that with S = const, di = dp/ρ,

$$\left(\frac{\partial\rho}{\partial i}\right)_S=\rho\left(\frac{\partial\rho}{\partial p}\right)_S=\frac{\rho}{a^2}, \tag{1.13}$$

where

$$a^2=\left(\frac{\partial p}{\partial\rho}\right)_S$$

(a is the local velocity of sound in gas). Let us differentiate (1.8) on θ and Φ:

$$\left.\begin{aligned}\frac{\partial i}{\partial \theta} &= -\left(u\frac{\partial u}{\partial \theta} + v\frac{\partial v}{\partial \theta} + w\frac{\partial w}{\partial \theta}\right), \\ \frac{\partial i}{\partial \Phi} &= -\left(u\frac{\partial u}{\partial \Phi} + v\frac{\partial v}{\partial \Phi} + w\frac{\partial w}{\partial \Phi}\right).\end{aligned}\right\} \quad (1.14)$$

Page 12.

We eliminate ρ from (1.1) with the aid of (1.11), (1.13), (1.14) and we utilize (1.2); the equation of continuity takes the form

$$(a^2 - v^2)\sin\theta v_\theta = (w^2 - a^2)\ w_\Phi + vw\,(\sin\theta w_\theta + v_\Phi) + \\ + u\,(v^2 + w^2 - 2a^2)\sin\theta - a^2 v\cos\theta. \quad (1.15)$$

(Derivatives are designated in indices).

From (1.12) it follows

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial S}\right)_i = T, \ -\frac{1}{\rho}\left(\frac{\partial p}{\partial i}\right)_S = 1,$$

therefore, taking into account (1.14),

$$-\frac{1}{\rho}\frac{\partial p}{\partial \Phi} = -\frac{1}{\rho}\left[\left(\frac{\partial p}{\partial S}\right)_i\frac{\partial S}{\partial \Phi} + \left(\frac{\partial p}{\partial i}\right)_S\frac{\partial i}{\partial \Phi}\right] = \\ = T\frac{\partial S}{\partial \Phi} + u\frac{\partial u}{\partial \Phi} + v\frac{\partial v}{\partial \Phi} + w\frac{\partial w}{\partial \Phi},$$

and equation (1.4) he will be written in the form

$$TS_\Phi = v\sin\theta w_\theta - uu_\Phi - vv_\Phi - \sin\theta w(u + v\operatorname{ctg}\theta). \quad (1.16)$$

Consequence (1.5), (1.16), (1.2) is the equation

$$TS_\theta = -uu_\theta - ww_\theta + w\operatorname{cosec}\theta v_\Phi + uv - w^2\operatorname{ctg}\theta. \quad (1.16a)$$

As a result of transformations instead of the system (1.1)-(1.5), (1.7) is obtained a simpler system of equations (1.15), (1.2), (1.16), (1.5) for determining of u, v, w, S [T = T(S, i),

a =

a(S, i); i is determined from (1.8)].

1.3. Cartesian coordinates. In series of problems (for example, in the case of flat/plane wings) to conveniently utilize a Cartesian system of coordinates O_1xyz (Fig. 2).

Page 13.

Here the parameters of gas in conical flow depend only on the relation of coordinates $\xi = x/z$, $\eta = y/z$. Plane ξ, η has the simple physical sense: this is plane z = 1 in space xyz, a ξ, η - correspondingly, coordinate x and y takes the form

$$\begin{aligned}
&\rho(u_\xi + v_\eta - \xi w_\xi - \eta w_\eta) + \\
&\quad + (u - \xi w)\rho_\xi + \\
&\quad\quad + (v - \eta w)\rho_\eta = 0, \\
&(u-\xi w)u_\xi + (v-\eta w)u_\eta = \\
&\quad = -\frac{1}{\rho}p_\xi, \\
&(u-\xi w)v_\xi + (v-\eta w)v_\eta = \\
&\quad = -\frac{1}{\rho}p_\eta, \\
&(u-\xi w)w_\xi + (v-\eta w)w_\eta = \\
&\quad = \frac{1}{\rho}(\xi p_\xi + \eta p_\eta), \\
&(u-\xi w)S_\xi + (v-\eta w)S_\eta = 0.
\end{aligned}$$

Here u, v, w - the **components of vector of V** in direction increase x, y, z; derivatives of designations, with indices.

From these equations by the way of transformations, the completely analogous to transformations in the case of spherical

coordinates, we obtain system of equations:

$$(u-\xi w)\left(\frac{u^2+v^2+w^2}{2}\right)_\xi+(v-\eta w)\left(\frac{u^2+v^2+w^2}{2}\right)_\eta+ \\ +a^2(\xi w_\xi+\eta w_\eta-u_\xi-v_\eta)=0, \quad (1.17)$$

$$\xi\,[(u-\xi w)\,u_\xi+(v-\eta w)\,u_\eta]+\eta\,[(u-\xi w)\,v_\xi+ \\ +(v-\eta w)\,v_\eta]+(u-\xi w)\,w_\xi+(v-\eta w)\,w_\eta=0, \quad (1.18)$$

$$(u-\xi w)\,w_\xi+(v-\eta w)\,w_\eta+\xi\left(\frac{u^2+v^2+w^2}{2}\right)_\xi+ \\ +\eta\left(\frac{u^2+v^2+w^2}{2}\right)_\eta+T\left(\xi\frac{\partial S}{\partial\xi}+\eta\frac{\partial S}{\partial\eta}\right)=0, \quad (1.19)$$

$$(u-\xi w)\,S_\xi+(v-\eta w)\,S_\eta=0 \quad (1.20)$$

[a = a(S, i), T = T(S, i), i it is determined from (1.8)].

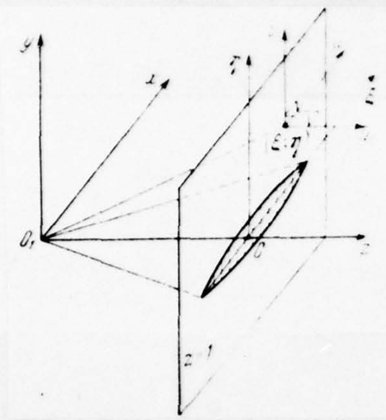


Fig. 2.

Page 14.

For ideal gas $p = \rho RT$ (R - the characteristic constant of gas), specific heat capacities at constant pressure c_p and a constant volume c_v are constant $(R = c_p - c_v)$ and

$$S = c_v \ln (p\rho^{-\gamma}) + \text{const}, \quad \gamma = \frac{c_p}{c_v}, \qquad i = c_p T = \frac{a^2}{\gamma - 1}.$$

$a^2 = \gamma (p/\rho) = \gamma RT$. Hence it follows that $T = (a^2)/\gamma R$, and integral (1.8) assumes the form $(V^2/2) + (a^2)/(\gamma - 1) = i_0$, i.e., in systems of equations (1.15), (1.16), (1.5) and (1.17)-(1.20) T and a^2 there are the known (linear) functions $V^2 = u^2 + v^2 + w^2$.

1.4. Generalized spherical coordinates. During the study of the

flow of hypersonic flows about the conical bodies of gas it is convenient to utilize a system of the orthogonal generalized spherical coordinates R, θ, Φ, in which one family of coordinate surfaces are spheres $R^2 = x^2 + y^2 + z^2 = \text{const}$; the second and third families are constructed as follows. On the surface of sphere R = 1, are carried out two families of orthogonal coordinate lines, moreover so that to body surface corresponds one of these lines. Then to these coordinate lines stretch themselves the conical surfaces, which are accepted as coordinate surface θ = const, Φ = const (Fig. 3).

If we designate by u, v, w the components of vector of rate of V in direction increase with respect to R, θ, Φ, then the equation of continuity, momentum, energy they will be written in the form [7]

$$2\rho u + \frac{v}{A_1}\rho_\theta + \frac{w}{A_2}\rho_\Phi + \frac{\rho}{A_1A_2}[(vA_2)_\theta + (wA_1)_\Phi] = 0, \quad (1.21)$$

$$\frac{v}{A_1}u_\theta + \frac{w}{A_2}u_\Phi - v^2 - w^2 = 0, \quad (1.22)$$

$$\frac{v}{A_1}v_\theta + \frac{w}{A_2}v_\Phi + uv + vw\frac{(\ln A_1)}{A_2} - w^2\frac{(\ln A_2)_\theta}{A_1} = -\frac{1}{\rho A_1}p_\theta, \quad (1.23)$$

$$\frac{v}{A_1}\left(i + \frac{u^2 + v^2 + w^2}{2}\right)_\theta + \frac{w}{A_2}\left(i + \frac{u^2 + v^2 + w^2}{2}\right)_\Phi = 0, \quad (1.24)$$

$$\frac{v}{A_1}S_\theta + \frac{w}{A_2}S_\Phi = 0. \quad (1.25)$$

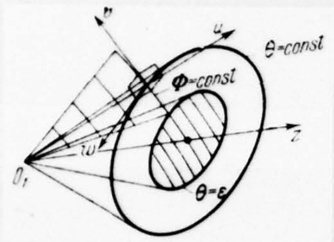


Fig. 3.

Page 15.

Here $A_1 = \frac{1}{R}\sqrt{x_\theta^2 + y_\theta^2 + z_\theta^2}$, $A_2 = \frac{1}{R}\sqrt{x_\Phi^2 + y_\Phi^2 + z_\Phi^2}$ Lame's coefficients, calculated cn the surface of sphere R = 1; one equation of Euler is replaced by the Bernoulli integral in differential form (1.24). Let us note the important special case when as surfaces Φ = const is selected the family of planes, normal to the surface it is streamlined of body.

In this system of coordinates θ and Φ is arc length, calculated of sphere R = 1 respectively along the normal tc the surface of the streamlined body and along it; $A_1 = 1$, $A_2 = \cos\theta - K_b'(\Phi)\sin\theta$, where $K_b'(\Phi)$ is equal to R, multiplied by the surface curvature of body, positive, when surface is ccncave to the side of increase θ (see [8]).

1.5. Irrotational flows. If the flow of gas is irrotational, then there is a velocity potential ϕ (V = grad ϕ) and $S = S_0$ = const. For conical flow ϕ, it is represented in the form ϕ = RF (θ, Φ), in the case of spherical coordinates. Function F let us call the conical potential. The velocity components will be determined according to the formulas

$$u = \varphi_R = F,\ v = \frac{1}{R}\varphi_\theta = F_\theta,\ w = \frac{1}{R\sin\theta}\varphi_\Phi = \frac{1}{\sin\theta}F_\Phi, \tag{1.26}$$

while system of equations (1.15), (1.2), (1.16), (1.5) will come to one equation for determining of F (see [20]).

$$\sin^2\theta(a^2 - v^2)F_{\theta\theta} - 2vw\sin\ \ \ \Phi + (a^2 - w^2)F_{\Phi\Phi} + $$
$$+ \sin^2\theta(2a^2 - v^2 - w^2)F + \frac{1}{2}\sin 2\theta(a^2 + w^2)F_\theta\ \ 0, \tag{1.27}$$
$$\left[a = a(i, S_0), \frac{u^2 + v^2 + w^2}{2} + i = i_0,\ (1.8)\right].$$

Page 16.

For the Cartesian coordinates

$$\left.\begin{array}{l}\varphi = zF(\xi, \eta),\\ u = \varphi_x = F_\xi, v = \varphi_y = F_\eta, w = \varphi_z = F - \xi F_\xi - \eta F_\eta\end{array}\right\} \tag{1.28}$$

System of equations (1.17)-(1.20) is reduced to equation (see [9])

$$L[F] = AF_{\xi\xi} + 2BF_{\xi\eta} + CF_{\eta\eta} = 0, \tag{1.29}$$

where

$$\begin{array}{l}A = a^2(1 + \xi^2) - (u - \xi w)^2,\\ B = a^2\xi\eta - (u - \xi w)(v - \eta w),\\ C = a^2(1 + \eta^2) - (v - \eta w)^2,\end{array} \tag{1.30}$$
$$\left[a = a(i, S_0), \frac{u^2 + v^2 + w^2}{2} + i = i_0\right.$$

see (1.8)].

If we in plane ξ, η introduce polar coordinates by formulas $\sqrt{\xi^2+\eta^2}$, $\operatorname{tg}\theta = \eta/\xi$, then equation (1.29) is converted to form [10]

$$\{a^2(1+r^2) - [rF - (1+r^2)F_r]^2\}F_{rr} + $$
$$+ 2\left[F - \left(r + \frac{1}{r}\right)F_r\right]F_\theta\left(\frac{1}{r}F_{r\theta} - \frac{1}{r}F_\theta\right) +$$
$$+\left(a^2 - \frac{1}{r^2}F_\theta^2\right)\left(\frac{1}{r^2}F_{\theta\theta} + \frac{1}{r}F_r\right) = 0, \quad (1.31)$$

moreover

$$u = \cos\theta F_r - \sin\theta\frac{1}{r}F_\theta,$$
$$w = \sin\theta F_r + \cos\theta\frac{1}{r}F_\theta, \; w = F - rF_r. \quad (1.32)$$

1.6. Transformation of hodograph. Since for the conical flow

$$u = \varphi_x = u(\xi, \eta), \quad v = \varphi_y = v(\xi, \eta), \; w = w(\xi, \eta) = \varphi_z,$$

it is possible to write that $\xi = \xi(u, v)$, $\eta = \eta(u, v)$ and hence $w = w(u, v)$ (on the condition that the jacobian of transformation $\frac{D(u, v)}{D(\xi, \eta)} = F_{\xi\xi}F_{\eta\eta} - F_{\xi\eta}^2 \neq 0$), i.e., to each conical flow corresponds certain surface in the space of hodograph u, v, w (case $F_{\xi\xi}F_{\eta\eta} - F_{\xi\eta}^2 \equiv 0$ will be examined later than).

Page 17.

The differential equation, to which it satisfies w = w(u, v), is most simply obtained with the aid of the application of the Legendre transformation [11, 12, 13] to conical potential $F(\xi, \eta)$:

$$\left.\begin{aligned}\chi(u, v) + F(\xi, \eta) &= u\xi + v\eta,\\ u = F_\xi, \; v = F_\eta, \; \xi &= \chi_u, \; \eta = \chi_v.\end{aligned}\right\} \quad (1.33)$$

From (1.28) it follows that Legendre potential $\chi(u, v)$ in our case is simply - w(u, v) and, therefore,

$$\xi = -w_u, \quad \eta = -w_v. \quad (1.34)$$

Equation for w(u, v) will be obtained from (1.29), if we in (1.30) substitute (1.34), and the second derivatives of F in terms of ξ in terms of η to replace in terms of known formulas (see [13])

$$F_{\xi\xi} = -w_{vv}g, \; F_{\xi\eta} = w_{uv}g, \quad F_{\eta\eta} = {}_{uu}\cdot g,$$

where $g = F_{\xi\xi}F_{\eta\eta} - F_{\xi\eta}^2$.

After reduction for g equation (1.29) it will take the form

$$A_1 w_{vv} - 2B_1 w_{uv} + C_1 w_{uu} = 0, \tag{1.35}$$

where

$$\left.\begin{aligned} A_1 &= a^2(1 + w_u^2) - (u + ww_u)^2, \\ B_1 &= a^2 w_u w_v - (u + ww_u)(v + ww_v), \\ C_1 &= a^2(1 + w_v)^2 - (v + ww_v)^2. \end{aligned}\right\} \tag{1.36}$$

1.7. Canonical variables. In the case $AC - B^2 > 0$ equation (1.29) can be reduced to the canonical system of equations, which contains as the unknown functions ξ, η, u, v, w [see (1.28)], if we introduce new independent variables ρ, σ by the aid of equations (see [14])

$$A\eta_\rho - B\xi_\rho + \sqrt{AC - B^2}\,\xi_\sigma = 0, \tag{1.37}$$

$$A\eta_\sigma - B\xi_\sigma - \sqrt{AC - B^2}\,\xi_\rho = 0. \tag{1.38}$$

The remaining equations of canonical system take the form

$$Au_\rho + Bv_\rho + \sqrt{AC - B^2}\,v_\sigma = 0, \tag{1.39}$$

$$Au_\sigma + Bv_\sigma - \sqrt{AC - B^2}\,v_\rho = 0, \tag{1.40}$$

$$\Delta w + \eta\Delta v + \xi\Delta u = 0 \tag{1.41}$$

($\Delta = \frac{\partial^2}{\partial\rho^2} + \frac{\partial^2}{\partial\sigma^2}$ — the operator of Laplace).

Page 18.

Furthermore, on the boundary of the region in which are defined ξ, η, u, v, w as functions ρ, σ, must be fulfilled the condition

$$dw + \eta\, dv + \xi\, du = 0. \tag{1.42}$$

From system of equations (1.37)-(1.41) by crossed differentiation it is possible to obtain the system of equations, which contains old

derivatives of ξ, η, u, v, w for ρ and σ, in the form of the operators of Laplace (see [14]). System of equations (1.37)-(1.42) of invariant of relatively conformal mapping of the plane of independent variables. (Another method of the reduction of equation (1.29) to canonical systems in cases $AC - B^2 < 0$ and $AC - B^2 > 0$ is given in work [15]).

1.8. Characteristics of equations of conical flows. For study of the type of the system of equations of the flow of gas, it is possible to use any coordinate system. Let us examine, for example, the Cartesian coordinate system, in which these equations [(1.17)-(1.20)] they have most symmetrical form.

From equations (1.18) and (1.20) it immediately follows that the lines of the constant entropy which we will call flow lines on plane ξ, η, determined by equation $d\xi/u - \xi w = d\eta/v - \eta w$, are dual system performances of equations (1.17)-(1.20). Multiplying the equations of this system to some indefinite factors, store/adding up them and requiring, so that the obtained combination would contain the derivatives of the unknown functions only in one direction (or taking into account that the characteristic is line of weak (discontinuity/interruption), we will obtain that those who were remaining two characteristics are determined by the same equations that and in the case of the irrotational motion

$$A\,(d\eta)^2 - 2B\,d\eta\,d\xi + C\,(d\xi)^2 = 0, \qquad (1.43$$

where A, B, C are given by formulas (1.30). Relationship/ratios on characteristics are derived in p. 12.2.

Page 19.

Equation (1.29) - elliptical (hyperbolic) type, if the projection of speed of V on the plane, perpendicular to the radius-vector of point in space xyz, is less (more) the local velocity of sound a [especially simply this is obtained from equation (1.27)]; respectively system of equations (1.17)-(1.20) will have two merging real characteristics and two - apparent/imaginary (all four characteristics are real).

Let us show that system performances (1.17)-(1.20) are traces from the intersection of conical characteristic surfaces and space xyz by plane z = 1. Characteristic surfaces for the steady flow of gas are determined by the condition that the projection of velocity vector on standard to characteristic surface, V_n. is equal in modulus to the local velocity of sound a, i.e., $V_n = \pm a$. Let us be subsequently interested only in conical characteristic surfaces. Their equations can be written in the form

$$\Gamma\left(\frac{x}{z}, \frac{y}{z}\right) = 0 \text{ or } \Gamma(\xi, \eta) = 0\left(\xi = \frac{x}{z}, \eta = \frac{y}{z}\right).$$

The single standard n to this surface in space xyz has the components

$$n\left(\frac{\Gamma_\xi}{N}; \frac{\Gamma_\eta}{N}; -\frac{\Gamma_\xi \xi + \Gamma_\eta \eta}{N}\right), N = \sqrt{\Gamma_\xi^2 + \Gamma_\eta^2 + (\Gamma_\xi \xi + \Gamma_\eta \eta)^2},$$

a the velocity vector of V - with respect to u, v, w. Condition $V_n = V \cdot n = \pm a$ let us write now in the form

$$[u\Gamma_\xi + v\Gamma_\eta - w(\xi\Gamma_\xi + \eta\Gamma_\eta)] [\Gamma_\xi^2 + \Gamma_\eta^2 + (\xi\Gamma_\xi + \eta\Gamma_\eta)^2]^{-1/2} = \\ = \pm a. \quad (1.44)$$

After the erection of relationship (1.44) into square, replacements Γ_ξ on $d\eta, \Gamma_\eta$ - on $(-d\xi)$ (since $\frac{d\eta}{\Gamma_\xi} = -\frac{d\xi}{\Gamma_\eta}$ along the curve :gu = of 0) and simple transformations we will obtain equation (1.43). Turning the given above reasonings, we will obtain that to each characteristic on plane ξ, η corresponds the conical characteristic surface in space xyz. Flow lines are traces from the intersection of conical stream surfaces, which are also the characteristic surfaces of the steady motion of gas, plane z = 1.

Page 20.

Actually, if the equation of conical stream surface is $(\xi, \eta) = 0$, then the projection of speed V on standard n to it is equal to zero, i.e.,

$$u\Gamma_\xi + v\Gamma_\eta - w(\xi\Gamma_\xi + \eta\Gamma_\eta) = 0 \text{ [см. (1.44)]}.$$

that after replacement respectively Γ_ξ on $d\eta, \Gamma_\eta$ - on $(-d\xi)$, it is possible to write in the form

$$\frac{d\xi}{u-\xi w}=\frac{d\eta}{v-\eta w}.$$

(in spherical coordinates of characteristic essence also traces from the intersection of conical characteristic surfaces with sphere R = 1).

System of equations (1.17)-(1.20) has the great similarity to the equations of motion of gas in two-dimensional vortex/eddy problem.

Actually, if we for flat/plane vortex flow as the unknown functions take the projections of speed on the axis of Cartesian system of coordinates u, v and $s = S\,[c_v\gamma\,(\gamma-1)]^{-1}$, where S is the specific enthropy, then equations of motion take the form

$$\left.\begin{array}{l}(a^2-u^2)\,u_x-uv\,(u_y+v_x)+(a^2-v^2)\,v_y=0,\\ us_x+vs_y=0,\\ v\,(u_y-v_x)-a^2s_x=0,\end{array}\right\}\quad(1.45)$$

where a is the speed of sound; $a^2=\frac{\gamma-1}{2}(V_{\text{np}}^2-u^2-v^2)$, $\left(f_x=\frac{\partial f}{\partial x},\, f_y=\frac{\partial f}{\partial y}\right)$. The flow lines, determined by equalization dx/u = dy/v, are here system performances of equations (1.45) (along them S = const; therefore such characteristics they are called entropy). The type of

flow (supersonic, subsonic), and the ranges of effect are completely defined by remaining two system performances (1.45), their equation taking the same form as in irrotational task. Exactly the same is matter with system of equations (1.17)-(1.20), only here for one unknown function it will be more; therefore will be addded one additional equation, and flow lines on plane ξ, η (lines of the constant entropy), determined by equation $d\xi/u - \xi w = d\eta/v - \eta w$, are dual characteristics.

Page 21.

The type of flow and range of effect (on plane ξ, η) they are completely determined by the remaining two characteristics; see (1.43). For this reason at the points where the last/latter characteristics are real (are apparent/imaginary), flow will be called conically supersonic (conical subsonic).

The noted nearness between two-dimensional problem of gas dynamics and tasks of conical flow theory on plane ξ, η (or to single sphere) makes it possible in many instances correct to foresee the properties of conical flows, being oriented toward known facts in two-dimensional problem.

1.9. Shock waves. If we designate by n normal to the surface of

the streamlined conical body, then the condition of its nonseparated flow is $V_n = V \cdot n = 0$, i.e., body surface must be stream surface. (Specifically, on plane ξ, along line, corresponding to body surface, is satisfied condition $d\xi/u - \xi w = d\eta/v - \eta w$).

On the surface of shock cone, are fulfilled the laws of conservation of mass, energy, momentum/impulse/pulse, which, if we designate the parameters of gas ahead of the shock by index 1, and after the shock - by index 2, they are record/written in the form

$$\left.\begin{array}{c} \rho_1 V_{n1} = \rho_2 V_{n2}, \quad i_1 + \frac{V_1^2}{2} = i_2 + \frac{V_2^2}{2}, \\ V_{\tau_1 1} = V_{\tau_1 2}, \qquad V_{\tau_2 1} = V_{\tau_2 2}, \\ p_2 + \rho_2 V_{n2}^2 = p_1 + \rho_1 V_{n1}^2. \end{array}\right\} \quad (1.46)$$

Here V_n, V_{τ_1}, V_{τ_2} - with respect to the projection of speed of V on single standard even two any tangential directions to the shock layer. Subsequently to relationships (1.46) will be given the forms, convenient for the solution of the problems of the corresponding section. Let us here examine only the case when shock wave is assigned by equation $\eta = \eta_s(\xi)$ and gas - ideal, i.e., $i = \gamma/\gamma - 1\ p/\rho$.

Page 22.

Let us assume that the shock wave borders on the flow, which has component speeds along the axes of the Cartesian system of coordinates O_1xyz, equal to u_1, v_1, w_1, and the speed of sound a_1. The velocity components after shock wave u_2, v_2, w_2 is possible

easily to obtain from the continuity condition of the component of the speed, tangential to the surface of jump, and Prandtl's conditions for normal components in the case of oblique shock. Omitting simple calculations, let us write the final formulas:

$$u_2 = u_1 - \eta_s' P, \quad v_2 = v_1 + P, \quad w_2 = w_1 + (\xi\eta_s' - \eta_s) P,$$

where

$$\eta_s' = \frac{d\eta_s}{d\xi},$$

$$P = \frac{2}{\gamma+1}\left[\frac{a_1^2}{v_1 - \eta_s' u_1 + w_1(\xi\eta_s' - \eta_s)} - \frac{v_1 - \eta_s' u_1 + w_1(\xi\eta_s' - \eta_s)}{1 + \eta_s'^2 + (\xi\eta_s' - \eta_s)^2}\right]. \tag{1.47}$$

Increase $s = S[\gamma(\gamma-1)c_v]^{-1}$ **during transition through the jump is determined** with the aid of usual formula for an increase in the entropy on shock wave

$$S_2 - S_1 = \frac{1}{\gamma(\gamma-1)}\left\{\ln\left[\frac{2\gamma q_n^2 - \gamma + 1}{\gamma+1}\right] - \gamma\ln\left[\frac{2 + (\gamma-1)q_n^2}{(\gamma+1)q_n^2}\right]\right\}, \tag{1.48}$$

where q_n is dimensionless velocity component in the direction of standard to shock wave; it is determined by the relationship/ratio

$$q_n^2 a_1^2 = [v_1 - u_1\eta_s' + w_1(\xi\eta_s' - \eta)]^2 [1 + \eta_s'^2 + (\xi\eta_s' - \eta_s)^2]^{-1}. \tag{1.49}$$

Let us assume now that u_1, v_1, w_1 - the components of the **speeds of irrotational** conical flow; then occurs the relationship/ratio $dw_1 + \eta_s dv_1 + \xi\, du_1 = 0$, **which** is the consequence of formulas (1.28). **Let us assume also** that flow behind shock wave irrotational, **then** $u = F_\xi$, $v = F_\eta$, $w = F - \xi F_\xi - \eta F_\eta$, **where** F, the conical potential of this **flow**,. **When** $\eta = \eta_s(\xi)$ **according to** (1.47) are fulfilled the equalities

$$\left.\begin{aligned}(F_\xi)_2 &= u_1 - \eta_s' P, \ (F_\eta)_2 = v_1 + P,\\ F_2 - \xi(F_\xi)_2 - \eta_s(F_\eta)_2 &= w_1 + (\xi\eta_s' - \eta_s)P.\end{aligned}\right\} \quad (1.50)$$

Page 23.

Functions $F_2, (F_\xi)_2, (F_\eta)_2$, those who were determined from (1.50), satisfy the conditions of strip, potential F is continuous during transition through the shock wave. Actually, let us multiply first equation (1.50) on ξ, the second - on η_s let us add with the third; as a result we will obtain

$$F_2 = w_1 + v_1\eta_s + u_1\xi = F_1.$$

Let us differentiate F_2 along shock wave:

$$dF_2 = dw_1 + \eta_s\,dv_1 + \xi\,du_1 + v_1\,d\eta_s + u_1\,d\xi = \\ = v_1\,d\eta_s + u_1\,d\xi. \quad (1.51)$$

Let us multiply first equation (1.50) by dξ, the second on $d\eta_s$ let us add them; as a result we will obtain

$$(F_\xi)_2\,d\xi + (F_\eta)_2\,d\eta_s = u_1\,d\xi + v_1\,d\eta_s. \quad (1.52)$$

From formulas (1.51), (1.52) follows the condition of the strip:

$$dF_2 = (F_\xi)_2 d\xi + (F\eta)_2\,d\eta_s. \quad (1.53)$$

This means that during the study of flows after shock waves in the irrotational approach/approximation of condition for the components of speed on jump they are satisfied by irrotational flow accurately, and it is not required to derive/conclude some approximations, valid only for irrotational flows.

In series of problems, it is necessary to examine contact discontinuity/interruptions in the form of conical surfaces; on such surfaces are satisfied the normal conditions

$$p_2 = p_1, \quad V_{n1} = V_{n2} \qquad (1.54)$$

(here n - standard to the contact surface).

§2. Some particular forms of the conical flows of gas.

In §2 will be in essence examined the simplist forms of the conical flows for which the equations of motion are reduced to ordinary differential equations or to the systems of final equations.

End section.

Page 24.

a) The axisymmetric flow about the round cone.

2.1. Lead-in observations. The task of the determination of axisymmetric supersonic flow about round cone was examined for the first time by A. Busemann in 1929 in the report from which in press/printing appeared only small delay [1]. Then appeared the very short reference to the solution of this problem in M. Burkard's note [16] and almost simultaneously with it was published the work of G. Taylor and D. Makkol [17] in whom the task of cone was investigated with large completeness both theoretically and experimentally. (Late D. Makkol [18] expanded and refined the calculations, carried out in [17], and were obtained the shadow photographs of the flying projectiles with cone head one of which is reproduced in Fig. 4.) A. Busemann solved problem in the space of hodograph, G. Taylor and D. Makkol utilized spherical coordinates in physical space.

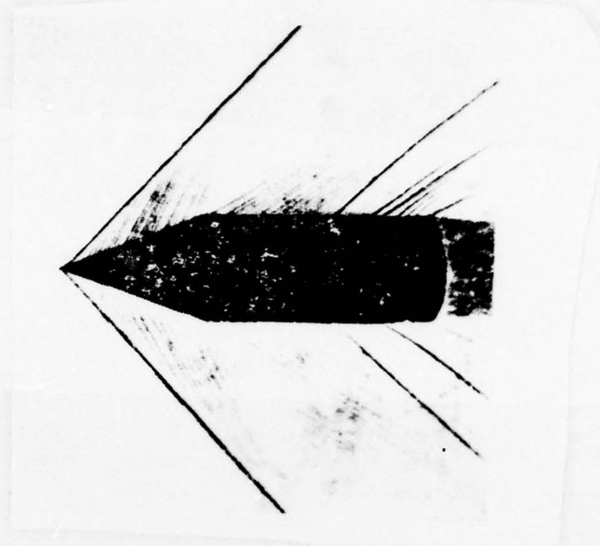


Fig. 4.

Page 25.

Since then for the task of the axisymmetric flow about the round cone, were comprised vast tables, were carried out the experiments, more precisely formulating the possible mode/conditions of the flow about the cone, were obtained numerous approximate solutions, were found empirical formulas for the different mode/conditions of the flow about the cone. Many of these results entered into textbocks on gas dynamics. For this reason well known things will examined briefly, and primary attention will be allotted to the less known to

the wide circle of the readers event/reports, which relate mainly to the transonic and hypersonic mode/conditions of the flow about the cone.

Let us examine round cone with half-angle ε in the uniform supersonic flow of gas, which has density ρ_1, temperature T_1, specific enthropy and enthalpy respectively S_1 and i_1, Mach number M_1 and speed V_1, directed along the axis of cone (Fig. 5).

If the parameters of uniform flow and value ε are such, that in the range between the leading shock wave and the surface of cone the flow is supersonic, then cone can be examined by unlimitedly continued downstream (then since the slight disturbances in the supersonic flow of gas they are not spread upstream). Since under conditions of task is not contained the reference length, the parameters of gas can depend only on angle θ, if we utilize spherical coordinates, or $\sqrt{\xi^2+\eta^2} = \frac{\sqrt{x^2+y^2}}{z}$, if we use Cartesian coordinates, i.e., flow will be conical.

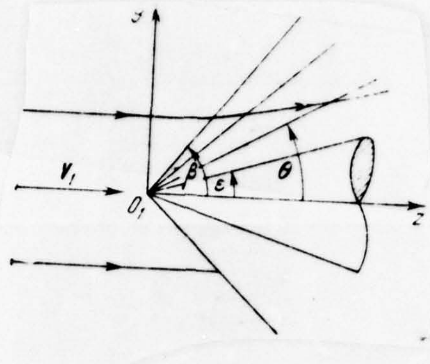


Fig. 5.

Page 26.

(About other mode/conditions of the flcw about the cone will be said later) all cell/elements of bow shock are inclined toward the direction of undisturbed flow at identical angle, therefore, after gallop $S = S_2$ = const and flow irrotational.

2.2. Hodograph analysis. Following A. Busemann [1, 11], let us examine the solution of problem in the space of hodograph. Since flow about cone is axisymmetric, the surface in the space of hodograph, corresponding to this flow, must be surface of revolution relative to axis w, and its equation can be written in the form

$$w = w(\omega), \quad \omega = \pm\sqrt{u^2 + v^2}, \qquad (2.1)$$

where ω is velocity component, perpendicular to the axis of symmetry.

Since the surface of revolution is determined by any meridional section of it, let us examine the axial section of surface by plane u = 0. Differentiating (2-1) with respect to u, v and set/assuming then u = 0, we will obtain the following relationship/ratios:

$$\left.\begin{array}{l}\omega = v,\ w_u = 0,\ w_v = \dfrac{dw}{d\omega} = \dfrac{dw}{dv},\\ w_{uu} = \dfrac{1}{\omega}\dfrac{dw}{d\omega} = \dfrac{1}{v}\dfrac{dw}{dv},\ w_{uv} = 0,\ w_{vv} = \dfrac{d^2w}{d\omega^2} = \dfrac{d^2w}{dv^2}.\end{array}\right\} \quad (2.2)$$

According to (2.2), equation (1.35) can be written now in the following form:

$$a^2\frac{d^2w}{dv^2} + \left\{a^2\left[1 + \left(\frac{dw}{dv}\right)^2\right] - \left(v + w\frac{dw}{dv}\right)^2\right\}\frac{1}{v}\frac{dw}{dv} = 0. \quad (2.3)$$

If we examine inverse dependence, i.e., to count v by function w, then equation (2.3) he will be written in the form:

$$vv'' = 1 + (v')^2 - \frac{(w + vv')^2}{a^2}. \quad (2.4)$$

(Prime indicates differentiation with respect to w).

Page 27.

That entering in (2.4) the speed of sound a there is the known function S and i (Mollier's diagram): $S = S_2$ = const after bow wave, and v, as function $V^2 = v^2 + w^2$ (for section u = 0) it is determined from Bernoulli's integral (1.8)

$$\frac{v^2 + w^2}{2} + i = i_0. \quad (2.5)$$

communication/connection of the space of hodograph with physical space is realized with the aid of the formulas (1.34), which for section u = 0 take the form

$$\operatorname{tg}\theta = \eta = \frac{y}{z} = -\frac{dw}{dv} = -\frac{1}{v'}, \quad \xi = \frac{x}{z} = 0. \qquad (2.6)$$

From (2.6) it follows that the direction of the beam in plane yO_1z, on which the components of velocity are equal to with respect 0, v, w, coincides with the direction of standard to meridional curve at point A (v, w) (Fig. 6). (We are speaking about the axial section u = 0). For graphoanalytical integration equation (2.4) is more convenient to write in another form. If we introduce the designations

$$R = -\frac{[1+(v')^2]^{3/2}}{v''}, \quad U = \frac{|w+vv'|}{\sqrt{1+(v')^2}}, \quad N = v\sqrt{1+(v')^2},$$

that equation (2.4) it is represented in the form

$$R = -\frac{N}{1-\frac{U^2}{a^2}}. \qquad (2.7)$$

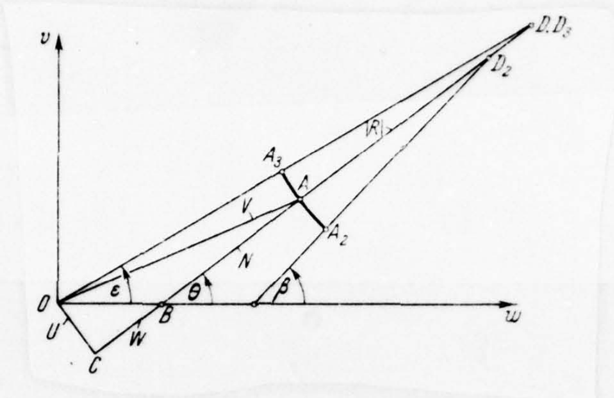


Fig. 6.

Page 28.

Values R, N, U make simple geometric sense (see Fig. 5).

Module/modulus R is, the radius of curvature of meridional curve at point A (v, w), and, if R > 0, then curved it is convex in the direction of increase v, if R < 0, then vice versa; N (with v > 0) there is length of cut AB of standard to curve at point A; U is length of perpendicular OC, omitted from the origin of coordinates to this standard. Let us designate even by W the length of the cut of standard AC; W, U are determined the components of vector of the speed respectively in direction of the team, inclined toward the axis

of cone at an angle θ, and in perpendicular to it direction.

Let us examine now the conditions of the nonseparated flow of cone and condition on bow shock. On the surface of cone tg θ = tg ε = v/w or U = 0, since velocity vector is directed here along the appropriate ray/beam of the surface of cone.

Bow shock in this task to eat the round cone the half-angle of which let us designate through β. Formulas (1.46) can be written in this form:

$$\text{with}\quad \theta = \beta\colon W_1 = W_2,\ \rho_1 U_1 = \rho_2 U_2,$$
$$i_1 + \frac{U_1^2}{2} = i_2 + \frac{U_2^2}{2},\quad p_1 + \rho_1 U_1^2 = p_2 + \rho_2 U_2^2. \quad (2.8)$$

Here $i_1 = i\ (p_1, \rho_1)$, $i_2 = i\ (p_2, \rho_2)$ [or it is possible to utilize dependences $p = p\ (S, i)$, $\rho = \rho\ (S, i)$].

Furthermore, must be fulfilled the kinematic relationship/ratios:

$$\left.\begin{array}{c} U_1 = V_1 \sin\beta, \quad W_1 = V_1 \cos\beta, \\ U_2 = V_2 \sin(\beta - \delta), \quad W_2 = V_2 \cos(\beta - \delta), \\ V^2 = U^2 + W^2, \end{array}\right\} \quad (2.8a)$$

where δ is an angle of rotation of velocity vector after shock wave.

Page 29.

For an ideal gas $i = c_p T = \frac{a^2}{\gamma - 1} = \frac{\gamma}{\gamma - 1} \frac{p}{\rho}$,

$$p = RT\rho,\ \gamma = \frac{c_p}{c_v} = \text{const},\ S = c_v \ln(p\rho^{-\gamma}) + S_0,$$

and relationship/ratio (2.8) it is possible to write (for example, see [19]) in the following form:

$$\frac{U_1}{U_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1) M_1^2 \sin^2\beta}{(\gamma-1) M_1^2 \sin^2\beta + 2}, \quad (2.9)$$

$$\frac{p_2 - p_1}{p_1} = \frac{2\gamma}{\gamma+1} (M_1^2 \sin^2\beta - 1), \quad (2.10)$$

$$\frac{T_2}{T_1} = \frac{a_2^2}{a_1^2} = 1 + \frac{2(\gamma-1)}{(\gamma+1)^2} \frac{M_1^2 \sin^2\beta - 1}{M_1^2 \sin^2\beta} (\gamma M_1^2 \sin^2\beta + 1), \quad (2.11)$$

$$\frac{S_2 - S_1}{R} = \ln\left\{\left[1 + \frac{2\gamma}{\gamma+1}(M_1^2 \sin^2\beta - 1)\right]^{1/\gamma-1} \times \left[\frac{(\gamma+1) M_1^2 \sin^2\beta}{(\gamma-1) M_1^2 \sin^2\beta + 2}\right]^{-\gamma/\gamma-1}\right\} = \ln\frac{p_{01}}{p_{02}}, \quad (2.12)$$

where $M_1 = V_1/a_1$ - the Mach number of undisturbed flow; p_{01}, p_{02} - stagnation pressure do shock wave

$$W_2/W_1 = 1, \quad (2.13)$$

communication/connection between δ and β is given by the formula

$$\operatorname{tg}\delta = 2\operatorname{ctg}\beta \frac{M_1^2 \sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}. \quad (2.14)$$

We will obtain now from formulas (2.8)-(2.14) a series of useful relationship/ratios.

Subsequently it is convenient to relate the components of velocity to the maximum (maximum) speed V_{np}, which corresponds to zero enthalpy (expansion into vacuum) in the equation of Bernoulli (1.8):

$$\frac{V^2}{2} + i = i_0 = \frac{V_{np}^2}{2}. \quad (2.15)$$

From formulas (2.8a) it follows that

$$\frac{W_2}{V_{np}} = \frac{V_1}{V_{np}} \cos\beta. \quad (2.16)$$

Page 30.

For an ideal gas equation (2.15) can be written in the form

$$\frac{1}{M^2} = \frac{\gamma-1}{2}\left[\left(\frac{V_{np}}{V}\right)^2 - 1\right]. \qquad (2.17)$$

From equations (2.8a) and (2.9) follows the formula

$$\frac{U_2}{U_1} = \frac{\dfrac{U_2}{V_{np}}}{\dfrac{V_1}{V_{np}}\sin\beta} = \frac{2}{\gamma+1}\left(\frac{1}{M_1^2\sin^2\beta} + \frac{\gamma-1}{2}\right). \qquad (2.18)$$

Since value M_1 is connected with V_{np}/V_1 equation (2.17), (2.18) it is possible to write in the form

$$\frac{U_2}{V_{np}} = \frac{V_1}{V_{np}}\sin\beta\,\frac{2}{\gamma+1}\left[\left(\frac{V_{np}^2}{V_1^2} - 1\right)\frac{\gamma-1}{2}\,\frac{1}{\sin\beta} + \frac{\gamma-1}{2}\right].$$

After substituting here (2.16), we will obtain

$$\frac{U_2}{V_{np}} = \frac{\gamma-1}{\gamma+1}\,\frac{W_2}{V_{np}}\operatorname{ctg}\beta\left[\left(\frac{V_{np}}{W_2}\right)^2 - 1\right]. \qquad (2.19)$$

In the fixed/recorded sense V_1/V_{np} (i.e. with fixed/recorded value M_1) the components of the vector of velocity after shock V_2 $(0, v_2, w_2)$ satisfy the known equation of the shock polar whose equation can be written (see for example, [20]) in the form

$$\left(\frac{v_2}{V_{np}}\right)^2 = \left(\frac{V_1}{V_{np}} - \frac{w_2}{V_{np}}\right)^2 \frac{\dfrac{w_2}{V_{np}} - \dfrac{\gamma-1}{\gamma+1}\dfrac{V_{np}}{V_1}}{\dfrac{\gamma-1}{\gamma+1}\dfrac{V_{np}}{V_1} + \dfrac{2}{\gamma+1}\dfrac{V_1}{V_{np}} - \dfrac{w_2}{V_{np}}}. \qquad (2.20)$$

In view of the fact that the exact solution of the task of cone does

not succeed in obtaining in the locked form, this problem is solved numerically, moreover in are possible two methods. In the first method, suitable only for an ideal gas, they are assigned by the half-angle of cone ε and by value V_3/V_{np}, where V_3 - speed on the surface of cone.

Page 31.

According to these data construct point A_3 (see Fig. 6) on the diagram of hodograph (where all speeds they are related to V_{np}). Equation (2.15) for an ideal gas can be written in the form

$$\left(\frac{a}{V_{np}}\right)^2=\frac{\gamma-1}{2}\left[1-\left(\frac{V}{V_{np}}\right)^2\right]. \tag{2.21}$$

In terms of value V_3/V_{np} from (2.21) they find that $(a_3/V_{np})^2$. Furthermore, for point A_3 are valid the equalities: $U_3/V_{np}=0$ (condition of continous flow about cone) and $\frac{N_3}{V_{np}}=\frac{V_3}{V_{np}}$. According to data from equalation (2.7), written in the form

$$\frac{R}{V_{np}}=-\frac{\frac{N}{V_{np}}}{1-\left(\frac{U}{V_{np}}\right)^2\cdot\left(\frac{V_{np}}{a}\right)^2}, \tag{2.22}$$

is determined that R_3/V_{np} and during the continuation of cut OA_3 construct center of curvature (point D_3) by meridional curve for a point A_3 ($A_3D_3=|R_3|/V_{np}$).

The end point A (Fig. 6) of small circular arc with a center at point D_3 and the radius, equal to $|R_3|/V_{np}$, determines approximately

point in meridional curve. After constructing at point A standard to meridional curve (standard to the appropriate circumference), find the lengths of cuts AB, AC CC, OA (Fig. 6), which determine $\frac{N}{V_{np}} \cdot \frac{W}{V_{np}}$, $\frac{U}{V_{np}}$, $\frac{V}{V_{np}}$, and further with the aid of equations (2.21), (2.22) is determined that R/V_{np} at point A is constructed center of curvature - point D. The end/lead of the small circular arc with center into D and by the radius, equal to $|R|/V_{np}$, will determine the following point of meridional curve, and so forth. Calculations are carried out to as long as will not be satisfied condition (2.19), that is fulfilled on bow shock when $\theta = \beta$ (point A_2 in Fig. 6).

Page 32.

The practical method of determination β consists in the fact that for a series of values θ they construct $\frac{U}{V_{np}}$ according to the diagram of hodograph and $\frac{U^0}{V_{np}}$, determined by the formula

$$\frac{U^0}{V_{np}} = \frac{\gamma-1}{\gamma+1} \frac{W}{V_{np}} \operatorname{ctg} \theta \left[\left(\frac{V_{np}}{W}\right)^2 - 1\right],$$

where right side is located also through values, removed from the diagram of hodograph. The point of intersection of these two curves will determine β. In terms of known values β and $\frac{W_2}{V_{np}}$ from formula (2.16) is determined that $\frac{V_1}{V_{np}}$ also, from equation (2.17) - the Mach number of undisturbed flow. From formulas (2.9), (2.10) then are determined ρ_2/ρ_1, p_2/p_1, after which the pressure and density at each

point of flow after bow shock they can be found from the conditions of the isentropicity of the flow

$$\left(\frac{p}{p_1}\right)\left(\frac{\rho}{\rho_1}\right)^{-\gamma}=\left(\frac{p_2}{p_1}\right)\left(\frac{\rho_2}{\rho_1}\right)^{-\gamma} \qquad (2.23)$$

and of Bernoulli's integral (2.15), written in the form

$$\left(\frac{V}{V_{np}}\right)^2+\frac{2}{\gamma-1}\left(\frac{V_1}{V_{np}}\right)^2\frac{1}{M_1^2}\frac{p}{p_1}\frac{\rho_1}{\rho}=1. \qquad (2.24)$$

(Let us note that the described method of the construction of meridional curve can be carried out, also, without graphic constructions, but only it is numerical) (see [17].).

In the second method, suitable both for ideal and for the inadequate gas, is assigned the angle β - the semiangle of bow wave and the parameters of undisturbed flow unknown is the half-angle of the streamlined cone ε.

Let us examine first the case of ideal gas. On that which was assigned M_1 from formula (2.17) it is determined that $\frac{V_1}{V_{np}}$; then by formulas (2.8a) $-\frac{U_1}{V_{np}}$, $\frac{W_1}{V_{np}}$ and further by formulas (2.9), (2.13), (2.14)- $\frac{U_2}{V_{np}}$, $\frac{W_2}{V_{np}}$, tg δ. In terms of known values $\frac{V_2}{V_{np}}=\sqrt{\left(\frac{U_2}{V_{np}}\right)^2+\left(\frac{W_2}{V_{np}}\right)^2}$ and δ is constructed point A_2 on the diagram of hodograph (Fig. 6).

Page 33.

The direction of standard to meridional curve is assigned by angle β.

The further course of calculations in no way differs from calculations for the first method. The end point A_3, which corresponds to the surface of cone, is determined by condition $U_3 = 0$. (Standard to meridional curve at point A_3 passes through the origin - point O; Fig. 6.

The demonstrative representation of the results of integration can be obtained with the aid of the shock polar. (See [11]). The circuit of this construction is given in Fig. 7.

With the assigned Mach number of the incident flow M_1, the terminuses of the velocity vector after shock wave lie/rest on the shock polar (curved $GA''_2A'_2A_2H$ in Fig. 7), determined by equation (2-20), [M_1 and V_1/V_{np} are connected by equation (2-17)]. On the shock polar is taken a series of points A_2, A'_2, A''_2, ..., that correspond to the different angles β of the slope/inclination of bow shock; for each of them, is constructed the meridional curve A_2A_3, $A'_2A'_3$..... The end/leads of these curves, point A_3, A'_3, A''_3, ..., that correspond to the different half-angles of cone ε, are formed certain curve, named by A. Busemann "apple-shaped." The grid of "apple-shaped" curves for different Mach numbers M_1 is constructed in [11].

Let us examine now the case of the inadequate gas.

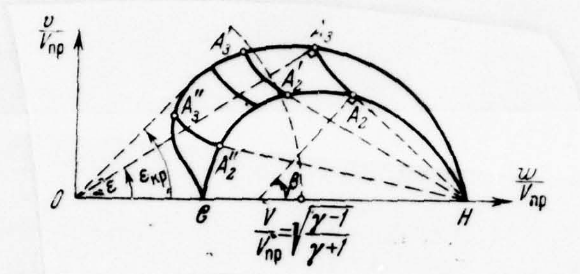


Fig. 7.

Page 34.

In conical flow theory basic interest are of the deviations of the properties of gas from the properties of ideal gas at the high temperatures, when are excited oscillatory degree of the freedom of molecules, occurs their dissociation and ionization, are formed new chemical compounds. (So for example, in the air, which represents in the base compound of nitrogen, oxygen and argon, besides the reactions of dissociation and ionization, occurs the formation of oxide of nitrogen and so forth). If relaxation times for these reactions are short in comparison with characteristic time of specific gas-dynamic problem, then it is possible to consider gas in local thermodynamic equilibrium. Another limiting case is the case of the so-called "frozen" thermodynamic equilibrium when the relaxation times of some reactions are great in comparison with characteristic time of task.

In both cases there are equations of state of gas and are possible the mode/conditions of conical flow about the bodies, limited by conical surfaces. For the intermediate case when the relaxation times of the reactions, which occur in gas, are comparable with characteristic time of task, flow differs from the conical, and parameter determination of gas is conducted taking into account the equations of chemical kinetics (see, for example [22], [23]). Flow is not conical, also, in the case, when in energy balance the significant role plays radiation from the heated gas and the streamlined body.

For the theoretical determination of the properties of gas in the case of thermodynamic equilibrium (or the "frozen" equilibrium) are utilized the methods of statistical physics in its quantum-mechanical form along with the data of spectroscopy. The results of the corresponding calculations for air at the values of the parameters of gas, which are of interest in gas dynamics, can be found in works [24, 25]; see also [26]. In these works the equation of state of gas in the form of the different dependences between the parameters of gas is represented in the form of tables and diagrams. For calculations by ETsVM either according to tables are determined the approximating expressions, which relate the parameters of gas (for example, see [27, 28]), or the equations of chemical kinetics for the case of thermodynamic equilibrium they are connected to the

equations of motion of gas (for example, see [29]).

Page 35.

The problem of parameter determination of gas in the range between the bow shock and the surface of cone, and also the angle of partial opening of cone ε from the assigned parameters of undisturbed flow, in the case of the inadequate gas is solved into two stages. During the first stage are determined the parameters of gas directly after bow shock.

Since relationship/ratios (2.8) in this case cannot be presented in the form of simple formulas of the type (2.9)-(2.12), determination of the parameters of gas after shock wave is conducted with the aid of iterative process or with the aid of the construction of auxiliary curve/graph and interpolation. Following [30], let us examine, for example, the order of calculations in the latter case when are given V_1, T_1, p_1 and β. First are determined the missing parameters of undisturbed flow; from diagram i - p, where are plotted/applied to line constant T and ρ, they are determined ρ_1 and i_1, then from diagram i - p - a it is determined a_1 ($M_1 = V_1/a_1$) and from diagram i - S from the known p_1 and i_1 is determined S_1. Further relationship/ratios (2.8) and (2.8a) are represented in the form

$$U_2 = V_1 \cos\beta \operatorname{tg}(\beta - \delta), \quad (2.25)$$

$$i_2 = i_1 + \frac{U_1^2 - U_2^2}{2}, \quad (2.26)$$

$$p_2 = p_1 + U_1\rho_1 (U_2 - U_1), \quad (2.27)$$

$$\rho_2 = \rho_1 \frac{U_1}{U_2}. \quad (2.28)$$

Being given a series of values of the angle of deflection of velocity vector after shock wave δ, according to formulas (2.25)-(2.28) is determined a series of values $U_2^\times, i_2^\times, p_2^\times, \rho_2^\times$ and construct curve/graph $\rho_2^\times = \rho_2^\times(\delta)$. Then in terms of the obtained values $i_2^\times, p_2^\times$ according to diagram i - p is determined the function of ρ'_2 = of $\rho'_2(\delta)$ and construct its curve/graph. The point of intersection of these curves gives to us δ and ρ_2. Then in terms of value δ from formula (2.25) it is located U_2, and from formulas (2.26), (2.27) - with respect to i_2, p_2. Temperature T_2 is determined from diagram i - p from the known values of i_2, p_2, the speed of sound a_2 - according to diagram i - p - a, entropy S_2, with the aid of p_2, T_2, according to diagram p - T.

Page 36.

(According to diagram i - S it is possible to determine the parameters of the isentropically stagnation gas by value S_2 and $i_0 = i_1 + (V^2_1/2)$). In terms of the obtained values δ and $V_2 = \sqrt{U^2_2 + W^2_2}$ (where $W_2 = V_1 \cos\beta$) is constructed point A_2, (Fig. 6); standard to

meridional curve composes angle β with axis w. The second stage of calculations differs from the case of ideal gas only in the facts that the speed of sound a in equation (2.7) is determined according to diagram a – i, where are plotted/applied to the line of constant entropy, in terms of the known value S_2 and value i, which for each ray/beam is determined from **the equaticn of Bernoulli:**

$$i = i_0 - \frac{V^2}{2}.$$

After the construction **of the diagram of hodograph** p, ρ, T in field of flow, are determined from S – i- diagram from the known values S_2 and i.

2.3. Methods, based on use of spherical and cylindrical coordinates. Following G. Taylor and D. Makkol [17], let us examine the solution of the problem of cone in spherical coordinates. If we combine the spherical coordiante axis with axis of the symmetry of cone (and by direction of undisturbed flow), then the parameters of gas will depend on the strength of symmetry only on angle θ (see Fig. 5) and $w \equiv 0$ (see Fig. 1). Furthermore, in flow $S = S_2$ = const; therefore equation (1.15), (1.12) **they take the form**

$$(a^2 - v^2)\frac{dv}{d\theta} = u(v^2 - 2a^2) - a^2 v \operatorname{ctg}\theta, \qquad (2.29)$$

$$\frac{du}{d\theta} = v, \qquad (2.30)$$

a equations **(1.16), (1.15) are satisfied identically.**

In the case of ideal gas, the speed of sound a is determined

directly from the equation of Bernoulli (1.8), written in the form (2.21):

$$\left(\frac{a}{V_{\text{пр}}}\right)^2 = \frac{\gamma-1}{2}\left[1-\left(\frac{u}{V_{\text{пр}}}\right)^2-\left(\frac{v}{V_{\text{пр}}}\right)^2\right].$$

For the inadequate gas a, it is determined from diagram i - S from values S_2 and i, moreover i is located from (1.8), if is known value $V^2 = u^2 + v^2$.

Page 37.

Boundary conditions on bow shock are assigned by formulas (2.8)-(2.14), where it is necessary only to consider that U = -v, W = u. The condition of the nonseparated flow of cone is v = 0 when $\theta = \varepsilon$.

In the case of ideal gas the solution of problem can be begun, being given values ε and $\frac{u}{V_{\text{пр}}}$ when $\theta = \varepsilon$ and then to define β and M_1, both the were done G. Teplor and D. Makkol [17], or, being given values β and the parameters of undisturbed flow, to find ε. For the inadequate gas there is only second possibility. During the integration of system of equations (2.29), (2.30) it is possible to utilize finite-difference methods, without turning to the diagram of hodograph [17], or to utilize it for graphoanalytical solution [20].

The task of cone was solved also by the "method of

establishment" [29] which will be examined later in connection with the solution of the problem of round cone at an angle of attack, and also the method of integral relationship/ratios [32].

2.4. Transonic flow about cone. The examined methods make it possible in principle to obtain with the assigned accuracy the solution of the problem of cone for any value $M_1 > 1$ (on the condition that flow conical). But if the Mach number of undisturbed flow M_1 is close to unity, then with small θ the components of velocity sharply change and the application/use of the described methods becomes difficult. With $M_1 - 1 \ll 1$ conical flow is possible only at the sufficiently small half-angles of cone ε.

For the overcoming of these difficulties, K. Oswatitsch and L. S"yedin [33] introduced the new unknown functions and independent variables.

If we use Cartesian coordinates (see Fig. 5) and to introduce the designations

$$\left.\begin{aligned} \operatorname{ctg}\alpha_1 = \sqrt{M_1^2 - 1},\quad y^{\times} = y \operatorname{ctg}\alpha_1, \\ w^{\times} = \frac{w - w_1}{w_1 - a_*},\quad v^{\times} = \frac{v}{w_1 - a_*}\cdot \operatorname{tg}\alpha_1, \end{aligned}\right\} \quad (2.31)$$

where a_* - the critical speed of the sound,

$$\left(a_*^2 = \frac{\gamma - 1}{\gamma + 1} V_{np}^2\right), \quad w_1 = V_1.$$

that the equation of the axisymmetric irrotational flow of gas in transonic approach/approximation **will be written in the form:**

$$(1+w^{\times})\frac{\partial w^{\times}}{\partial z}+\frac{1}{y^{\times}}\frac{\partial (y^{\times}v^{\times})}{\partial y^{\times}}=0, \qquad (2.32)$$

$$\frac{\partial w^{\times}}{\partial y^{\times}}-\frac{\partial v^{\times}}{\partial z}=0. \qquad (2.33)$$

Page 38.

For conical flow $v^{\times}, w^{\times}$ **they depend only on**

$$\eta^{\times}=\frac{y^{\times}}{z}=\frac{y}{z}\operatorname{ctg}\alpha_1=\eta\operatorname{ctg}\alpha_1, \qquad (2.34)$$

and equations (2.32), **(2.33) accept this form:**

$$(\eta^{\times})^2(1+w^{\times})\frac{dw^{\times}}{d\eta^{\times}}+\frac{d(v^{\times}\eta^{\times})}{d\eta^{\times}}=0, \qquad (2.35)$$

$$\frac{dw^{\times}}{d\eta^{\times}}+\frac{d(v^{\times}\eta^{\times})}{d\eta^{\times}}-\frac{v^{\times}\eta^{\times}}{\eta^{\times}}=0. \qquad (2.36)$$

From equation (2.35) it follows that if $w^{\times}$ when $\eta^{\times}\to 0$ does not have the "powerful" special feature/peculiarity, **then** $v^{\times}\eta^{\times}\to$ const **fast enough, when** $\eta^{\times}\to 0$. Let us designate by $\Psi(\eta^{\times})$ the product $v^{\times}\eta^{\times}$, **then** $\Psi(\eta^{\times})\to\Psi(0)$, (if $\eta^{\times}\to 0$,) **and from (2.36) it follows that**

$$w^{\times}=\Psi(0)\ln\eta^{\times}+\ldots \qquad (2.37)$$

Let us make now **a replacement of the unknown functions by the** formulas

$$\left.\begin{aligned}\Psi(\eta^{\times})&=v^{\times}(\eta^{\times})\eta^{\times},\\ \chi(\eta^{\times})&=w^{\times}(\eta^{\times})-\Psi(\eta^{\times})\ln\eta^{\times}.\end{aligned}\right\} \qquad (2.38)$$

[**Functions** $\Psi(\eta^{\times}), \chi(\eta^{\times})$ are limited when $\eta^{\times}\to 0$].

Equations (2.35), **(2.36) are converted then to the** form

$$\frac{d\Psi}{d\eta^{\times}}=\Psi\frac{\eta^{\times}(1+\chi+\Psi\ln\eta^{\times})}{(\eta^{\times})^2(1+\chi+\Psi\ln\eta^{\times})-1}, \qquad (2.39)$$

$$\frac{d\chi}{d\eta^{\times}}=-\frac{d\Psi}{d\eta^{\times}}(1+\ln\eta^{\times}). \qquad (2.40)$$

Page 39.

In transonic approach/approximation the equation of the shock polar and the continuity condition the tangential to the surface of jump component of vector of speed they are record/written in the form

$$(v_2^\times)^2 = (w_2^\times)^2\left(1 + \frac{1}{2} w_2^\times\right),$$

$$\eta_2^\times = \operatorname{ctg}\alpha_1 \operatorname{tg}\beta = -w_2^\times/v_2^\times = \left(1 + \frac{1}{2} w_2^\times\right)^{-1/2}$$

(index 2 below about the **sign of function means that is** taken its value immediately after shock wave). Hence, after transition to variables Ψ and χ from formulas (2.38) and some transformations, we will obtain finally following conditions on bow shock:

$$\eta_2^\times = \left(1 - \frac{1}{2}\Psi_2\right)^{-1/2}, \quad \chi_2 = -\Psi_2(1 + \ln\eta_2^\times). \quad (2.41)$$

Being given now **certain value** Ψ_2 according to formulas (2.41) are determined the appropriate values $\eta_2^\times, \chi_2$ and they produce with finite-difference methods the integration of equations (2.39), (2.40) on cut from $\eta_2^\times$ to zero. The half-angle of cone ε is determined as follows. On the surface of the cone

$$\frac{v}{w} = \frac{y}{z} = \operatorname{tg}\varepsilon \text{ and } \eta^\times = \eta_3^\times = \operatorname{tg}\varepsilon \cdot \operatorname{ctg}\alpha_1 = \operatorname{tg}\varepsilon \cdot \sqrt{M_1^2 - 1}.$$

From equations (2.38) and (2.31) we obtain

$$\Psi(0) \approx \Psi(\mathrm{tg}\,\varepsilon\,\mathrm{ctg}\,\alpha_1) = \left(\eta^{\varkappa}\frac{v}{w}\,\mathrm{tg}\,\alpha_1\frac{w}{w_1 - a_*}\right)_{\eta^{\varkappa} = \eta_3^{\varkappa}} = = \mathrm{tg}^2\,\varepsilon \cdot \frac{M_1^*}{M_1^* - 1}\left(1 - \frac{w_1 - w}{w_1}\right)_{\eta^{\varkappa} = \eta_3^{\varkappa}}, \qquad (2.42)$$

where $M_1^* = \frac{w_1}{a_*}$ there is the given speed. (Dependence M_1^* on M_1 is located from Bernoulli's integral; it takes the form

$$\frac{1 - M_1^{*2}}{M_1^{*2}} = \frac{2}{\gamma + 1} \cdot \frac{1 - M_1^2}{M_1^2}.)$$

For slender cones $(w_1 - w)/w_1$, there is a value small.

Page 40.

Disregarding this sense in comparison with unity, we will obtain from (2.42) equation for $\mathrm{tg}\,\varepsilon$:

$$\Psi(0) \approx \mathrm{tg}^2\,\varepsilon\frac{M_1^*}{M_1^* - 1}. \qquad (2.43)$$

More precise value $\mathrm{tg}\,\varepsilon$ can be obtained from equation (2.42) by means of iterations.

From formulas (2.31), (2.38) it follows that

$$\left.\begin{aligned} \frac{w - w_1}{w_1} &= \frac{M_1^* - 1}{M_1^*}(\chi + \Psi \ln \eta^{\varkappa}), \\ \frac{v}{w_1} &= \frac{M_1^* - 1}{M_1^*}\,\mathrm{ctg}\,\alpha_1\frac{1}{\eta^{\varkappa}}\Psi. \end{aligned}\right\} \qquad (2.44)$$

On the surface of cone when $y/z = \mathrm{tg}\,\varepsilon$

$$\left.\begin{aligned} \frac{w_3 - w_1}{w_1} &\approx \frac{M_1^* - 1}{M_1^*}\,[\chi(0) + \Psi(0)\ln(\mathrm{tg}\,\varepsilon\,\mathrm{ctg}\,\alpha_1)], \\ \frac{v_3}{w_1} &\approx \frac{v_3}{w_3} = \mathrm{tg}\,\varepsilon. \end{aligned}\right\} \qquad (2.45)$$

Pressure coefficient in flow $C_p = \frac{p - p_1}{\rho_1 w_1^2/2}$ is determined from the formula

$$C_p = -2\frac{w - w_1}{w_1} - \left(\frac{v}{w_1}\right)^2. \tag{2.46}$$

Since bow shock must be arrange/located upstream from the Mach cone of the undisturbed flow to which corresponds, according to (2.34), the value $\eta^\times = 1$, or $1 < \eta_2^\wedge < \infty$, whence, with the aid of formula (2.41), we find that $0 < \Psi_2 < 2$. The authors [33] were given values Ψ_2 = 0.10; 0.20; 0.40; 0.60; 0.80; 1.00; 1.20; 1.40. (High values Ψ_2 they are of practical use, since maximum value $\Psi(0)$ and, consequently, also ε they are reached at the value Ψ_2, which satisfies inequality $1{,}00 < \Psi_2 < 1{,}20$. (See on this question also the following point/item). Integration of equations (2.39), (2.40) for those intervals where χ and Ψ they changed rapidly, it was conducted by Runge - Kutta's method, in the remaining cases was utilized the simplest method of averages value.

Page 41.

The results of calculations are given in appendix at the end of the book. The greatest value of angle ε which corresponds to the assigned parameters of undisturbed flow, will designate ε_{kp}; it is determined from relationship/ratio (2.43) and the results of calculations from the formula

$$\operatorname{tg} \varepsilon_{\text{кр}} = \sqrt{\frac{M_1^* - 1}{M_1^*} \Psi(0)_{\max}} = 0{,}86 \sqrt{1 - \frac{1}{M_1^*}}. \quad (2.47)$$

Since with $M_1 \longrightarrow 1$

$$1 - \frac{1}{M_1^*} = \frac{1}{\gamma+1}(M_1^2 - 1) + \ldots = \frac{1}{\gamma+1} \operatorname{tg}^2 (90^\circ - \alpha_1) + \ldots,$$

of (2.47) after **simple calculations and the replacement of the** tangent of small angle **by the value of angle itself** finally we will obtain

$$\varepsilon_{\text{кр}}^\circ = \frac{0{,}86}{\sqrt{\gamma+1}} (90^\circ - \alpha_1^\circ). \quad (2.48)$$

[When $\gamma = 1{,}400$ $\varepsilon_{\text{кр}}^\circ = 0{,}55\ (90^\circ - \alpha_1)$.] **The presented theory** gives good results up to M_1 = 1.2.

2.5. Analysis of results of flow-field analyses of cones and results of experiments. The basic characteristic features of the flow about the cones with the moderate Mach numbers M_1 of undisturbed flow were establish/installed in the works of G. Taylor and D. Makkol [17, 18].

The velocity vector of the particle of gas after bow shock is turned with respect to direction of undisturbed flow to angle δ less than the half-angle of cone ε; pressure grow/rises to certain value $p_2 > p_1$.

During further isentropic particle motion of gas asymptotically approaches a surface of cone; the angle between its velocity vector and the surface of cone, but also velocity modulus continuously they decrease, and pressure grow/rises, approaching pressure on the surface of cone $p_3 > p_2$. Location and the form of flow lines for the case $M_1 = 1.65$; $\varepsilon = 20°$ are shown in Fig. 8 [18]. Broken line here designated waves of Mach (characteristic), which go from the surface of cone.

Page 42.

With assigned M_1 and ε_1 just as in the case of wedge, are possible two solutions of problem. (First solution is characterized by from the second smaller values β and p_3). This fact one can see well in Fig. 7, where ray/beam OA_3 intersects "apple-like curve" at two points. As a rule, cone is the part of the streamlined body. If the remaining parts of the tested body are not the source of a powerful pressure increase, for example, the cone it follows the cylindrical part, which has the same diameter, as the basis/base of cone, then is realized in actuality the solution with smaller values β and p_3. But if, for example, the cone follows the cylinder, which has larger diameter than the basis/base of cone, then bow shock can be intense, and flow after it will be subsonic. The front of this jump, as a rule, is bent and flow about cone will not be conical, although formally there is solution, which describes subsonic conical flow. (this solution is realized locally only near the apex of the cone).

Since after bow shock particle speed and, consequently, also

local Mach number $M = V/a$ continuously decrease with approach/approximation to the surface of cone, smallest value M in flow is reached on the surface of cone, and at a specific ratio between M_1 and ε Mach number on the surface of cone $M_3 = V_3/a_3$ becomes equal to one. (in this case $\frac{V_3}{V_{np}} = \sqrt{\frac{\gamma-1}{\gamma+1}} \approx 0{,}41$ for $\gamma = 1.4$). If we now somewhat decrease M_1 (or increase ε), that on certain ray/beam $\theta = \theta^*$, where $\varepsilon < \theta^* < \beta$, we will obtain $M = 1$ and when $\varepsilon < \theta < \theta^*$ the flow becomes subsonic: here $M < 1$.

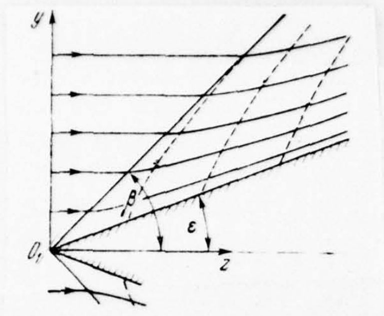


Fig. 8.

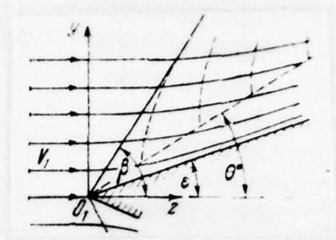


Fig. 9.

Page 43.

This position corresponds to the curve $A'_3A'_2$ in Fig. 7. Let us note that during the determination of the solution in this case no difficulties appear, since the denominator of the relation, which stands in the right side of the equation (2.7), grow/rises during motion from bow wave to the surface of cone.

Figures 9 depicts by dashed lines the ray/beam $\theta = \theta^*$ and the Mach lines, which go from this ray/beam, for the case

$$\varepsilon = 20°,\ M_1 = 1{,}31\ \left(\frac{V_s}{V_{np}} = 0{,}40;\ \cancel{\text{см}}\ \text{see}\ [17]\right).$$

During further decrease of M_1 (or an increase ε) subsonic region rapidly increases and occupies entire zone of flow $\varepsilon < \theta < \beta$.

If with certain M_1 and ε. on the surface of cone M = 1, then, strictly speaking, with smaller M_1 (or large ε) the solution of problem in the form of conical flow becomes unsuitable.

However, as show experiments [34], [35], conical flow is realized in these cases locally, and of the vicinity of the apexes of the cone, and a conical flow theory can be utilized, for example, for the approximate determination of the greatest angle $\varepsilon_{\text{кр}}$ for the datum M_1, by which the bow wave will move away from the apex of the cone. Figures 10a-c gives the results of experiments [34] for the case of the combination of cylinder with the cone for which $\varepsilon = 25°$ with M_1 = 1.401; 1.273; 1.229. (On the surface of cone M = 1 with M_1 = 1.5).

The authors [34] note that the measured values of pressure near apex/vertexes coincided with the values, calculated according to conical flow theory; bow wave will move away from body at somewhat large values ε, how this it is predicted according to conical flow theory. In work [35] it was established that the departure/withdrawal of bow wave, for example, with M_1 = 2.45, occurs when $\varepsilon_{\text{кр}} = 46°$, and the calculated value $\varepsilon_{\text{кр}} = 45°40'$.

Page 44.

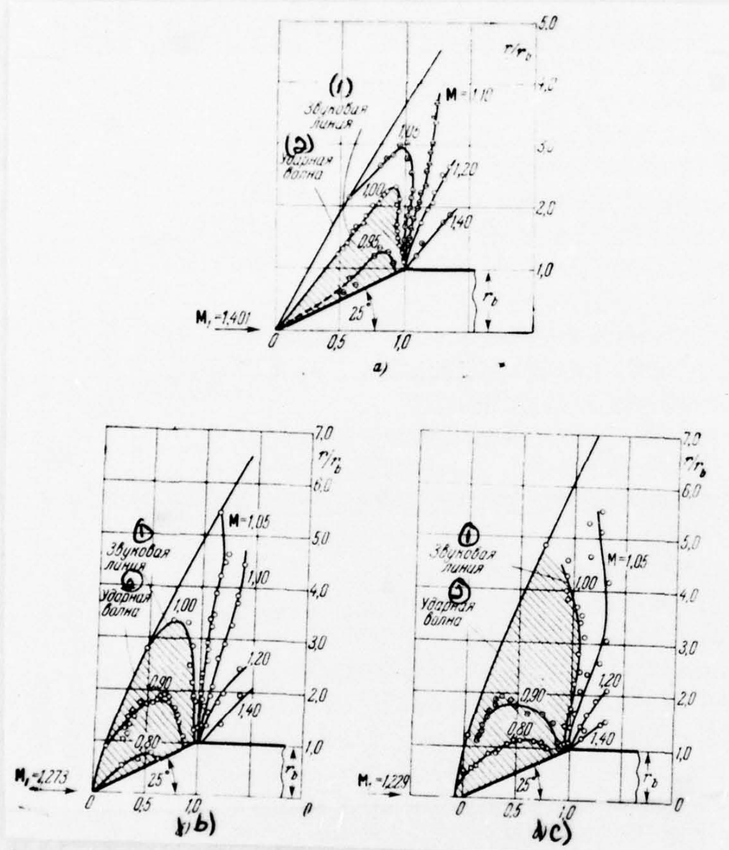


Fig. 10.

Key: (1). Sonic line. (2). Shock wave.

Page 45.

Determination $\varepsilon_{кр}$ - the greatest angle ε with which is possible the

solution in the class of conical flows with assigned M_1, can be produced on "apple-like" curve (see Fig. 7) or by calculations. For the first time such calculations were made in work [18], in particular, was establish/installed that the values $\varepsilon_{кр}$ with any M_1 for ideal gas, γ = 1.405, do not exceed value 57.6°.

Figures 11 gives the theoretical boundaries of mode/conditions during the flow of ideal gas about the cones [30] (with γ = 1.4).

Since the angle of deflection of velocity vector after jump δ of less half-angle ε, then $\varepsilon_{кр}$ for a cone is obtained by substantially greater than for a wedge with assigned M_1; see [17], [20].

In Fig. 12, undertaken from work [35], is well observable the process of the establishment of the mode/conditions of conical flow, in the case when M_1 = 2.45. Thus far ε is greater than value 42°30', with which, according to theory, on the surface of cone M = 1, the pressure is changed along the chord of model. During decrease ε the pressure is equalized and when $\varepsilon = 42°30'$ becomes constant, which indicates the emergence of conical flow. By small circles are noted values C_p, determined by conical flow theory.

When between the bow shock and the surface of cone the flow the supersonic, between theory and experiment is observed exclusively

good agreement. Once in the work of G. Taylor and D. Makkol, [17] it was established that the theory predicts well the half-angle of bow wave β with assigned M_1 and ε and distribution pressure p on the surface of cone (although p was determined on the models, which did not exceed 0.74 inch).

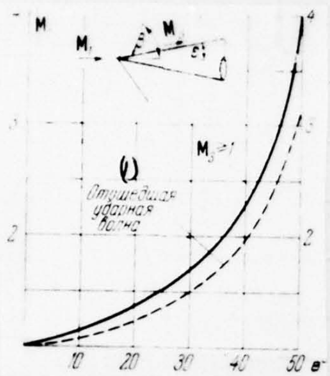


Fig. 11.

Key: (1). Detached shock wave.

Page 46.

Then D. Makkol [18] refined the calculations [17] and were obtained the shadow photographs of projectiles with cone head (diameter of which was more than 5 cm) in free-air conditions, on which with high accuracy it was possible to determine β and M on the surface of head cone. (On the surface of cone were made the small roughness, which made it possible to see the Mach lines, exiting/waste from the surface of cone, and to measure the appropriate mach angle). Theory and experiment render/showed in very good agreement. In Fig. 13, undertaken from work [18], dark small

circles (•) designated the points, obtained from photographs, light (◦) - the points, obtained from calculations.

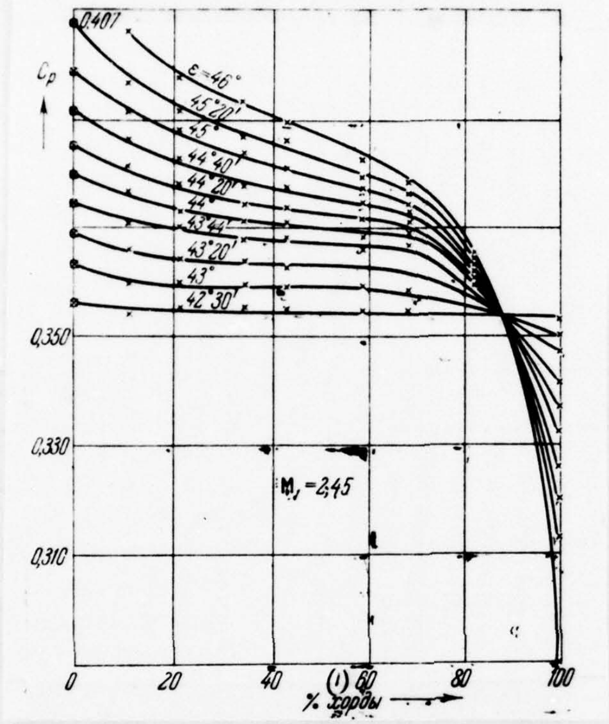


Fig. 12.

Key: (1). o/o of chord.

Page 47.

During the determination of Mach number M on the surface of cone especially clearly it was possible to trace the disappearance of the Mach lines, going from the surface of cone at the values M_1 smaller

than that value at which according to theory $M = 1$ on the surface of cone.

With the large Mach numbers M_1 of undisturbed flow $(M_1 \sin \varepsilon > 4$; see [36]), when become essential the effects of the excitation of the vibrational degrees of freedom of the molecules of gas (air), their dissociation and ionization, the calculations of equilibrium flows about cones were conducted independently by a series of the authors [25, 30, 37, 38], etc.

These calculations showed that if after bow shock occurs the dissociation (and ionization) of the molecules of gas, then, as compared with the case of ideal gas with $\gamma = 1.4$, the shock layer it is arrange/located considerably nearer to the surface of cone and difference $\beta - \varepsilon$ composes a total of several degrees; pressure on the surface of cone barely depends on the presence of dissociation; the temperature on the surface of cone in the dissociating gas is considerably less, but density is greater than in ideal gas; in the region between the shock wave and the surface of cone, the module/modulus of full speed, Mach number M, the speed of sound are virtually constant. Substantially grow/rises also value $\varepsilon_{\text{кр}}$, with which it occurs the departure/withdrawal of bow wave from the apex of the cone. For example [37], during the motion of cone in standard atmosphere at height/altitude 100,000 feet, where $T_1 = 218°K$, and $p_1 =$

0.01 atm. with a velocity of V_1, larger than 26,000 feet/s, $2\varepsilon_{кр} \leqslant 135°$, while in a perfect gas at $\gamma = 1.405$ $2\varepsilon_{кр} < 115.2°$ with any M_1 [8].

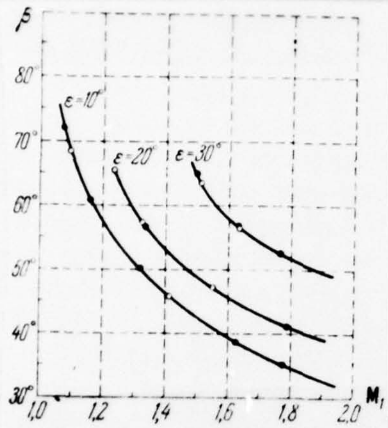


Fig. 13.

Page 48.

In following table 1, are given the results of the flow constructions of air about cones by approximation method [36] for the cases $\varepsilon = 30°$, M_1 **= 7.860 and 16.62;** T_1 **= 218°;** p_1 **= 0.01 atm. (which corresponds to flight in standard atmosphere at height/altitude 100,000 feet) and for a comparison are given the corresponding values, calculated according to the same method for an ideal gas with** γ **= 1.405.**

In table 1 indices 1, 2, 3 designated the values, undertaken in undisturbed flow, immediately after bow wave, and on the surface of

cone corresponding m is effective molecular weight of air.

From table is evident that the difference in results for the inadequate and ideal gases is obtained because of the excitation of the vibrational degrees of freedom of the molecules of air (when M_1 = 16.62 degree of dissociation of the molecules of oxygen of approximately 1.5o/o, but nitrogen - virtually zero). However, already under these conditions difference $\beta - \varepsilon$ is 3-4°, the parameters of gas after jump are changed little. Temperature on the surface of cone is substantially lower than in ideal gas. With an increase in the values $M_1 \sin \varepsilon$ these phenomena become expressed more sharply.

Table 1.

M_1	7,860			16,62		
(1) Величины	(2) Совершенный газ	(3) Несовершенный газ	(4) Относит. ошибка, %%	(2) Совершенный газ	(3) Несовершенный газ	(4) Относит. ошибка, %%
T_2/T_1	4,750	4,481	+6	17,36	12,91	+34
T_3/T_2	1,018	1,015	+0,5	1,015	1,005	+1
T_3/T_1	4,837	4,548	+3,5	17,63	12,98	+36
p_2/p_1	22,48	22,42	+0,5	97,34	97,32	+0,5
p_3/p_2	1,065	1,060	+0,5	1,054	1,030	+2,5
p_3/p_1	23,94	23,76	+1	102,2	100,2	+2,5
ρ_2/ρ_1	4,733	5,082	−5,5	5,606	7,435	−25
ρ_3/ρ_2	1,046	1,045	+0,5	1,038	1,025	+1
ρ_3/ρ_1	4,949	5,224	−5,5	5,821	7,617	−24
β	34,07°	33,80°	+1	33,35°	32,45°	+3
M_2	3,017	3,185	−5	3,352	4,200	−20
M_3	2.975	3.149	−5,5	3,315	4,182	−21
T_2(°K)	1036	977	+6,1	3785	2815	+33,8
T_3(°K)	1054	991	+6,6	3843	2829	+35,6
m_2	28,97	28,97	0	28,97	28,58	+1,5

Key: (1). Values. (2). Ideal gas. (3). Inadequate gas. (4). Rel. error.

Page 49.

Thus, for instance, according to the data [25], in the case $\varepsilon = 50°$ M_1 = 26.84 (V_1 = 8 km/s), p_1 = 0.000070 atm., T_1 = 220°K, for air β = 52.30°, T_3 = 5720°K, while for an ideal gas with γ = 1.4; $\beta \approx 58°$ and $T_3 \approx 24500°K$.

Figure 14, undertaken work [32], gives the dependence of the angular thickness of the compressed layer $\beta - \varepsilon$ on the Mach number M_1 of the undisturbed flow of air during the equilibrium flow about

the cone in the case p_1 = 0.001 atm., T_1 = 273°K. Here small circles designated the results of precise calculation, by solid line - the results of calculations by the method of integral relationship/ratios with one band.

Figure 15, undertaken work [37], gives the dependence of pressure coefficient C_p on the surface of cone during the equilibrium flow of air about it in cases $V_1 \times 10^{-3}$ = 10, 15, 26 feet/s, p_1 = 0.01 atm., T_1 = 218° (which corresponds to height/altitude 100000 feet in standard atmosphere), and also are given results for an ideal gas with γ = 1.4, $M_1 = \infty$, and the results, obtained by Newton's formula $C_p = 2\sin^2\varepsilon$. (Let us note that Newton's formula gives values C_p, which less by 3-6% of those values which are obtained for the dissociating gas). More complete information of the relatively equilibrium flow of dissociating air about the cones can be found in work [25]. The comparison of the parameters of gas in equilibrium and frozen flows about cone is carried out in note [39].

During the comparison of the results of the theoretical calculations of the equilibrium flow of nonviscous flows about the acute cones of gases in the case $M_1 \sin\varepsilon \gg 1$ with the results of experiments it is necessary to keep in mind following facts.

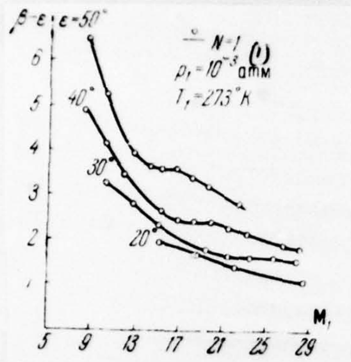


Fig. 14.

Key: (1). atm.

Page 50.

If in the particle of gas it occurs an abrupt change in its thermodynamic parameters, for example, particle intersects shock wave, then for the establishment of the thermodynamic equilibrium, which corresponds to new state of the gas, is required finite time, called relaxation time. When internal energy of gas is determined mainly by the forward/progressive and rotational degrees of freedom of the molecules of gas, relaxation time very little and in aerodynamic problems it can be disregarded. When internal energy of gas depends substantially on the processes of the excitation of the

vibrational degrees of freedom of atoms within molecules, dissociation and the ionization of molecules, relaxation time can become comparable with characteristic time of problem.

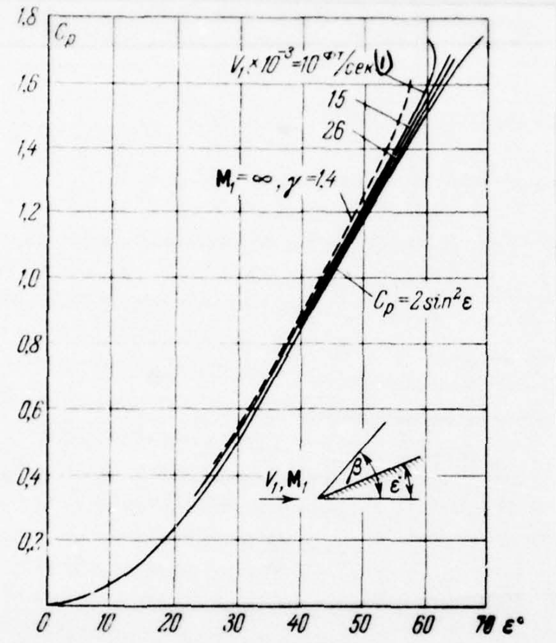


Fig. 15.

Key: (1). s.

Page 51.

If we instead of the relaxation time τ introduce the relaxation length $l = V\tau$, where V is a characteristic particle speed of the gas after bow shock, and to designate the length of the cone by L, then flow can be considered finding in the state of local thermodynamic equilibrium only in the case when $l/L \ll 1$. (Near the apex of the

cone always are regions the "frozen" and nonequilibrium flows of gas). For example, for vibrational degrees of freedom relaxation time for the molecules of nitrogen N_2 approximately 10 times more than for the molecules of oxygen O_2, and since almost 80o/o of air consists of the molecules of nitrogen, then relaxation effects during the excitation of the vibrational degrees of freedom of the molecules of air are determined mainly by the relaxation time τ_{N_2} of vibrational degrees of freedom N_2 during the collisions of the molecules of nitrogen between themselves. From the results [40] it follows that $\tau_{N_2}=\frac{940}{p}$ ms with 1000°K (p - pressure in the atmosphere); $\tau_{N_2}=\frac{58}{p}$ ms with 3300°K. For the relaxation length l, determined taking into account these formulas, for a cone from $\varepsilon = 30°$, flying of height/altitude H above sea level in standard atmosphere with Mach number M_1, are obtained the values, given in table 2; see on this question [22], [23].

Relative to the nonequilibrium flow about the cones see [41].

Further, the examined theories are valid for cones with ideally sharp apex/vertexes. In actuality always it is necessary to deal with the bodies, slightly blunted in front (both due to the limitations, connected with technology of their production, and because the very sharp apex/vertex rapidly is destroyed on the strength of intense heat fluxes with $M_1 >> 1$).

Table 2.

H (фт) (1)		35 000	70 000	100 000
l (фт) (1)	$M_1 = 7,9$	1,1	5,9	25
	$M_1 = 16,6$	0,04	0,21	0,76

Key: (1). feet.

Page 52.

If the half-angle of cone ε is not small, then even the considerable blunting of cone changes the distribution of pressure according to the surface of cone at a distance altogether only of several diameters of blunting downstream. The distributions of other parameters of gas (density, speed, etc.) during downstream also rapidly approach distributions for the pointed cone (with the same ε) everywhere in the region between the bow shock and the body surface, with the exception of the region, which adjoins the surface of cone and called entropy layer. In this region move the particles, passed through the subnormal shock wave and therefore which have lower density, than particle in remaining flow. The thickness of entropy layer decreases approximately inversely proportional to distance from the apex of the cone, and at certain distance it is absorbed by usual viscous boundary layer.

Figure 16, undertaken work [42], depicts dependence $C_p/(C_p)_{max}$, where C_p - pressure coefficient on body surface, and $(C_p)_{max}$ · its value at deceleration point, from distance x, counted off from the stagnation point along body surface and in reference to a radius of blunting r.

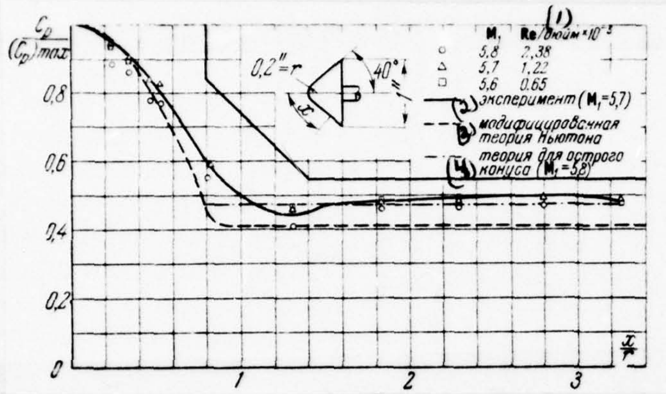


Fig. 16.

Key: (1). inch. (2). experiment. (3). the improved theory of Newton. (4). theory for sharp cone.

Page 53.

The geometry of body and its size/dimension are given in figure. The Mach number of undisturbed flow M_1 varies within the limits of 5.6-5.8, Reynolds number Re, designed by 1 inch from the parameters of undisturbed flow - within limits $0.65 \cdot 10^5 - 2.38 \cdot 10^5$.

In figure also are given the results of calculation for sharp cone (see [17]) and the results of calculation according to the improved theory of Newton $\frac{C_p}{(C_p)_{max}} = \sin^2 \varepsilon$, where ε is an angle

between the tangent to body surface at the point in question and direction of undisturbed flow.

Work [43] gives schlieren photographs for 17 bodies, which are cones with spherical blunting, in the flow of helium with M_1 = 22 and Reynolds number of undisturbed flow, designed to one inch, $Re \approx 0.79 \cdot 10^6$.

Of all models the ratio of a radius of spherical blunting to a radius of the basis/base of cone was equal to three, and the half-angle of cone ε varied consecutively from 90° to 0°. When is located of 30° < ε < 50°, bow shock immediately after blunting is conical segment, which indicates the nearness of flow (downstream from blunting) to conical flow. When $\varepsilon > 50°$ and $\varepsilon < 30°$ the bow shock either will move away up to considerable distance from body or at a distance, equal to the length of model, flow does not manage to be evened and still noticeably it differs from conical flow. Quantitative results relative to parameter distribution in airflow about the blunting of cones can be found in works [44], [45], [29], where the problem was solved by numerical methods.

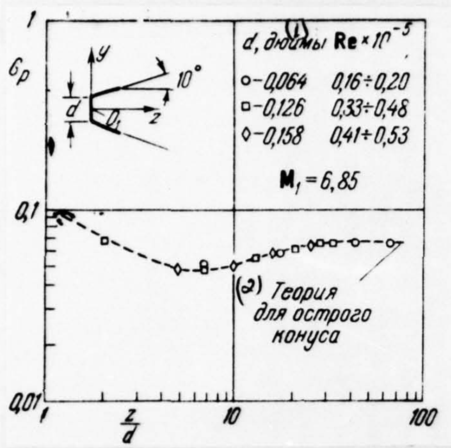


Fig. 17.

Key: (1). inches. (2). Theory for jail cone.

Page 54.

For slender cones with $\varepsilon = 5° \div 10°$ the small blunting of apex/vertex can have defining effect both on the total resistance of body and on parameter distribution of gas in the region between the surface of bow shock and the body surface [46]. Pressure balance on body surface occurs already at a distance of dozen diameters of blunting downstream from the apex of the cone, see, for example, [29], [44].

Figure 17, undertaken work [47], gives the distribution of pressure coefficient C_p on the surface of cone with $\varepsilon = 10°$ and by the flat/plane blunting of diameter d, in airflow with M_1 = 6.85 and Reynolds number Re_d (for reference length undertook value d), by changing within limits 0.16 x 10^5-0.53 x 10^5, depending on distance x, counted off along the axis of cone from his apex/vertex.

According to the law of similarity for the fine/thin blunt-nosed cones (without taking into account of the form of blunting) [46, 48], the distribution of pressure on the surface of cone is described by the formula

$$\frac{p - p_1}{p_1 M_1^2 \operatorname{tg}^2 \varepsilon} = f\left[\sqrt{\frac{2}{c_{x0}}} \operatorname{tg}^2 \varepsilon \frac{x}{d},\ M_1 \operatorname{tg} \varepsilon,\ p_1,\ \rho_1,\ X_{i1}\right],$$

where c_{x0} there is a drag coefficient of blunting, d - its diameter, X_{i1} - the concentration of the components of gas in undisturbed flow. With the large M_1, when K = M_1 tg $\varepsilon \gg 1$, the essential parameter is only $\sqrt{\frac{2}{c_{x0}}} \operatorname{tg}^2 \varepsilon \frac{x}{d}$ (for one the same gas). For K of the order of one, the pressure balance on the surface of cone in air occurs with x, which satisfy condition $\sqrt{\frac{2}{c_{x0}}} \operatorname{tg}^2 \varepsilon \frac{x}{d} = O(1)$ (see [47], [46]). For large K the given estimation is retained (see [29], [44], [49]). For these reasons cone can be considered ideally acute/sharp, if its length L satisfies condition $\sqrt{\frac{2}{c_{x0}}} \operatorname{tg}^2 \varepsilon \frac{L}{d} \gg 1.$ After reject/throwing factor $\sqrt{\frac{2}{c_{x0}}} = O(1)$, we will finally obtain that $\frac{L}{d} \gg \operatorname{ctg}^2 \varepsilon.$

Page 55.

The representation of the thickness of entropy layer can be obtained from works [45, 29], and also from work [50], where is given bibliography according to the analytical methods, which consider entropy layer.

In conclusion let us note one additional special feature/peculiarity of the hypersonic flows of gas. With $M_1 >> 1$ viscous boundary layer occupies the large part of the region between the bow shock and the body surface how this occurs at the moderate values of M_1 (for example, see [5]). Because of this the boundary layer with $M_1 >> 1$ has noticeable effect on external (inviscid) flow with large Reynolds numbers of undisturbed flow, than this occurs in flows with the moderate values of M_1.

Especially sharply this effect is expressed for thin cones. The basic parameter, which characterizes the interaction of boundary layer and inviscid external flow, it is [5]

$$\bar{\chi}_L = M_н^3 \sqrt{\frac{C}{(\mathrm{Re}_L)_н}}.$$

Here

$$C = \left(\frac{\mu}{T}\right)_в \left(\frac{T}{\mu}\right)_н$$

there is a coefficient of chepmen - Rubezin's ductility/toughness/viscosity, by mark "v" are noted the values,

taken on boundary layer edge, by the mark of "p" - the values, undertaken on body surface; M_B and $(Re_L)_B$- Mach number and Reynolds (that which was designed along the length of body L).

Flow about cone (outside boundary layer) can be calculated from the formulas of the theory of inviscid flow in cases when $\bar{\chi}_L \ll 1$. Further information relative to the viscosity effect and blunting on the flow about the slender cones with M_1 >> 1 can be found, for example, in works [51], [52].

2.6. Approximate analytical solutions. The problem of the determination of the approximate analytical solutions for a round cone was examined in many works. Transonic conditions (M_1 - 1 << 1) of the flow about the cone he was studied in works [11], [53]. In work [11] equation (2.4) in the space of hodograph was simplified in the vicinity of the axis of the symmetry of flow and the solutions to the simplified equation were located in the locked form.

Page 56.

In monograph [53] initial is the equation for the velocity potential of disturbance/perturbation ϕ' in the transonic approach/approximation:

$$\varphi'_{yy} + \frac{\varphi'_y}{y} - (M_1^2 - 1)\varphi'_{zz} = (\gamma + 1)\varphi'_z\varphi'_{zz}.$$

The location of the axes is shown in Fig. 5. The velocity vector of the particles of the gas is determined from formula of $V = V_1 \mathrm{grad}\,(z + \phi')$. The condition of the nonseparated flow of cone and condition on leading shock wave they are record/written in the form

$$\lim_{y\to 0} y\varphi'_y = \varepsilon^2 z,$$

Key: (1). with.

$$\left.\begin{aligned} \varphi' &= 0, \\ -\varphi'_z &= \frac{2}{\gamma+1}\frac{M_1^2 \sin^2\beta - 1}{M_1^2} \end{aligned}\right\} \overset{(1)}{\text{при}}\ \frac{y}{z} = \mathrm{tg}\,\beta.$$

Pressure coefficient on the surface of cone is determined from the formula

$$C_p = -2\varphi'_z - \varepsilon^2.$$

During the determination of the solution of problem, is utilized the perturbation method of coordinates (Poincare-Lighthill-Ho method). Expansions for ϕ' and an independent variable $\frac{y}{z} = \eta$ are accepted in the form

$$\frac{1}{z}\varphi'(z, y, \varepsilon) \sim \varepsilon^2 f_1(\eta^\times) + \varepsilon^4 f_2(\eta^\times) + \varepsilon^6 f_3(\eta^\times) + \ldots,$$

$$\sqrt{M_1^2 - 1}\,\frac{y}{z} \sim \eta^\times + \varepsilon^4 y_3(\eta^\times) + \varepsilon^6 y_4(\eta^\times) + \ldots$$

The position of leading shock wave is determined by the expansion

$$\sqrt{M_1^2 - 1}\,\mathrm{tg}\,\beta \sim 1 + k_3\varepsilon^4 + k_4\varepsilon^6 + \ldots$$

In work [53] are located (in the locked form) the extracted terms of expansions and thereby with good approach/approximation it is determined ϕ'.

Page 57.

Specifically, for pressure coefficient on the surface of cone we obtain

$$\frac{C_p}{\varepsilon^2}+2\ln\left(\sqrt{M_1^2-1}\cdot\varepsilon\right)\sim(2\ln 2-1)+\frac{(\gamma+1)\varepsilon^2}{M_1^2-1}+\left(\frac{\pi^2}{12}-\frac{1}{4}\right)\left[\frac{(\gamma+1)\varepsilon^2}{M_1^2-1}\right]^2+\ldots \quad (2.49)$$

This formula is asymptotic expansion for C_p at the larger values of the transonic parameter of similarity $\frac{M_1^2-1}{(\gamma+1)\varepsilon^2}$, which it gives the numerical values, close to the results of job estimates [33] when $\frac{M_1^2-1}{(\gamma+1)\varepsilon^2}\geqslant 2$ (at smaller values $\frac{M_1^2-1}{(\gamma+1)\varepsilon^2}$ the error in determination $\frac{C_p}{\varepsilon^2}+2\ln\left(\sqrt{M_1^2-1}\cdot\varepsilon\right)$ it does not exceed 25o/o).

At average values of $_1$, there are two types of approximate solutions. To the first type let us relate the solutions in which are more precisely formulated the results of linear theory (theory of the extended bodies) [54], [55]; they give approach/approximation to accurate results at the moderate values of the half-angle of cone $\varepsilon\ (\varepsilon<15°)$ and with an increase of M_1 their accuracy rapidly falls. (See on this question also [56]).

To the second type let us relate different variations, actually one and the same solution, based on following simple idea. If we

utilize the spherical coordinates, depicted on Fig. 1, then on the surface of cone (with $\theta = \varepsilon$, Fig. 5) $v^2/a^2 = 0$. This sense is also small in the region between the bow shock and the surface of cone $(\varepsilon \leqslant \theta < \beta)$, and the greater the M_1, the less here v^2/a^2. Disregarding ratio v^2/a^2 in comparison with unity in equation (2.29) and eliminating v with the aid of the equation (2.30), for determining the radial component of speed u we obtain the equation

$$\frac{d^2u}{d\theta^2} + \operatorname{ctg}\theta \frac{du}{d\theta} + 2u = 0, \qquad (2.50)$$

which by replacement $\cos\theta = \xi$ it is reduced to the equation of Legendre [13] whose solution it takes the form

$$u = A\xi + B\left(\frac{1}{2}\xi \ln \frac{1+\xi}{1-\xi} - 1\right), \qquad (2.51)$$

where A, B are arbitrary constants (determined from boundary conditions on body and the shock wave).

Page 58.

Assumption $v^2/a^2 \approx 0$ when $\varepsilon \leqslant \theta \leqslant \beta$ is equivalent to the assumption that gas density is constant in this region. Actually, the equation of continuity (1.1), taking into account (2.30), he is record/written in the form

$$\frac{d^2u}{d\theta^2} + \operatorname{ctg}\theta \frac{du}{d\theta} + 2u + \frac{du}{d\theta} \cdot \frac{d\rho}{d\theta} \cdot \frac{1}{\rho} = 0,$$

from which, under condition $d\rho/d\theta = 0$, there is obtained equation (2.50).

The examined solution both for supersonic and for hypersonic speeds ($M_1 >> 1$) was found, apparently, by the independently different authors [5, 37, 57-60]. The simplified version of this solution when in equation (2.50) they disregard another term ctg θ du/dθ = ctg $\theta \cdot v$, is in works [36, 61]. As show the curve/graphs, given in work [58], solution (2.51) for an ideal gas with $\gamma = 1.4$ gives the results, close to precise with $\varepsilon \geqslant 10°$ and $M_1 > 2$. (For obtaining concrete/specific/actual results with the formulas [58] it is necessary to assign values ε and β; remaining values, including M_1, are the elementary functions of these parameters, γ and θ.

Let us note another work [62], in which the unknown parameters of gas are considered functions of the angle of rotation of velocity vector relative to direction of undisturbed flow, and communication/connection between modulus of velocity and pressure is approximated by linear function in the region between the bow wave and the body surface. Under the indicated assumptions also it is possible to obtain solution in the locked form. For an ideal gas (γ = 1.4) this solution approximates well exact solution both at the moderate and large values of M_1.

At hypersonic speeds ($M_1 >> 1$), the problem of the determination of approximate solutions is considerably simplified by the facts that the region between the surfaces of the bow shock and body becomes

fine/thin, but the parameters of gas change in it little.

Page 59.

For these reasons it is logical to approximate the parameters of gas in region $\varepsilon \leqslant \theta \leqslant \beta$ by the polynomials of low degrees from the difference β - θ whose coefficients are selected so as to satisfy in any sense equations of motion and boundary conditions. For slender cones $(\varepsilon \ll 1,\ M_1\ \mathrm{tg}\ \varepsilon > 1)$ this solution in the locked form for an ideal gas is obtained in work [63] (see also [46]), where for approximation v was utilized the polynomial of the first degree from θ - ε, while for u - by the second. Similar solutions are obtained in [64], [65] without limitation $\varepsilon \ll 1$.

Solution of the type described above for the inadequate gas, virtually which does not differ from precise, is given in work [25]. (The discussion concerns the examined/considered in work [25] range of a change in the parameters of the gas: V_1 = 2.5-10 km/s, H = 0-70 km, where H is flight altitude in standard atmosphere).

For the approximation of the radial component of speed u (see Fig. 1), is utilized the polynomial of cube relatively β - θ:

$$u = A_0 + \frac{A_1}{1!}(\beta - \theta) + \frac{A_2}{2!}(\beta - \theta)^2 + \frac{A_3}{3!}(\beta - \theta)^3, \quad (2.52)$$

respectively,

$$v = \frac{du}{d\theta} = -A_1 - A_2(\beta - \theta) - \frac{A_3}{2}(\beta - \theta)^2.$$

Here θ - the semiangle of bow wave (see Fig. 5).

After assigning the parameters of undisturbed flow and the half-angle of bow wave β, find the parameters of gas immediately after gallop u_2, v_2, a_2, S_2, ... (see Section 2.2). Then coefficients A_k in formula (2.52) are determined from the conditions: when $\theta = \beta$

$$u = u_2, \quad \frac{du}{d\theta} = v_2, \quad \frac{d^2u}{d\theta^2} = -u_2 - \frac{v_2 \operatorname{ctg} \beta + u_2}{1 - \frac{v_2^2}{a_2^2}}$$

[boundary conditions on the surface of bow shock and equation (2.29), written for $\theta = \beta$].

Page 60.

When $\theta = \varepsilon$

$$v = 0, \quad \frac{d^2u}{d\theta^2} = -2u$$

[condition of the nonseparated flow of cone and equation (2.29), written when $\theta = \varepsilon$].

From these conditions it follows that

$$A_0 = u_2 = V_1 \cos \beta, \quad A_1 = -v_2,$$
$$A_2 = -u_2 - \frac{v_2 \operatorname{ctg} \beta + u_2}{1 - \frac{v_2^2}{a_2^2}},$$
$$A_1 + A_2(\beta - \varepsilon) + 0{,}5 \cdot A_3 (\beta - \varepsilon)^2 = 0,$$
$$2A_0 + 2A_1(\beta - \varepsilon) + A_2[1 + (\beta - \varepsilon)^2] + $$
$$+ A_3(\beta - \varepsilon)\left[1 + \frac{(\beta - \varepsilon)^2}{3}\right] = 0.$$

Coefficients A_0, A_1, A_2 are determined of the given above formulas from the parameters of gas after shock wave. Two Last/latter equations they are reduced to the form

$$A_3 = -\frac{2}{(\beta-\varepsilon)^2}[A_1 + A_2(\beta-\varepsilon)],$$

$$\beta-\varepsilon = \frac{2A_1\left[1-\frac{2}{3}(\beta-\varepsilon)^2\right]}{2A_0 - A_2\left[1-\frac{1}{3}(\beta-\varepsilon)^2\right]}.$$

Solving last/latter equation by the method of iterations, we find that $\beta-\varepsilon$, and, consequently, ε. Knowing coefficients A_k, we we obtain possibility to determine field of flow about cone.

[Value of enthalpy i (θ) is determined from the equation of energy $i + V^2/2 = i_1 + V^2_1/2$. Through the known values of i and $S = S_2$, are located the remaining thermodynamic parameters of gas].

The most laborious operation in the examined method is parameter determination of gas after oblique shock wave. In work [25] are produced the calculations for a series of values V_1 and H and results

are represented in the form of diagrams and tables, convenient for use.

Page 61.

For slender cones $(\varepsilon \ll 1)$ the application/use of a method of the slight disturbances in the case of $M_1 \operatorname{tg} \varepsilon > 1$ is led to certain simplification in the problem (see [46, 66-68]), but in the final analysis problem is reduced to the nonlinear ordinary differential equation of the second order for conical potential (either another function), which is necessary to solve numerically, or approximately, analytically, in the same way as this was done, for example, in work [63].

Let us note another method of shock layer (the "method of the boundary layer"), proposed in work [69] (see also [234]), which makes it possible to obtain in the locked form several terms in the expansion of solution in a series according to the degrees of small parameter $\varepsilon = \gamma-1/\gamma+1$ (γ - adiabatic index) on the condition that $M^2{}_1 > (\gamma-1)$ const > 0, when $\gamma \to 1, M_1 \to \infty$. (in the case of the inadequate gas the role ε plays the relation ρ_1/ρ_2, where ρ_1, ρ_2 is gas density respectively before and after bow shock).

2.7. Laws of similarity. For slender cones $(\varepsilon \ll 1)$ the solution of

the problem of flow can be simplified by deletion in the equations of motion of gas of the secondary terms, which have the higher orders of smallness along ε. than those ones that are retained in equations. As a result of such action are obtained simpler, than initial, the equations of the "theory of the slight disturbances", which for transonic and supersonic regimes of the flow about the cones remain still fairly complicated and cannot be solved in the locked form. However, the analysis of these equations makes it possible to establish that the relationship between the which interest us values they take the form of the so-called laws of the similarity in which these values enter not separately, but in the form of some combinations. For example, for transonic speeds (M_1 - 1 << 1) pressure coefficient C_p on the surface of cone is expressed by formula (see [70])

$$\frac{C_p}{\mathrm{tg}^2\varepsilon}+2\ln\left(\sqrt{M_1^2-1}\cdot\mathrm{tg}\,\varepsilon\right)=f\left[\frac{(\gamma+1)\,\mathrm{tg}^2\varepsilon}{M_1^2-1}\right], \quad (2.53)$$

where the first terms of the expansion of function f (ξ) according to degrees $\xi=\frac{(\gamma+1)\,\mathrm{tg}^2\varepsilon}{M_1^2-1}$ are given by formula (2.49).

End Section.

Page 62.

For the moderate supersonic speeds (see the book [19])

$$\frac{C_p}{\mathrm{tg}\,\varepsilon} = g\left(\mathrm{tg}\,\varepsilon \sqrt{M_1^2 - 1}\right). \qquad (2.54)$$

For hypersonic speeds ($M_1 \gg 1$, $M_1\,\mathrm{tg}\,\varepsilon > 1$) (see [71], [66])

$$\frac{C_p}{\mathrm{tg}\,\varepsilon} = h(\mathrm{tg}\,\varepsilon \cdot M_1, \gamma) \qquad (2.55)$$

(in the range of the accuracy of the theory of slight disturbances $\mathrm{tg}\,\varepsilon = \sin\varepsilon = \varepsilon$).

Works [72, 68] show, that instead of the laws, expressed by formulas (2.54), (2.55), it is possible to write the united law of similarity in the form

$$\frac{C_p}{\mathrm{tg}^2\,\varepsilon} = q\left(\mathrm{tg}\,\varepsilon \sqrt{M_1^2 - 1}, \gamma\right), \qquad (2.56)$$

which is valid with super and hypersonic speeds.

If we convert pressure on cones with $\varepsilon = 5°, 10°, 15°$ in the coordinates of formula (2.56), then we will in practice obtain one

curve for those cases when flow about cone the everywhere supersonic. Since in Newton's theory pressure coefficient C_p is proportional to $\sin^2 \varepsilon$, formula (2.56) to more expedient write in the form

$$\frac{C_p}{\sin^2\varepsilon} = q\left(\operatorname{tg}\varepsilon\sqrt{M_1^2-1}, \gamma\right). \qquad (2.57)$$

If we construct the dependence $\frac{C_p}{\sin^2\varepsilon}$ on $\operatorname{tg}\varepsilon\cdot\sqrt{M_1^2-1}$, (where C_p undertaken from exact solution with γ = 1.40), for cones with ε = 5°, 10°, 15°, 20°, 25°, 30°, 40°, 50°, then the corresponding points lie down to one "universal" curve with the spread, which does not exceed 5-6o/o with tg $\varepsilon\cdot\sqrt{M_1^2-1}$ > 0.5 (for example, see [73]).

The execution of the law of similarity in the form (2.57) for cones with the half-angles on the order of 50° cannot be explained on the basis of the theory of the slight disturbances, and this result one should consider as semi-empirical.

Page 63.

2.8. Semi-empirical formulas. For the calculations, conducted on electronic digital computers (ETsVM [(ЭЦВМ) - digital computer]), it is desirable to have the simple analytical expressions, which approximate with sufficient accuracy the parameters of gas at the flow about the cones. In work [74] are found by selection the approximate analytical expressions for pressure coefficient C_p on

the surface of cone and sin β (β - the half-angle of bow wave) depending on γ, M_1, sin ε, which can be utilized for all M_1 and ε, when the flow of gas about cone remains supersonic. This same to question are dedicated [75-78], and also [38], in which examined equilibrium flow of dissociating air about the cone.

2.9. Establish/installed flow of flow about cone of detonating gas. In connection with the problem of the combustion of fuel/propellant in supersonic flow of gas, is of interest the problem of the flow of flow about the cone of gas, capable of the detonation which solved within the framework of conical flow theory G. Chernyy and S. Kvashina [79, 80]. The wave of detonation during the establish/installed flow about the cone is also round cone with the angle of half-aperture β_d. Chemical reactions in detonation wave are considered by means of the examination of the relation of temperatures of stagnation behind wave and before it, this sense let us designate through λ; it characterizes fuel heating value. The flow of the gas between the detonation wave ($\theta = \beta_d$) and the surface of cone ($\theta = \varepsilon$) is assumed to be conical. Equation (2.4), that connects component speeds w (in the direction of the axis of the symmetry of cone) and v (along the normal to it), remains without changes, but the speed of sound a will be determined from the formula

$$a^2 = \frac{\gamma+1}{2}\lambda a_*^2 - \frac{\gamma-1}{2}(v^2 + w^2). \qquad (2.58)$$

where γ is specific heat ratio, a_* - the critical speed of the sound of the gas before the wave of detonation.

Page 64.

The equation which with known velocities of incident flow V_1 and λ connects the components of velocity vector after detonation wave (equation of the detonation polar), takes the form

$$v^2 = (V_1 - w)^2 \frac{V_1 w (V_1 - w) - (\lambda V_1 - w) a_*^2}{\frac{2}{(\gamma + 1) V_1^2} - V_1 w + a_*^2}. \qquad (2.59)$$

When λ = 1 equation (2.59) it transfer/converts to the equation of usual shock polar (2.20).

In Fig. 18 section GI of the detonation polar GA'_2BA_2IH corresponds to powerful (or supercompressed) detonation waves whose normal component of gas velocity behind wave is less than the local velocity of sound; section IH corresponds to weak (or undercompresden) detonation waves whose normal component of gas velocity behind wave is greater than the local velocity of sound. Point I, which divides on a polar of two types of detonation, answers the detonation of Chapman-Jouguet, during which the normal component of gas velocity behind wave is equal to the local velocity of sound. (Point I is obtained by carrying out tangent to detonation polar from point H_1, on the axis Ow, which corresponds to speed V_1, the incident

flow). Weak detonation waves under normal conditions are not encountered and therefore section IH on polar must be excluded from examination. In the case of the flow about the wedge with half-angle δ, just as for inert gas, there are two mode/conditions of flow, determined by points A_2 and A'_2 in Fig. 18. For points A_2, the gas velocity behind detonation wave is supersonic (with the exception of small vicinity of point B), while for points A'_2 - subsonic. The possible mode/conditions of the flow about the cone is most simple reveal/detect/exposed by the construction of "apple-like" curve for the detonating gas.

After taking any point A_2 on section GI of detonation polar as initial (Fig. 19), we carry out straight line through it and point H_1) (corresponding to the incident flow).

AD-A066 205 FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO F/G 20/4

NONLINEAR CONICAL FLOWS OF GAS, (U)

MAR 78 B M BULAKH

UNCLASSIFIED FTD-ID(RS)T-0266-78 NL

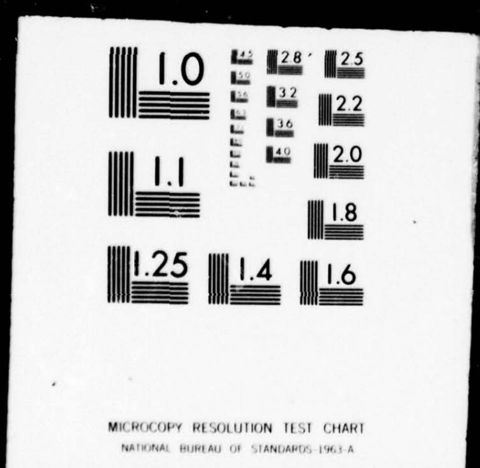
2 OF 7

AD

A066205

2 OF 7

AD
A066205



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

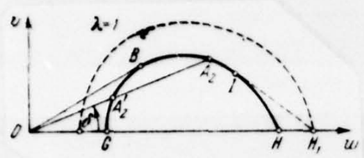


Fig. 18.

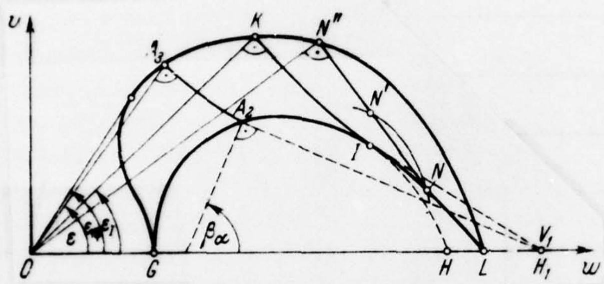


Fig. 19.

Page 65.

The direction of this straight line determines the direction of tangent to the hodograph of flow at point A_2. (Generatrix of detonation wave is normal to cut A_2H_1, since the tangential to detonation wave velocity component does not change during transition through wave front). The further constructions of the hodograph of flow do not differ from the case of inert gas. Since for powerful detonation the normal to wave component of speed is less than the

local velocity of sound, R in equation (2.7) has negative sign at point A_2, which corresponds to curve, convex to the side of decrease in v. The same picture is retained up to point A_3, which corresponds to the surface of cone ($\theta = \varepsilon$). The character of the flow about the cone in this case is analogous to the case of the inert gas: after detonation wave the gas continuously is compressed during its motion to the surface of cone. After constructing "apple-like" curve on section GA_3K, where point K corresponds to point I on detonation polar, is obtained possibility to describe the flows of gas about cones with the half-angles ε, which satisfy inequality $\varepsilon_I < \varepsilon \leqslant \varepsilon_{кр}$ (see Fig. 19). Let us note that just as for a wedge, for a cone with assigned ε are possible two mode/conditions of the flow: from more powerful and weaker by the detonation waves.

Page 66.

For the construction of the solution of problem in the cases $\varepsilon < \varepsilon_I$ it is not possible to utilize a section of polar IH; therefore in these cases the solution of problem for the detonating gas differs significantly from the same for inert gas. For to the solution to equation (2.4) would corresponding certain flow of gas in physical space, it is necessary, in order to standard to curve, depicting in hodograph plane vw this solution, it was turned clockwise, during motion along curve from the point, corresponding to

detonation (or impact) wave, to the point, corresponding to the surface of cone. Point I in Fig. 19 is point of inflection for the curve KIL, that depicts the solution to equation (2.4), passing through point I in the direction of cut IH_1. During motion from one point I to next L, the standard on curve is turned clockwise, just as during motion to point K; therefore the section of the curve IL corresponds to the flow of expansion, to following behind detonation wave. Since in this flow the velocity component, normal to ray/beam θ = const, is more the local velocity of sound, each such ray/beam can be accepted as wave front of shock. After taking any point N for the participation of curve IL, let us construct the shock polar, which corresponds to the velocity vector of this point: Fig. 19 depicts part NN' of this polar. Velocity vector behind shock wave is determined by point N', which lies on the intersection of shock polar NN' and tangential to the curve INL at point N, so kA the ray/beam θ = const, which corresponds shock wave, from one side is perpendicular to tangent to the curve IL at point N, on the other hand, to the cut, which connects points N and N'. The velocity component, normal to shock wave, behind its front is less than the local velocity of sound; therefore the hodograph of the flow of compression from point N' to point N", the appropriate surface of cone, has convexity to the side of decrease v. The shock-wave intensity with the motion of point N from one point I to next L first grow/rises from zero at point I, and then decreases again to zero at point L.

Thus, during the flow of gas about the cone, capable of detonation, are possible such mode/conditions of the flow about the cone when in flow simultaneously there are moving along the particles of gas to one and the same side of two discontinuity/interruptions: the wave of detonation O_1A (Fig. 20), that is spread in connection with particles of gas after it from the normal component velocity, equal to the speed of sound, and the following behind the wave of detonation shock wave O_1B, which propagateds through the particles of the gas before it at supersonic speed.

Page 67.

In region AO_1B, the flow is expanded, in region BO_1C - it is compressed. Let us note that during a change in the angle ε the wave front of the detonation O_1A will not change its position (there is in form the case $\varepsilon < \varepsilon_l$), changes only the position of shock wave.

б) axisymmetric conical flows.

2.10. Preliminary observations. The problem of the axisymmetric

flow about the round cone, which is one of the most important problems of aerodynamics of bodies of revolution, is characteristic fact that for the fixed/recorded form of body, i.e., at the fixed/recorded half-angle of cone ε, its solution always is obtained in the class of conical flows, if only $M_1 > 1$, $\varepsilon < \varepsilon_{\text{кр}}$. There is another series of the axisymmetric conical flows about which it will be said further. But they all can be considered as flows about some bodies (or within channels) only with the fixed/recorded Mach number of the incident flow M_1. Briefly stated, one of the flow lines of conical flow is accepted as the surface of the streamlined body (or the wall of channel), which changes its form with change M_1. But if the form of wall is fixed that then flow at the changing value M_1 will be no longer conical.

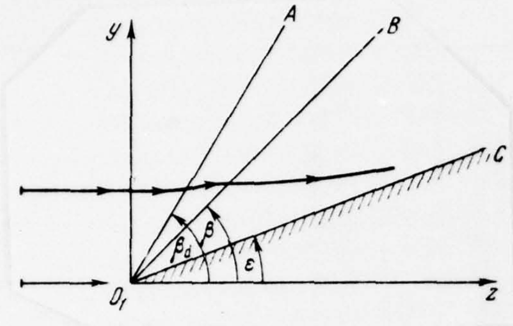


Fig. 20.

Page 68.

2.11. Nozzle of compression. Flow about the throttle/tapered cylindrical pipe. Flows in channels. Let us examine the class of the conical flows which adjoin the uniform flow along the appropriate Mach cone or shock wave in the form of circular cone. Let the Mach number of uniform flow will be $M_1 = V_1/a_1$, where V_1 and a_1 - respectively the speed of this flow and the speed of sound in it; flow direction let us accept for the direction of axis O_1z. Research on such flows is convenient to conduct on hodograph plane, after using formulas (2.4)-(2.7).

On hodograph plane point H (Fig. 21), which corresponds uniform flow ($v = 0$, $w = V_1$), he is person for equation (2.4) and if flow is

continuous, then dependence of v on w in its vicinity is given by expansion [11],

$$v=\pm\left\{\sqrt{M_1^2-1}\,(w-V_1)+\frac{V_1(\gamma+1)M_1^2}{2\sqrt{M_1^2-1}\,a_1^2}(w-V_1)^2+\right.$$
$$+\frac{(\gamma+1)M_1^4}{6a_1^2(M_1^2-1)^{5/2}}\left[(2\gamma+5)-M_1^2(\gamma+4)\right](w-$$
$$\left.-V_1)^3\ln\left|\frac{w-V_1}{V_1}\right|+c(w-V_1)^3+\ldots\right\}, \quad (2.60)$$

where c is an arbitrary constant. With $w = V_1$ $v' = \pm\sqrt{M_1^2-1}$ and $\operatorname{tg}\theta = \eta = y/z = -\frac{1}{v'}=\mp\frac{1}{\sqrt{M_1^2-1}}$, i.e., to uniform flow the conical flow can adjoin along the Mach cone, constructed upstream from point O_2 $\left(v' = +\sqrt{M_1^2-1},\ \eta=-\frac{1}{\sqrt{M_1^2-1}}\right)$ and along the Mach cone, constructed downstream from point O_1 $\left(v'=-\sqrt{M_1^2-1},\ \eta=+\frac{1}{\sqrt{M_1^2-1}}\right)$.

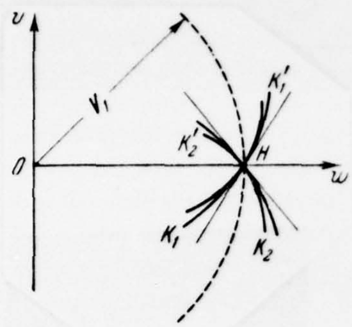


Fig. 21.

Page 69.

Respectively with $w = V_1 v'' = \frac{V_1(\gamma+1)M_1^2}{v' a^2}$ has the same sign, as V' and in the vicinity of point H the behavior of the integral curves of equation (2.4) is represented in Fig. 21.

Along each direction, determined $V' = \pm\sqrt{M_1^2 - 1}$, point H emerges the one-parameter family of solutions of equation (2.4), having at point H identical curvature [see equation (2.60)].

Since the standard to the hodograph of flow at point (v, w) determines direction of beam in physical space, corresponding to these v and w, during motion from point H along the integral curve of equation (2.4) standard to it must be turned clockwise, if is

examined the region of the physical space $y > 0$ (and vice versa, if $y < 0$). Furthermore, integral curve must not have point of inflection, since otherwise in physical space flow is obtained bipinnate, which is inadmissible. Since they are studied axisymmetric flows, it suffices to examine the part of space $y > 0$. During this limitation physical sense make the curves, that emerge from point H (see Fig. 21) in the negative direction of axis Ov. Let us examine first certain curve HK_1 from the family, determined by condition $v' = \sqrt{M_1^2 - 1}$

at point H. Uniform flow is separate/liberated from conical flow by the Mach cone, turned upstream (straight line O_1A in Fig. 22).

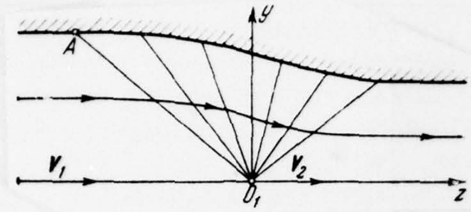


Fig. 22.

Page 70.

During motion along the hodograph of flow from one point H to next K_1, the speed of flow decreases, and velocity vector is turned to the side of the axis of symmetry, i.e., to the Mach cone, turned upstream, can adjoin only flow of compression. This flow of compression theoretically it can be continued until on its hodograph appears point of inflection. A. Busemann's reasonings [11] show that there are flow conditions during which this flow can be locked by jump O_1B, which again converts conical flow into the uniform, parallel axes O_1z and having speed $V_2 < V_1$. As a result in us, is obtained the conical flow in certain nozzle of compression.

Let us examine now curved HK_2, that belongs to family of solutions with $V' = -\sqrt{M_1^2 - 1}$ at point H. Uniform flow is

separate/liberated from conical flow by the Mach cone, constructed downstream from point O_1 (straight line O_1A in Fig. 23).

During motion along the hodograph of flow from one point H to next K_2, the speed of flow grow/rises, and velocity vector is turned to the side of the axis of symmetry, i.e., to the Mach cone, constructed downstream, they can adjoin only flows of expansion. Each such flow can be considered as result of external flow of the undisturbed supersonic flow about certain body of revolution, which is the semi-infinite cylinder which from certain section gradually becomes narrow, [81]. In work [81] A. Nikol'skiy investigated in detail this case and showed that with the aid of such flows it is not possible to construct the flow about the locked bodies [i.e. bodies whose point B (Fig. 23) it would lie/rest on the axis O_1z] since always there exists during such maximum ray/beam OB further which solution does not exist.

Fig. 23.

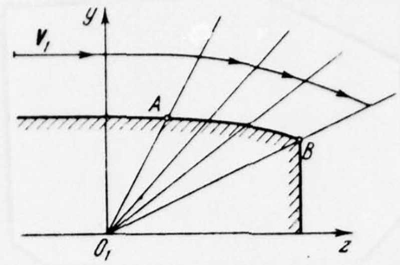
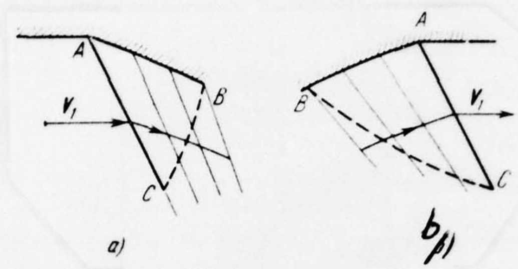


Fig. 24.



Page 71.

In this work are also produced the numerical calculations of flows and is constructed a series of the airfoil/profiles of the streamlined bodies.

Besides the examined cases where conical flows play the significant role are possible the cases when the limited conical fields "are built in" into conical flows. Figures 24 shows examples of such cases which can be met during the flow of gas in channels (see [82], [198]).

In Fig. 24a the uniform flow with a speed of V_1 intersects oblique shock AC, after which follows the conical flow, limited by the characteristic of flow (removable discontinuity) BC; in Fig. 24b

picture of flow reverse/inverse.

c) the flow about the pyramidal bodies.

2.12. Preliminary observations. Let us examine in the supersonic uniform flow of gas, which has speed V_1, density ρ_1, the Mach number M_1 and of so forth, the shock wave whose plane front Σ composes with direction V_1 angle β (Fig. 25).

On surface Σ let us select any curve ABC and through its points let us conduct the rectilinear flow lines, which correspond to the direction of speed V_2 of supersonic uniform flow after gallop Σ.

As a result we will obtain stream surface ABCD, which can be accepted for body surface during flow of flow about which with a a velocity of of V_1 at an angle of attack δ is formed the step shock of the packing/seal Σ, after which follows the uniform flow with a a velocity of of V_2.

Page 72.

If cuts AB and BC are rectilinear, then as a result will be obtained certain V-shaped wing. Connecting several such wings, it is possible

to obtain the exact solution of gas-dynamic problem (with that which was fix/recorded M_1) for certain pyramidal body, which has star-shaped transverse cross section. The described bodies were constructed by G. Makapar [83]; V- and W-shaped wings were proposed by T. Nonvayler [84, 85] for supersonic regime of the flow when airfoil lift is created virtually by flow after step shocks. In work [86] are investigated in detail the geometry and the characteristics of these bodies and wings; see also [87]. A. Gonor, solving the spatial problem of bodies of minimum drag in Newton's theory [88], it reveal/detected that the bodies with star-shaped cross section can possess substantially smaller resistance, than the equivalent to them by volume (or midship section) circular of cone. (that fact that the pyramidal bodies have smaller wave impedance than equivalent of cone, was noted in [83]). Utilizing systems of the regularly intersecting jumps, A. Gonor constructed exact solution for pyramidal bodies with the star-shaped section of more complex form, on surface of which the pressure is constant, but differently on the different parts of the body. (On possibility of the constructions of such bodies indicated G. Maykapar, see [90]). These bodies really/actually have the wave impedance which several times is less than resistance of equivalent cones. On the results of experiments and the possible mode/conditions of the flow about the pyramidal bodies, it will be said later. In work [91] are constructed also the exact solutions for pyramidal bodies with the aid of the system of shock waves with Mach interaction.

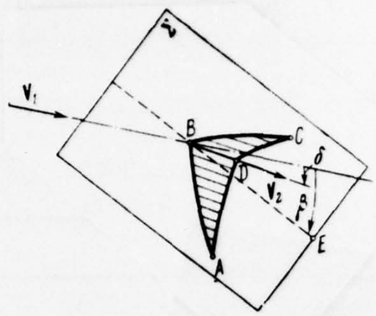


Fig. 25.

Page 73.

The more complex pyramidal bodies during construction of which are used, besides flows after the step shocks of packing/seal, simple conical waves (see the following section), are obtained in work [92].

2.13. Flow about pyramidal body with cross section in the form of correct concave polyhedron. According to G. Maykapar [83], let us examine flow about pyramidal body, depicted on Fig. 26. The surface of this body is formed by triangles BAD, CDB so forth, by the being parts of stream surfaces of uniform flows after the shock waves ABCE and of so forth whose intersection determines the fi /edges of body BA, BC and of so forth. On the strength of symmetry let us examine the flow about the part of the body whose cross section is shaded in

Fig. 26.

The velocity components after oblique shock wave ABCE are determined from usual formulas; they are equal to

$$\left.\begin{aligned} u_2 &= 0, \\ v_2 &= V_1 \operatorname{ctg} \beta (1 - \varepsilon)(\sin^2\beta - \sin^2\alpha_1), \\ w_2 &= V_1 [1 - (1 - \varepsilon)(\sin^2\beta - \sin^2\alpha_1)], \end{aligned}\right\} \quad (2.64)$$

where $\sin \alpha_1 = 1/M_1$, $\varepsilon = (\gamma - 1)/(\gamma + 1)$.

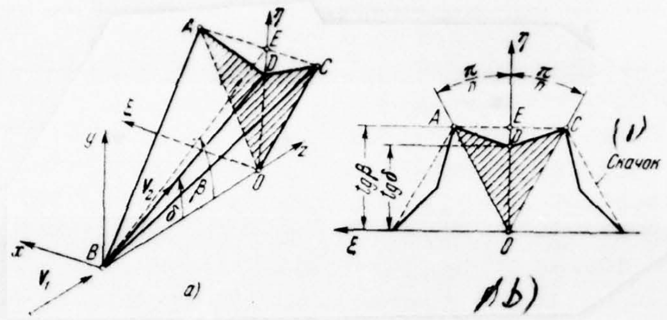


Fig. 26.

Key: (1). Jump.

Page 74.

From formulas (2.61) we obtain the slope tangent of the internal fin/edge BD

$$\operatorname{tg}\delta=\frac{v_2}{w_2}=\frac{\operatorname{ctg}\beta\,(1-\sigma)\,(\sin^2\beta-\sin^2\alpha_1)}{1-(1-\sigma)\,(\sin^2\beta-\sin^2\alpha_1)}. \quad (2.62)$$

Dependence of $\frac{\operatorname{tg}\delta}{\operatorname{tg}\beta}$ on β and M_1 with σ = 1/6 is depicted on Fig. 27.

The area of the bottom section of body with the length, equal to unity, is determined from the formula

$$S=n\operatorname{tg}\frac{\pi}{n}\cdot\operatorname{tg}\beta\cdot\operatorname{tg}\delta=n\operatorname{tg}\frac{\pi}{n}\,\frac{(1-\sigma)\,(\sin^2\beta-\sin^2\alpha_1)}{1-(1-\sigma)\,(\sin^2\beta-\sin^2\alpha_1)}. \quad (2.63)$$

Air pressure after shock wave ABCE is equal

$$p_2 = (1-\sigma)\rho_1 V_1^2\left(\sin^2\beta - \frac{\sigma}{1+\sigma}\sin^2\alpha_1\right),$$

and pressure coefficient on body surface

$$C_p = \frac{p_2 - p_1}{\rho_1 V_1/2} = 2(1-\sigma)(\sin^2\beta - \sin^2\alpha_1). \qquad (2.64)$$

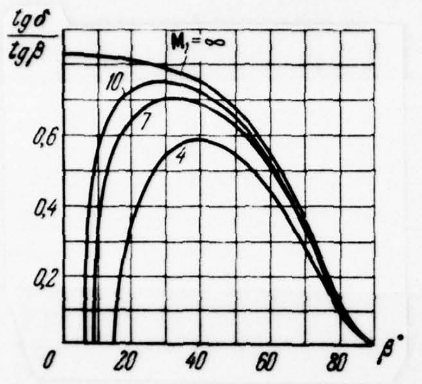


Fig. 27.

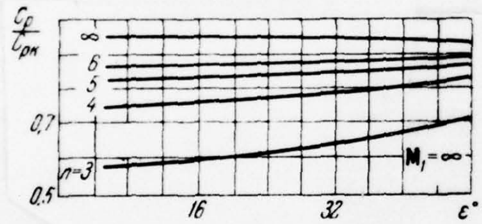


Fig. 28.

Page 75.

For the half-angle of round cone ε, of equivalent by area bottom section to pyramidal body, according to (2.63) we will obtain the relationship/ratio:

$$\operatorname{tg}\varepsilon=\sqrt{\frac{n}{\pi}\operatorname{tg}\frac{\pi}{n}\frac{(1-\varepsilon)(\sin^2\beta-\sin^2\alpha_1)}{1-(1-\varepsilon)(\sin^2\beta-\sin^2\alpha_1)}}. \quad (2.65)$$

From formulas (2.64) and (2.65) follows the simple dependence between

C_p, the being also coefficient of wave drogue drag, and

$$\operatorname{tg} \varepsilon = \sqrt{\frac{n}{\pi} \operatorname{tg} \frac{\pi}{n} \frac{C_p}{2 - C_p}}.$$

which can be written in the form

$$C_p = \frac{2 \operatorname{tg}^2 \varepsilon}{\operatorname{tg}^2 \varepsilon + \frac{n}{\pi} \operatorname{tg} \frac{\pi}{n}}. \tag{2.66}$$

on Fig. 28, undertaken from work [83], it is depicted dependence $\frac{C_p}{C_{pk}}$, where C_{pk} there is the pressure coefficient of equivalent cone, from ε and n with $\epsilon = \frac{1}{6}$ ($\gamma = 1.4$) and $M_1 = \infty$.

(Recall that during change M_1 the form of pyramidal body changes). As can be seen from Fig. 28, resistance of pyramidal body is lower than resistance of equivalent cone with n = 3, but with increase of n, the difference between C_p and C_{pk} rapidly decreases.

2.14. Flow about pyramidal body with cross section in the form of correct concave polygon of complex form.

Page 76.

Following A. Gonoru [89], let us examine two planes crossed shocks ABC and ABC' (Fig. 29) in the uniform flow of gas, which has speed V_1, the Mach number $M_1 > 1$ and of so forth.

The shock-waves junction, the straight line AB, composes angle ϕ with direction V_1 (by axis Bz); the traces of the intersection of jumps with plane z = 1 (by plane $\xi O \eta$) AC, AC' compose angle ψ with axis $O\eta$. On the strength of symmetry let us examine the flow of gas with x > 0. If we introduce auxiliary angle ϕ_1 by formula tg ϕ_1 = tg ϕ sin ψ, then the components of velocity vector V_2 after shock wave ABC will be determined from the expressions

$$\left.\begin{aligned} u_2 &= V_1[-(1-\sigma)\,\mathrm{ctg}\,\varphi_1(\sin^2\varphi_1-\sin^2\alpha_1)\cos\psi],\\ v_2 &= V_1[(1-\sigma)\,\mathrm{ctg}\,\varphi_1(\sin^2\varphi_1-\sin^2\alpha_1)\sin\psi],\\ w_2 &= V_1[1-(1-\sigma)(\sin^2\varphi_1-\sin^2\alpha_1)], \end{aligned}\right\} \quad (2.67)$$

where $\sigma = (\gamma - 1)/(\gamma + 1)$, $\sin \alpha_1 = 1/M_1$.

The angle δ of the rotation of velocity vector V_2 with respect to V_1 is calculated from the relationship/ratio

$$\mathrm{tg}\,\delta = \frac{-u_2\cos\psi + v_2\sin\psi}{w_2} = \mathrm{ctg}\,\varphi_1 \frac{(1-\sigma)(\sin^2\varphi_1-\sin^2\alpha_1)}{1-(1-\sigma)(\sin^2\varphi_1-\sin^2\alpha_1)}.$$

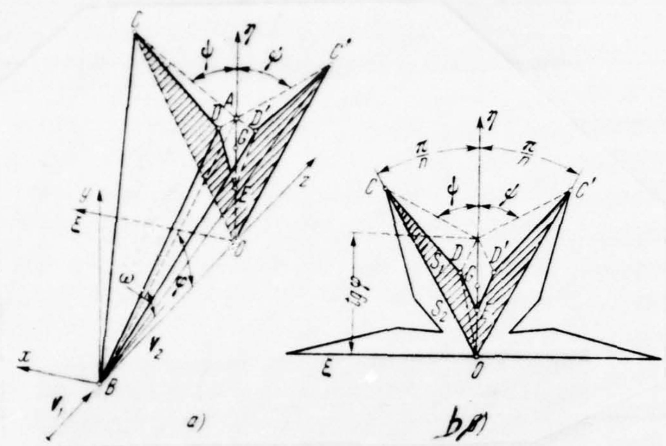


Fig. 29.

Page 77.

Knowing angle δ, we find M_2 through known formulas for an oblique shock wave. Let us assume that the shock waves ABC and ABC' intersect regularly, and flows after the jumps ABD, ABD' REFLECTED are parallel to the plane of symmetry yBz.

Angle ω between V_2 and cut $\overline{BA}$ is found from the formula

$$\cos\omega = \frac{(V_2\cdot\overline{BA})}{V_2\cdot BA} = \frac{V_1\cos\varphi}{\sqrt{u_2^2+v_2^2+w_2^2}} = \sin\delta\,\frac{\cos\varphi\,\mathrm{tg}\,\varphi_1}{(1-\varepsilon)(\sin^2\varphi_1-\sin\alpha_1)} = \frac{\cos(\varphi_1-\delta)\cos\varphi}{\cos\varphi_1}. \quad (2.68)$$

The necessary condition of the regular intersection of jumps takes the form

$$M_{2n} = M_2 \sin \omega > 1. \qquad (2.69)$$

Let us introduce into examination the plane Π, perpendicular to the intersection of jumps AB. The angle between projections on this plane of cut $\overline{OA}$ and of vector V_2 let us designate by δ_1. It characterizes the slope/inclination of the component of speed V_2 in plane Π to the plane of symmetry yBz. After passage to the coordinate system in which axis Bz' is combined with cut AB, and axis Bx' coincides with axis Bx, and simple lining/calculations we obtain the formula

$$\cos \delta_1 = (\operatorname{tg} \varphi - \operatorname{tg} \delta \sin \psi) \; [\operatorname{tg}^2 \varphi \, (1 + \cos^2 \psi \operatorname{tg}^2 \delta) + \operatorname{tg}^2 \delta - 2 \sin \psi \cdot \operatorname{tg} \varphi \cdot \operatorname{tg} \delta]^{-1/2}. \qquad (2.70)$$

Shock wave ABD must turn flow so that it would become parallel to the plane of symmetry, i.e., the component of speed V_2 in plane Π must turn itself to angle δ_1. The location of the shock wave ABD can be determined now by angle β between traces from the intersection of planes ABD and ABO by plane Π from the condition that the rotation of flow in this plane is assigned by expression (2.70).

The formulas of oblique shock wave, we will obtain

$$\operatorname{tg} \delta_1 = 2 \operatorname{ctg} \beta \frac{M_{2n}^2 \sin^2 \beta - 1}{M_{2n}^2 (\gamma + \cos 2\beta) + 2}. \qquad (2.71)$$

Page 78.

If the initial parameters of problem M_1, φ, ψ are such, that is satisfied the condition (2.69), then a sufficient condition for existence of the unknown flow is the possibility of the resolution of expression (2.71) relative to angle β. In work [93] produced the study of the existence domain of the examined flow for the assigned number M_1. For M_1 = 4 results of calculation are depicted on Fig. 30; the region of the allowed values of angles φ and ψ is shaded.

After accepting the flat surfaces of current BCD, BDE, BC'D', BD'E, for uniform flows after shock waves ABC, ABD, ABC', ABD' for solid surface, we will obtain the exact solution of gas-dynamic problem for one side of certain wing with the cross section, comprised of the line segments. Connecting several such wings, we obtain the solution of gas-dynamic problem for the pyramidal body whose cross section is represented in Fig. 29. The geometry of body is determined by the position of points E, D, C for coordinates of which after elementary calculations we obtain the following expressions:

$$\left.\begin{aligned}
\eta_E &= \operatorname{tg}\delta\left[\sin\psi - \frac{\cos\psi\cos\omega}{\sin\varphi\cos\psi\sin\delta - \operatorname{ctg}(\beta-\delta_1)\cos\delta}\right],\\
\xi_D &= \frac{\sin\varphi - \cos\varphi\cdot\eta_G}{\operatorname{ctg}(\beta-\delta_1) + \cos\varphi\cdot\operatorname{ctg}(\psi-\tau)},\\
\eta_G &= \frac{\operatorname{tg}\varphi_1\operatorname{tg}\delta\cos\left(\psi-\frac{\pi}{n}\right)}{\sin\left(\frac{\pi}{n}\right)\operatorname{tg}\varphi_1 + \sin\left(\psi-\frac{\pi}{n}\right)\cos\psi\operatorname{tg}\delta},\\
\operatorname{tg}\tau &= \left(1-\frac{\operatorname{tg}\delta}{\operatorname{tg}\varphi_1}\right)\operatorname{tg}\left(\psi-\frac{\pi}{n}\right),\\
\eta_D &= \frac{\eta_G\operatorname{ctg}(\beta-\delta_1) + \sin\varphi\cdot\operatorname{ctg}(\psi-\tau)}{\operatorname{ctg}(\beta-\delta_1) + \cos\varphi\operatorname{ctg}(\psi-\tau)},\\
\xi_c &= \frac{\operatorname{tg}\varphi_1\cdot\sin\left(\frac{\pi}{n}\right)}{\sin\left(\psi-\frac{\pi}{n}\right)},\\
\eta_c &= \xi_c\operatorname{ctg}\left(\frac{\pi}{n}\right).
\end{aligned}\right\} \quad (2.72)$$

Page 79.

Pressure coefficient $C_p^{(1)}$ for wall BCD (see Fig. 29a) is determined from the formula

$$C_p^{(1)} = 2\,(1-\sigma)\,(\sin^2\varphi_1 - \sin^2\alpha_1), \qquad (2.73)$$

for wall BDE - from the formula

$$C_p^{(2)} = C_p^{(1)} + 2\left(\frac{V_2}{V_1}\right)^2\frac{(\sin^2\omega\cdot\sin^2\beta - \sin^2\alpha_1)\,(1-\sigma)\,M_1^2\sin^2\varphi_1}{1+\sigma M_1^2(\sin^2\varphi_1 - \sin^2\alpha_1)}. \qquad (2.74)$$

For determining the coefficient of the wave impedance of body C_x the

area of its cross section is divide/marked off into triangles OCD, ODE and so forth (see Fig. 29b) whose area S_1, S_2 are determined from the formulas

$$2S_1 = \eta_G(\xi_c - \xi_D), \quad 2S_2 = \eta_E \xi_D,$$

then

$$C_z = \frac{C_p^{(1)} S_1 + C_p^{(2)} S_2}{S_1 + S_2}. \qquad (2.75)$$

Figures 31 depicts the cross sections of some pyramidal bodies.

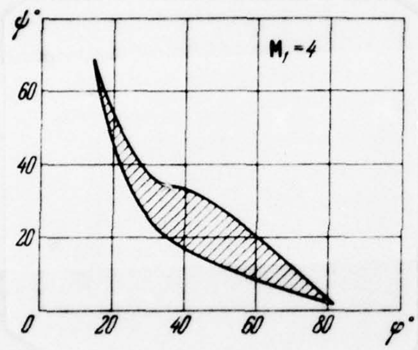


Fig. 30.

Page 80.

Figure 32a-c depicts dependence $\frac{C_z^0}{C_z}$, where C_z is a coefficient of the wave impedance of pyramidal body, C_z^0 - the equivalent along the length and area midsection of round cone, from M_1, φ, ψ for a series these of values of parameters (these materials kindly furnished to the author A. Gonor). Curve/graphs in Fig. 32 show that the wave impedance of pyramidal bodies with star-shaped cross section can be

ten times less than the wave impedance of equivalent along the length and midsection round cone. Let us emphasize that the word goes only about wave, but not about total drag of body.

2.15. Possible mode/conditions of flow about pyramidal bodies and results of experiments.

Let us examine the now theoretically possible mode/conditions of the flow about the pyramidal bodies, constructed in point/items 2.13 and 2.14. During the flow about the wedge (cone) by the supersonic uniform flow of gas there are two flow conditions: with "weak" and with by "powerful" shock waves, and, if wedge (cone) is located in "infinite" flow or in the free jet, then is realized flow with "weak" shock wave. For the body, depicted on Fig. 26, the solution of gas-dynamic problem also not is singular. (It is assumed that to body attacks the flow with those parameters M_1 and so forth for which it was constructed). Let us consider that for the construction of pyramidal body is used the "weak" shock wave ABCE (see Fig. 26), which turns uniform flow with a a velocity of of $\bar{V}_1$ to angle δ. The nonuniqueness of the solution of the problem because of emergence about the body of flow with the "powerful" shock wave, connected only to the apex/vertex of body (point B on Fig. 26), be examined will not further be. Thus, we assume that jumps ABCE so forth rest on the fin/edges of body BA, BC and so forth. Under the made assumptions are

theoretically possible two mode/conditions of the flow about the pyramidal body.

In the first mode/conditions jump ABCE flat/plane and flow after it uniform. In the second mode/conditions the shock wave ABCE is the conical surface after which flow conical, but not uniform.

Page 81.

The existence of two mode/conditions of the flow about the body is explained as follows. Shock wave ABCE can be examined, on one hand, as part of the single jump, which surrounded body, which was decomposed to individual sections with increase M_1; on the other hand, jump ABCE can be considered as arisen during the flow about the V-shaped wing ABCD with the supersonic edges BA, BC. During the flow about this wing on its edges, are formed the step shocks of the packing/seals after which follow the uniform flows, which then interact, forming the conical flow of general view about wing center section. In the particular case when the velocity vectors of the flows of gas after shock waves on leading wing edges are identical (are parallel to the plane of the symmetry of wing BODE), flow about wing is everywhere uniform.

During the solution of the problem of V-shaped wing shock waves

on its leading edges BA, BC can be taken both "weak" and "powerful". If the flows of gas after these oblique shock waves remain supersonic, then this will mean that the flow about the V-shaped wing of finite dimensions, (and, consequently, pyramidal body) is theoretically possible both with "weak" and with "powerful" shock waves on leading wing edges (on the fin/edges of body). (See discussion of this question in [94]).

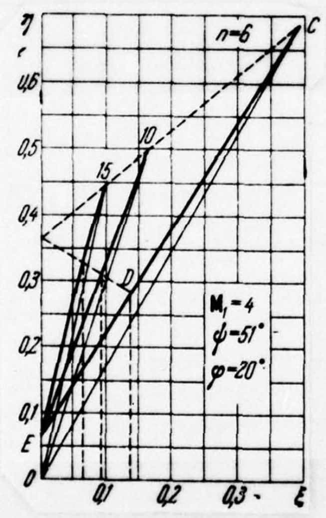


Fig. 31.

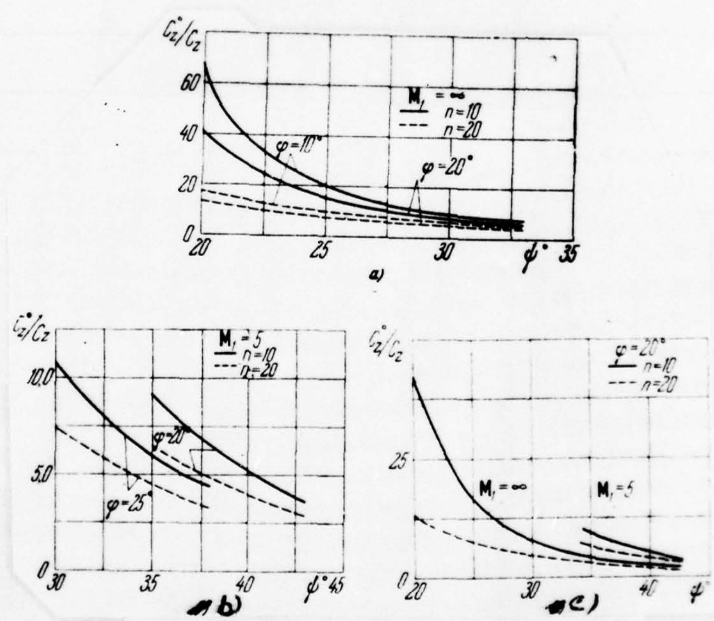


Fig. 32.

Page 82.

Let us explain now, being oriented toward results for the flat/plane and axisymmetric flows of gas, which types of flows one should expect

actually, during the flow about the pyramidal bodies.

If the angle between planes ABD and CBD of V-shaped wing (see Fig. 26) is sufficiently great, so that the step shock of packing/seal ABCE is "weak" for leading wing edges AB, BC, then it is possible to expect that is realized flow with the step shock of packing/seal ABCE. If wing planes closely spaced to each other, so that jump ABCE is "powerful" for leading wing edges, then one should expect in actuality of flow with "weak" shock waves on leading edges, i.e., the emergence of heterogeneous flow about wing, if, of course, viscous boundary layer it does not fill the significant part of the clearance between wing planes. For the bodies, examined in point/item 2.14, the position is analogous. Here one should expect flow with "weak" irregularly ABC, ABC', ABD, ABD' (see Fig. 29); is theoretically possible also flow with the bent "powerful" jump for edges BC, BC', which rests on these edges.

In L. Skvayer's experiments [95], who carried out experiments with V-shaped wing with $M_1 \approx 4$, by $Re \approx 3.5 \cdot 10^6$ (where Reynolds number Re was calculated along the length 30 cm, having the same order, as the length of models) was observed the mode/conditions of flow with the step shock of packing/seal, resting on wing edges. During off-design conditions the flow was conical and pressure on the surface of model remained close to pressure on plate, witn an angle

of attack, equal to the angle of the slope of chord BD (see Fig. 26) to direction of undisturbed flow.

Work [87] gives the results of D. Tredgol'd's experiments with M_1 = 4.3 for two models of V-shaped wing, which were constructed so that for one model the step shock, resting on wing edges, was for these edges "weak", for another model - "powerful." Pressures on the surfaces of these models, measured in one point, proved to be equal in the range of angles of attack ±10°. Although pressure measurements at one point of model is clear insufficiently for, drawing the of substantiated conclusion about the character of flow, all the same can be assumed that there is this parametric domain, which define undisturbed flow and the geometry of wing in which is possible the flow about the body both with "weak" shock waves on leading edges and with the step shock, which are "powerful" for wing edges, depending on the history of the emergence of concrete/specific/actual flow. This assumption they confirm that to a certain extent, A. Gonor's experiments with the V-shaped wing, described in point/item 2.13, with n = 10, M_1 = 3.9. On Fig. 33, a, b, are given the geometry of model and the results of the measurement of pressure on its surface.

Test model was carried out was concealed by form, that the step shock of packing/seal, resting on wing edges, was "powerful" for them. Figures 33b by broken line shows the value c_p which

corresponds to uniform flow after this step shock.

Page 83.

As it follows from Fig. 33b, pressure not is constant on the wing surface, approaching pressure after the step shock of packing/seal only in the domain of small r/R. This type of flow is explained, apparently, by gas overflow through leading wing edges. If we carry out experiments with pyramidal body, then, in all probability, will be obtained flow with the step shock of packing/seal or close to it. This point of view confirm recent experiments V. Keldysh [96].

In the works of A. Gonor and A. Shvets [93, 97], was carried out the study of the models of the V-shaped wings from which are comprised the pyramidal bodies, described in point/item 2.14, with $M_1 \approx 4$ and Reynolds number $Re \approx 6 \cdot 10^6$, calculated along the length of model and the parameters of undisturbed flow. In these experiments was realized the mode/conditions of the flow about the bodies with "weak" shock waves ABC, ABC' on wing edges BC, BC' (see Fig. 29) and by weak reflected irregularly ABD, ABD'. According to observed pressure distributions on the surfaces of models, were designed the coefficients of wave impedance C_x of pyramidal bodies for cases of n = 6, 10, 15. Experimental values C_x proved to be equal to:

$$0{,}029\ (n=6);\ 0{,}035\ (n=10);\ 0{,}042\ (n=15);$$

corresponding theoretical values c_z were equal to 0.026; 0.029 and 0.032. Certain increase in coefficients c_z, obtained in experiments, in comparison with theoretical values c_z, is explained, apparently, by the effect of viscous boundary layer on external "inviscid" flow.

The ratios of the coefficients of wave impedance c_z^0 for the cones, equivalent along the length and area of midsection to pyramidal bodies, to c_z the tested models, are given below: $\frac{C_z^0}{C_z}$ = 3.9; 2.2; 1.95 respectively for n = 6, 10, 15. Let us note that the theoretical results are located in good agreement with experimental, and the wave impedance of pyramidal bodies in the cases, forecasted by theory really several times is less than in equivalent cones.

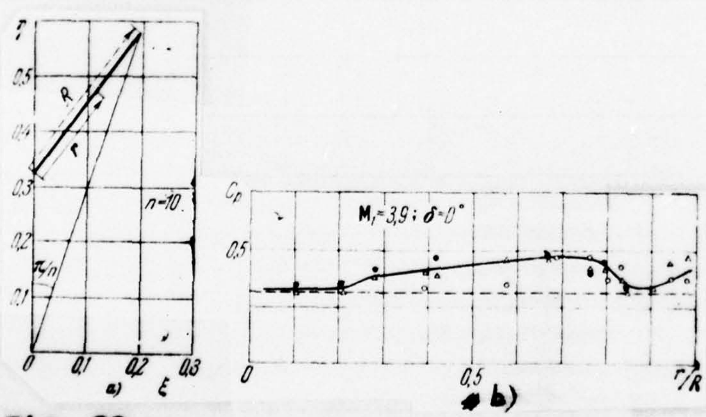


Fig. 33.

Page 84.

d) Simple Conical Waves.

2.16. Lead-in observations. The stationary irrotational (isentropic) flow of nonviscous gas is called simple wave, if it has one-dimensional hodograph. It is possible to shcw that if we conduct the conical surface through the origin of coordinates and the curve, which corresponds to simple wave in the space of hodograph and to expand/develop this surface to plane, then the mentioned curve will be converted into epicycloid; velocity vector and the thermodynamic functions of gas are constant on the planes of certain one-parameter family, which is characteristic for this flow (projection of velocity vector on standard to plane is equal to the local velocity of sound) [98, 99]. Thus, simple waves are a generalization of the known flows of Prandtlya - Mayer in the three-dimensional/space case. Simple waves were opened independently against each other by S. Vallender [100], by A. Nikol'skiy [98] and by D. Giz [99] in connection with

research on the flow about the solid walls, which are (See also [101]). Simple wave is conical, if all the characteristic planes on which century the torus of speed and the thermodynamic functions of gas are constant, they pass through one point - the pole of conical flow. In works [100, 99, 98], was considered (as special case) also the task of flow around of the wing in the form of conical segment with supersonic edges, during solution of which the conical flow pattern did not have vital importance. Research on the properties of simple conical waves as strictly conical flows was carried out by B. Bulakh [9] in connection with their important role when selecting the correct circuit of the flow about the triangular plate and in other questions. Simple conical waves were examined also in works [12, 101-103].

Subsequently simple conical waves will be examined on plane ξ, η ($z = 1$), where they have a family of rectilinear characteristics of the equations (1.29), during which the velocity vector, pressure and so forth retain constant values.

Page 85.

(These rectilinear characteristics on plane ξ, η are traces from the intersection of characteristic planes in space xyz by plane z = 1).

2.17. Properties of simple conical waves. Let us examine consecutively uniform flow, the flow of Prandtlya - Mayer and simple conical wave from the viewpoint of conical flows. Let the uniform flow of gas have a number $M_1 > 1$, a velocity vector of V and of so forth. It is oriented axis O_1z of certain Cartesian system in direction V_1; then the components V_1 along the axes of coordinates O_1x, O_1y, O_1z with respect are equal to 0, to 0, w_1. Uniform flow can be considered as conical flow with pole at point O_1 and the conical potential $F = w_1$ [see formulas (1.28)]. Equation (1.43) for determining the characteristics of flow on plane ξ, η will take the form

$$(1-M_1^2)\left(\xi\frac{d\eta}{d\xi}-\eta\right)^2+\left(\frac{d\eta}{d\xi}\right)^2+1=0. \qquad (2.76)$$

We search for its solution in the form $\eta = d\xi + c$, where d, c are constant. Substitution $\eta = d\xi + c$ into equation (2.76) gives the dependence between d and c in the form $(1-M_1^2)c^2 + d^2 + 1 = 0$, and the equation of characteristics they will be written in the following manner:

$$\eta = \pm\sqrt{(M_1^2-1)c^2-1}\cdot\xi + c. \qquad (2.77)$$

Both performance characteristics consist of straight lines. The envelope of each family is the circumference $\xi^2+\eta^2=(M_1^2-1)^{-1}$, which is simultaneously parabolic line ($AC-B^2 = 0$) for equation (1.29). This

circumference is trace from the intersection of Mach cone with apex/vertex at point O_1 by plane z = 1, and subsequently it for brevity will be called Mach cone. Figures 34 characteristics of one family depicts as solid lines, the characteristics of another - dash.

If $\xi^2+\eta^2>(M_1^2-1)^{-1}$, then equation (1.29) for a uniform flow is hyperbolic equation, but if $\xi^2+\eta^2<(M_1^2-1)^{-1}$, then eliptic equation; Mach cone $\xi^2+\eta^2=(M_1^2-1)^{-1}$ is the parabolic line $AC-B^2 = 0$ exactly as this occurs in usual linear conical flow theory.

Page 86.

From the picture of the location of characteristics, given in Fig. 34, it follows that on each rectilinear characteristic of uniform flow is a unique parabolic point, where $AC-B^2 = 0$. This is correct for the rectilinear characteristics of arbitrary conical simple wave, since each such characteristic can be considered belonging to the uniform flow, which has the speed, equal to the speed of flow on the characteristic of simple wave in question. From the geometric picture of the location of the characteristics of uniform flow, also follows that if we fix any rectilinear characteristic of random simple wave (it is more precise, characteristic plane in space xyz), to direct axis O_1z along velocity vector on it, and axis O_1y to select then, so that the characteristic would be depicted on plane ξ,η

direct/straight with equation $\eta = \eta_1$, then the parabolic point of rectilinear characteristic will lie/rest on the axis $O\eta$, and its coordinates will be: $\xi = 0$, $\eta = \eta_1 = (M_1^2 - 1)^{-1/2}$, where M_1 is a Mach number on characteristic.

Direct/straight and oblique flows of Prandtl - Mayer have on plane ξ, η a family of rectilinear characteristics, passing through the fixed point (ξ_0, η_0), i.e., they are here centered waves.

The conical potential F is record/written for them in the form

$$F = (\xi - \xi_0)\,\Phi(t) + F_0,$$

where $t = \frac{\eta - \eta_0}{\xi - \xi_0}$, F_0 are are constant; the components of velocity along the axes O_1x; O_1y; O_1z they are located through the formulas

$$\left.\begin{aligned} &u = F_\xi = \Phi - t\Phi', \quad v = F_\eta = \Phi', \quad \Phi' = \frac{d\Phi}{dt}, \\ &w = F - \xi F_\xi - \eta F_\eta = F - \xi u - \eta v = F_0 - \xi_0 u - \eta_0 v. \end{aligned}\right\} \quad (2.78)$$

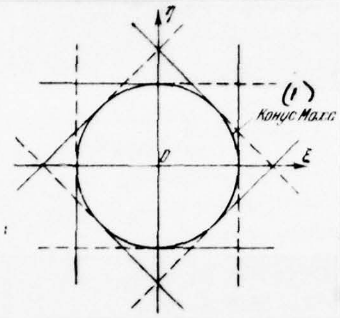


Fig. 34.

Key: (1). Mach cone.

Page 87.

Equation for Φ is obtained from equation (1.29) by the substitution of the second derivatives of F in terms of ξ and η, which are located with the aid of formulas (2.78).

After reduction on Φ'', $(\xi-\xi_0)^{-1}$ equation he is record/written in the form

$$At^2 - 2Bt + C = 0, \qquad (2.79)$$

or, in more detail, in the form

$$a^2 [1 + t^2 + (\xi_0 - t\eta_0)^2] - [w(\xi_0 t - \eta_0) - ut + v]^2 = 0.$$

From (2.79) follows equality $B^2-AC = 1/4(At-Ct^{-1})^2$, which makes it

possible to write the equations of both performance characteristics in the form

$$\left(\frac{d\eta}{d\xi}\right)_+ = t, \quad \left(\frac{d\eta}{d\xi}\right)_- = \frac{C}{At}. \qquad (2.80)$$

The first equation easily integrated: its solution is $\eta - \eta_0 = \text{const}\ (\xi - \xi_0)$; it corresponds to pencil of straight lines, passing through the point $(\xi_0; \eta_0)$.

Let us now move on to the study of arbitrary conical simple wave. Let us assume that potential F of this wave has continuous second derivatives in terms of ξ and η everywhere, with the exception of the points of the envelope of rectilinear characteristics. Let us derive the formulas, which determine the second derivatives of F in terms of ξ and η (through which it is expressed the particle acceleration of the gas) on the rectilinear characteristics of simple wave. Let us fix any rectilinear characteristic, it is oriented axis O_1z in the sense of the vector of speed V_1 on it, we select axis O_1y so that on the plane $\xi\eta$ characteristic would have equation $\eta = \eta_1$, then

$$\eta_1 = (M_1^2 - 1)^{-1/2},$$

where M_1 is a Mach number on it, and coordinates of parabolic point on characteristic are $\xi = 0$, $\eta = \eta_1$ (Fig. 35).

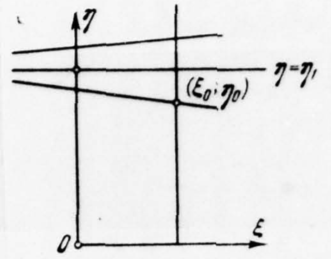


Fig. 35.

Page 88.

Derivatives $F_{\xi\xi}$ and $F_{\xi\eta}$ when $\eta = \eta_1$ are equal to zero (in the selected system of coordinates), since $F_\xi = u$, $F_\eta = v$ are here equal to zero.

Distribution $v = F_\eta$ in the vicinity of characteristic $\eta = \eta_1$ is given by the formula

$$v = f[\eta - k(\xi - \xi_0)], \qquad (2.81)$$

where $k = \frac{B + \sqrt{B^2 - AC}}{A}$, $f(\eta_0)$ is the continuously differentiated function of its argument, which gives distribution v on the straight line $\xi = \xi_0 \neq 0$ depending on η_0. Differentiating equality (2.81) on η and set/assuming then $\eta = \eta_1$, we will obtain the formula

$$v_\eta = f'(\eta_1)\left[1 - (\xi - \xi_0)\frac{C_\eta}{2B}\right],$$
$$\left[f'(\eta_1) = \left(\frac{df}{d\eta_0}\right)_{\eta_0 = \eta_1}\right].$$

whence taking into account the fact that when $\eta = \eta_1,\ w_{\eta} = -\eta_1 v_{\eta}$, $B = (a_1^2 - w_1^2)\,\xi\eta_1,\ C_{\eta} = \eta_1 [(1 + \eta_1^2)(1 + \gamma) v_{\eta} + 2 (a_1^2 - w_1^2)]$,

it follows the relationship/ratio

$$\lambda = f'(\eta_1)\left[\frac{\xi_0}{\xi} - \left(1 - \frac{\xi_0}{\xi}\right)\frac{1+\gamma}{2w_1}\frac{M_1^4}{(M_1^2-1)^2}\lambda\right],$$

where

$$\lambda = (v_{\eta})_{\eta=\eta_1} = (F_{\eta\eta})_{\eta=\eta_1}.$$

It hence follows that if $f'(\eta_1) = 0$, then $\lambda \equiv 0$ on characteristic, with the exception of point $\xi = 0$; but if $f'(\eta_1) \neq 0$, then

$$\lambda = \left[\frac{\gamma+1}{2w_1}\frac{M_1^4}{(M_1^2-1)^2} - \mathrm{const}\,\xi\right]^{-1}, \qquad (2.82)$$

where

$$\mathrm{const} = -\frac{1}{\xi_0}\left[\frac{1}{f'(\eta_1)} + \frac{\gamma+1}{2w_1}\frac{M_1^4}{(M_1^2-1)^2}\right].$$

Page 89.

From formula (2.82) it follows that if on characteristic $\lambda \not\equiv 0$, then with $\xi = 0$

$$\lambda = \frac{2w_1}{\gamma+1}\frac{(M_1^2-1)^2}{M_1^4}, \qquad (2.83)$$

i.e. for any simple wave the second derivatives of F in terms of ξ and η, through which is expressed the particle acceleration of the

gas, at parabolic point they depend only on γ, M_1, w_1, but not from the concrete/specific/actual form of simple wave. Value const in formula (2.82) is determined by the assignment of acceleration at one point of rectilinear characteristic. From (2.82) it follows that if the acceleration on characteristic not is equal to zero, then the envelope of rectilinear characteristics is not passed through the parabolic point. Envelope traverses that point where $\lambda = (F_{\eta\eta})_{\eta=\eta_1}$ it goes to infinity; this point will be determined from the condition of the equality of zero expression in the bracket in formula (2.82).

Let us compute the curvature of the second characteristic of equation (1.29) at the parabolic point E (Fig. 36), on the assumption that $\lambda \not\equiv 0$ when $\eta = \eta_1$.

Since $\lambda \not\equiv 0$ when $\eta = \eta_1$, there is certain vicinity of point E, where there are no points of the envelope of the rectilinear characteristics of simple wave, therefore, end position of the point of intersection of two close characteristics is located outside this vicinity. Let us write the equations of characteristics in the form

$$\left(\frac{dn}{d\xi}\right)_{+,-} = \frac{B \pm \sqrt{B - AC}}{A}. \qquad (2.84)$$

If we designate the angular coefficient of the rectilinear characteristics by k, curvilinear - by η', i.e., $k = \left(\frac{d\eta}{d\xi}\right)_+$, $\eta' = \left(\frac{d\eta}{d\xi}\right)_-$ then of (2.84) it follows that

$$\eta' + k = 2\frac{B}{A}. \qquad (2.85)$$

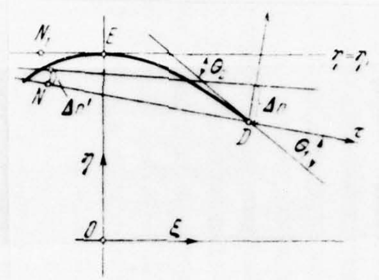


Fig. 36.

Page 90.

To find $\eta'' = \left(\frac{d^2\eta}{d\xi^2}\right)_-$ by differentiation with respect to ξ relationship/ratios (2.84) is inconvenient, since $B^2-AC = 0$ at point E; therefore we will use formula (2.85). Let us write equality $\eta' + k = 2\frac{B}{A}$ for the points of curved characteristics ED, passing through point E, and let us differentiate it lengthwise ED on ξ:

$$\eta'' = 2\left(\frac{B_\xi}{A} - \frac{B}{A^2}A_\xi\right) + 2\left(\frac{B}{A}\right)_{\eta}\cdot\eta' - \frac{dk}{d\xi}. \qquad (2.86)$$

Let us take any point D on characteristic ED and introduce the local coordinate system: axis $D\tau$ it is directed along rectilinear characteristic, axis Dn - along the normal in it, (see Fig. 36); let us designate by ds the cell/element of the arc of characteristic ED, then

$$\frac{dk}{d\xi} = \frac{\partial k}{\partial\tau}\frac{d\tau}{d\xi} + \frac{\partial k}{\partial n}\cdot\frac{dn}{d\xi} = \frac{\partial k}{\partial n}\cdot\frac{dn}{d\tau}\cdot\frac{d\tau}{ds}\cdot\frac{ds}{d\xi} = \frac{\partial k}{\partial n}\frac{\sin\theta_1}{\cos\theta_2},$$

since

$$\frac{\partial k}{\partial \tau}=0,\ \frac{dn}{d\tau}=\mathrm{tg}\,\theta_1,\ \frac{d\tau}{ds}=\cos\theta_1,\ \frac{ds}{d\xi}=\sec\theta_2.$$

Derivative $\frac{\partial k}{\partial n}$ is limited, when point D approaches according to characteristic DE point E. Actually, in the vicinity of point E, where no of envelope rectilinear characteristics, will be located point N, which lies on the rectilinear characteristic, passing through point D, such, that

$$\left|\frac{\partial k}{\partial n}\right|_N>\left|\frac{\partial k}{\partial n}\right|_D.$$

As point N, it is possible to take any point between point D and the point of the envelope of rectilinear characteristics, which lies on characteristic Nd, since increases n, Δn, which correspond to identical increase Δk, for such points are less than for point D (see Fig. 36). Hence it follows that

$$\lim_{D\to E}\left|\frac{\partial k}{\partial n}\right|_D<\lim_{N\to N_1}\left|\frac{\partial k}{\partial n}\right|_N.$$

Here N_1 is certain point on characteristic $\eta=\eta_1$ between point E and the point of the envelope of rectilinear characteristics.

Page 91.

From expression (2.84) it follows that in view of the made assumptions relative to F, which

$$\left|\frac{\partial k}{\partial n}\right|_{N_1} = \left|\frac{\partial k}{\partial \eta}\right|_{N_1} < \infty;$$

hence follows that

$$\lim_{D\to E}\frac{dk}{d\xi} = \lim_{D\to E}\frac{\partial k}{\partial n}\cdot\sin\theta_1\cdot\sec\theta_2 = 0,$$

since in point $E\left|\frac{\partial k}{\partial n}\right| < \infty$, $\sin\theta_1 = 0$, $\sec\theta_2 = 1$. On the strength of the fact that at point $E\ \eta' = B = 0,\ F_{\xi\xi}^2 + F_{\xi\eta}^2 + F_{\eta\eta}^2 < \infty$, from (2.86) we obtain, that

$$\lim_{D\to E}\eta'' = 2\left(\frac{B_\xi}{A}\right)_{\xi=0,\,\eta=\eta_1} = -2(M_1^2 - 1)^{1/2}, \qquad (2.87)$$

since with $\xi = 0$, $\eta = \eta_1$

$$B_\xi = -a_1^2(M_1^2 - 1)^{1/2},\quad A = a_1^2.$$

The radius of curvature of curved characteristics DE at parabolic point half the radius of curvature of Mach cone for a flow on characteristic $\eta = \eta_1$ and, which is important, characteristic DE is arrange/located in the vicinity of the parabolic point E along the same "side" of the rectilinear characteristic according to which lie/rests the velocity vector on it.

Let us study now the question concerning the location of the

parabolic points of simple wave. Let us introduce on plane $\xi\eta$ the unit vector of standard n to rectilinear characteristics. Each such characteristic is trace from the intersection of characteristic plane in space xyz by plane z = 1. If we place velocity vector on characteristic plane into the pole of simple wave, then it will lie/rest along the determined side from this plane. This side of characteristic plane let us call/name positive (opposite side - negative); to it will correspond the positive side of rectilinear characteristic on plane $\xi\eta$.

Page 92.

It is directed by n to positive side. To this will be unambiguously determined the field of standards n to rectilinear characteristics. Let us demonstrate first a series of lemmas, and then a theorem about the location of the parabolic points in simple wave.

Lemma 1. If simple wave has continuous first derivatives of the conical potential F in terms of ξ and η, then the field of standards n is continuous.

According to condition the velocity field of simple wave is continuous, since the components of the velocity expressed through the first derivatives of F in terms of ξ and η; consequently, the

angular coefficients of the rectilinear characteristics of simple wave are continuous. Let us take the vectors of standards n at two points and fix one point to another; within limit we will obtain on the strength of the continuity of the angular ccefficients of rectilinear characteristics, that the vectors of standards either will coincide or they will be oppositely directed. The latter can be only if velocity vector on characteristic lie/rests at the appropriate characteristic plane, which is impossible, since the projection of velocity vector on standard to characteristic plane is equal to the local velocity of sound.

Let us now move over any curve GH, of the intersecting the rectilinear characteristics simple wave, from one point G to next H (Fig. 37).

The unit vector τ, tangential to the curve GH, is directed to the side of motion. Let us call/name motion along the curve GH by "motion to the side of velocity", if vectors n and τ form angle less $\pi/2$, i.e., scalar product $(n \bullet \tau) > 0$.

Lemma 2. If motion along the curved characteristics of simple wave is initiated "to the side of velocity", then it remains the same to parabolic point. Vectors τ, n change continuously during motion along curved characteristics. Sign change $(\tau \bullet n)$ can occur only during

inversion ($\tau \cdot n$) into zero; this means that here the characteristic directions pour, i.e., this point is parabolic.

Let us fix any rectilinear characteristic of simple wave, axis O_1z it is directed along velocity vector on it, and axis O_1y so that the characteristic on plane $\xi\eta$ would have equation $\eta = \eta_1$. Let us call/name this coordinate system for each characteristic of special.

Lemma 3. If on each rectilinear characteristic of simple wave in the special coordinate system the second derivatives of F in terms of ξ and η are equal to zero, then flow is uniform.

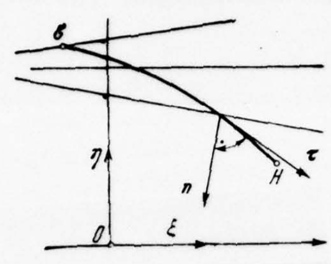


Fig. 37.

Page 93.

If velocity vector is designated by V, then acceleration it is expressed by the formula

$$\frac{dV}{dt} = u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z}, \text{ i.e.}$$
$$\frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \text{ and so forth.}$$

Derivatives in terms of x, y, z uniform are expressed as derivatives in terms of ξ and η:

$$\frac{\partial}{\partial x} = \frac{1}{z}\frac{\partial}{\partial \xi}, \quad \frac{\partial}{\partial y} = \frac{1}{z}\frac{\partial}{\partial \eta}, \quad \frac{\partial}{\partial z} = -\frac{1}{z}\left(\xi\frac{\partial}{\partial \xi} + \eta\frac{\partial}{\partial \eta}\right).$$

while derivatives of the components of speed in terms of ξ and η uniform are expressed as the second derivatives of F in terms of ξ

and η, since $u = F_\xi$, $v = F_\eta$, $w = F - \xi F_\xi - \eta F_\eta$.

Equality zero of second derivatives of F in terms of ξ and η on characteristic indicates the equality of zerc acceleration on it. Consequently, if the second derivatives of F in terms of ξ and η in the special coordinate systems are equal to zerc on each characteristic, then flow is uniform, since acceleration is everywhere equal to zero.

Lemma 4. If motion along the curved characteristics of simple wave from one point G tc next H is completed "tc the side of velocity", then also during the rotation of the axes of the system of coordinates O_1xyz it will be completed "to the side of velocity".

Actually, characteristic lines on plane $\xi\eta$ are traces from the intersection of characteristic conical surfaces in space xyz by plane z = 1. In the case of simple wave, there is a family of characteristic planes and a curvilinear characteristic surface, which connects the ray/beams, determined in space xyz by points G and H with plane $\xi\eta$. In order to hit to ray/beam with point H, it is necessary to move from ray/beam with point G over curved characteristics so as to be moved from disadvantage of characteristic plane, passing through G, to positive. During the rotation of the axes of the system of coordinates O_1xyz of characteristic on plane

$\xi\eta$ they will change their form; this will be already traces from the intersection of characteristic surfaces with the new plane $\tilde{z}$ = 1, turned relative to plane z = 1. But, as before in order to achieve point $\tilde{H}$ on new plane $\tilde{\xi\eta}$. it is necessary to move over characteristic $\tilde{G}\tilde{H}$ so as to be moved from disadvantage of characteristic plane, passing through point $\tilde{G}$ (and point G), to positive, i.e., on plane $\tilde{\xi\eta}$ the motion must be begun "to the side of velocity". Along lemma 2, motion will remain the same up to the parabolic point which lie/rests outside the cutting off of characteristic $\tilde{G}\tilde{H}$ (GH) [or it coincides with point $\tilde{H}$ (H)], since at parabolic point (n•r) = 0, and "motion to the side of velocity" it is determined by condition (n•r) > 0.

Page 94.

Theorem 1. If the conical potential of the simple wave F has continuous second derivatives in terms of ξ and η everywhere, with the exception of the points of the envelope of rectilinear characteristics, and simple wave does not contain the ranges of uniform flow, then during motion along the curved characteristics of simple wave, if motion is beginning "to the side of velocity", the encountered parabolic point, if the same generally exists, it is maximum for the points of the envelope of rectilinear characteristics. Acceleration on the rectilinear characteristic,

passing through this parabolic point, is equal to zero in all points, which do not coincide with parabolic.

Along lemma 2, motion along characteristic, initiated "to the side of velocity", will remain the same to parabolic point. Let point E is a parabolic point on the curved characteristics along which occurs the motion. It is directed axis O_1z along velocity vector V_1 on the rectilinear characteristic on which lie/rests point E, axis O_1y it is directed so that the rectilinear characteristic would be depicted on the plane $\xi\eta$ of straight line $\eta=\eta_1$ (Fig. 38); then $\eta_1=(M_1^2-1)^{-1/2}$ and point E has coordinates ξ = 0, $\eta=\eta_1$, M_1 - Mach number on rectilinear characteristic.

Along lemma 4, motion along curved characteristics to point E in rotated coorinate system occurs "to the side of velocity". Let us distinguish two cases:

1. Let us assume that on characteristic $\eta=\eta_1$ it is possible to select point N, different from E (see Fig. 38), such, that the limit $F_{\eta\eta}'$ with approach to it on the points of simple wave is zero. Let us examine the sequence of the rectilinear characteristics, which unlimitedly approach the characteristic, determined by equation $\eta'=\eta_1$, on which the acceleration is excellently from zero. This sequence always will be located that otherwise, according to lemma 3, flow

would be uniform. Let us find $F_{\eta\eta}$ on such characteristics. We record/fix the rectilinear characteristic of mentioned sequence, it is directed axis O_1z^* on velocity vector V^*_1 on it, and axis O_1y^*- so that this characteristic on plane $\xi^*\eta^*$ would be determined by equation $\eta^* = \eta_1^*$; then $\eta_1^* = (M_1^{*2} - 1)^{-1/2}$, where M^*_1 is a Mach number on this characteristic.

Communication/connection between the systems of coordinates O_1xyz and $O_1x^*y^*z^*$ is established by the relationship/ratios:

$$\left.\begin{aligned} x^* &= (1+\alpha_1^*)\,x + \beta_1^* y + \gamma_1^* z, \\ y^* &= \alpha_2^* x + (1+\beta_2^*)\,y + \gamma_2^* z, \\ z^* &= \alpha_3^* x + \beta_3^* y + (1+\gamma_3^*)\,z. \end{aligned}\right\} \tag{2.88}$$

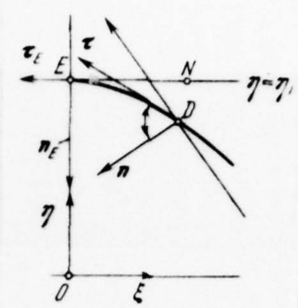


Fig. 38.

Page 95.

Here $\alpha_i^*, \beta_i^*, \gamma_i^*$ **are constant** which vanish (on the strength of the **continuity of the velocity** fields of the simple wave), when characteristic $\eta^* = \eta_1^*$ unlimitedly approaches a characteristic $\eta = \eta_1$.

From (2.88) by division x^* and y^* into z^* we obtain **communication/connection between** $\xi = x/z$, $\eta = \frac{y}{z}$ **and** $\xi^* = x^*/z^*$; $\eta^* = \frac{y^*}{z^*}$:

$$\xi^* = \frac{(1+\alpha_1^*)\xi + \beta_1^*\eta + \gamma_1^*}{\alpha_3^*\xi + \beta_3^*\eta + 1 + \gamma_3^*}, \quad \eta^* = \frac{\alpha_2^*\xi + (1+\beta_2^*)\eta + \gamma_2^*}{\alpha_3^*\xi + \beta_3^*\eta + 1 + \gamma_3^*}. \quad (2.89)$$

For the determination of communication/connection between F and F^* (F^* - the conical potential of simple wave in the system of coordinates $O_1x^*y^*z^*$) we will use by the facts that the projection of velocity vector on the radius-vector of point in space xyz is an

invariant during the rotation of the axes of the system. This projection is equal to $\frac{w+\eta v+\xi u}{\sqrt{1+\xi^2+\eta^2}}=\frac{F}{\sqrt{1+\xi^2+\eta^2}}$, whence follows the equality:

$$\frac{F}{\sqrt{1+\xi^2+\eta^2}}=\frac{F^*}{\sqrt{1+\xi^{*2}+\eta^{*2}}}$$

or

$$F=F^*\chi,\ \chi=\left(\frac{1+\xi^2+\eta^2}{1+\xi^{*2}+\eta^{*2}}\right)^{1/2}. \tag{2.90}$$

From (2.90) by differentiation with respect to η we obtain

$$F_{\eta_1\eta_1}=(F^*\chi)_{\eta^*\eta^*}\eta^{*2}_{\eta_1}+2(F^*\chi)_{\xi^*\eta^*}\xi^*_{\eta_1}\cdot\eta^*_{\eta_1}+ \\ +(F^*\chi)_{\xi^*\xi^*}\xi^{*2}_{\eta_1}+(F^*\chi)_{\eta^*}\eta^*_{\eta_1\eta_1}+(F^*\chi)_{\xi^*}\xi^*_{\eta_1\eta_1}. \tag{2.91}$$

Since on characteristic $\eta^*=\eta^*_1$ the acceleration not is equally identical to zero, $F^*_{\eta^*\eta^*}$ is expressed by formula (2.82):

$$F^*_{\eta^*\eta^*}=\left[\frac{\gamma+1}{2w^*_1}\frac{M^{*4}_1}{(M^{*2}_1-1)^2}-\text{const}^*\cdot\xi^*\right]^{-1},$$

where w^*_1 - speed on characteristic $\eta^*=\eta^*_1$, or $F^*_{\xi^*}=F^*_{\eta^*}=F^*_{\xi^*\eta^*}=F^*_{\xi^*\xi^*}=0$, $F^*=w^*+\eta^*v^*+\xi^*u^*=w^*_1$, when $\eta^*=\eta^*_1$.

If we fulfill differentiation in (2.91), to substitute there the values of derivatives of F^* and χ, and to express ξ^* in terms of formula (2.89), then as a result we will obtain

$$(F_{\eta_1\eta_1})_{\eta^*=\eta^*_1}=\left[\frac{\gamma+1}{2w^*_1}\frac{M^{*4}_1}{(M^{*2}_1-1)^2}- \right. \\ \left. -\text{const}^*\cdot\frac{(1+\alpha^*_1)\xi+\beta^*_1\eta+\gamma^*_1}{\alpha^*_3\xi+\beta^*_3\eta+1+\gamma^*_3}\right]^{-1}\chi\eta^{*2}_{\eta_1}+w^*_1\chi_{\eta_1\eta_1}. \tag{2.92}$$

Page 96.

From formulas (2.90), (2.89) we easily find that $\chi_{\eta_1} \to 0, \chi \to 1, \eta^*_{\eta} \to 1$, when $\alpha^*_i, \beta^*_i, \gamma^*_i \to 0$.

Let us examine now certain sequence of points (ξ; η) on rectilinear characteristics, with nonzero acceleration, convergent to point N on characteristic $\eta = \eta_1$.

Derivative $F_{\eta\eta}$ for such points is determined from formula (2.92), moreover $\alpha^*_i, \beta^*_i, \gamma^*_i \to 0$, when point ($\xi$, η) unlimitedly approaches point N. At point N $\xi_N \neq 0, \eta_N = \eta_1, F_{\eta\eta} = 0$ according to condition. On the strength of continuity $F_{\eta\eta}$ for the points of sequence $(\xi, \eta) F_{\eta\eta} \to 0$, when $(\xi; \eta) \to (\xi_N; \eta_N)$. From formula (2.92) it follows that this is possible only in such a case, when const* $\longrightarrow$ ∞, when $(\xi; \eta) \to (\xi_N, \eta_N)$ (is assumed that $M^*_1 \neq 1$). The points of the envelope of rectilinear characteristics will be located from the condition that the acceleration (i.e. $F_{\eta\eta}$) turns in them into

infinity. If we write the equation of characteristics $\eta^* = \eta_1^*$ in the form $\eta = (\eta_1 + \delta^*) + \varepsilon^*\xi$, where δ, $\varepsilon^* \to 0$, when $\alpha_i^*, \beta_i^*, \gamma_i^* \to 0$, then, solving the system of linear equations, comprised of equation $\eta = \eta_1 + \delta^* + \varepsilon^*\xi$ and the equation which is obtained by means of the equating of zero expression in the bracket in formula (2.92), let us find coordinates ξ_0, η_0 the points of the envelope of rectilinear characteristics. Formula for ξ_0 takes this form:

$$\left.\begin{aligned}\xi_0 &= \frac{\omega^* [\beta_3^*(\eta_1 + \delta^*) + 1 + \gamma_3^*] - [\beta_1^*(\eta_1 + \delta) + \gamma_1^*]}{1 + \alpha_1^* + \beta_1^*\varepsilon^* - \omega^*(\alpha_3^* + \beta_3^*\varepsilon^*)},\\ \omega^* &= \frac{1}{\text{const}^*}\cdot\frac{\gamma+1}{2w_1^*}\cdot\frac{M_1^{*2}}{(M_1^{*2}-1)^2}.\end{aligned}\right\} \quad (2.93)$$

From formulas (2.93) follows that when the characteristic $\eta = \eta_1^*$ unlimitedly approaches a characteristic $\eta = \eta_1$, i.e., $\alpha_i^*, \beta_i^*, \gamma_i^* \to 0$, const$^* \longrightarrow \infty$ and, therefore, $\omega^* \longrightarrow 0$, then $\xi_0 \longrightarrow 0$.

This means that the parabolic point E is maximum for the points of the envelope of the rectilinear characteristics of simple conical wave.

Since const$^* \longrightarrow \infty$, when $\alpha_i^*, \beta_i^*, \gamma_i^* \to 0$, then of formula (2.92) it follows that with approach to any point of the characteristic $\eta = \eta_1$, for which $\xi \neq 0$, $F_{\eta\eta} \to 0$.

Let us note still that from (2.92) follows the existence of such

sequences of points (ξ; η), convergent to point E, which during motion along them to point E, $F_{\eta\eta}$ approaches any preassigned number (since $F_{\eta\eta}$ on $\eta^* = \eta_1^*$ are taken the values from zero to infinity).

Page 97.

2. Let us assume that on characteristic $\eta = \eta_1$ it is possible to find point N, which does not coincide with point E (see Fig. 38) such, that limit $F_{\eta\eta}$ with approach to it on points of simple wave is not equal to zero and it is final. ($F_{\eta\eta}$ it cannot go to infinity on an entire characteristic $\eta = \eta_1$, as it follows from formula (2.92), since unlimitedly by approaching characteristic $\eta^* = \eta_1^*$ to characteristic $\eta = \eta_1$, not at which values of const* we will obtain on an entire characteristic $\eta = \eta_1$ $F_{\eta\eta} = \infty$.)

In this case $F_{\eta\eta}$ when $\eta = \eta_1$ it is located through formula (2.82):

$$(F_{\eta\eta})_{\eta=\eta_1} = \left[\frac{\gamma+1}{2w_1}\frac{M_1^4}{(M_1^2-1)^2} - \mathrm{const}\cdot\xi\right]^{-1}.$$

Since the derivative $F_{\eta\eta}$ with $\xi = 0$ is final, there is certain vicinity of point E (see Fig. 38), where there are no points of the envelope of the rectilinear characteristics of simple wave. Under these conditions takes place formula (2.87), and on the strength of continuity η' for point E can be written

$$\eta_E^* = -2(M_1^2-1)^{1/2}. \quad (2.94)$$

Vectors n and τ form in point E angle $\pi/2$ and are arrange/located in the manner that it is represented in Fig. 38, since axis O_1z is directed along velocity vector on characteristic $\eta = \eta_1$, and motion along curved characteristics DE is realized to the parabolic point E (" to the side of velocity"). If we designate angular coefficient of the rectilinear characteristic, passing through point D, by $k(\xi)$, then on the strength of the made assumptions relative to F the derivative $dk/d\xi = k'(\xi)$ exists and it is continuous, the same as $\frac{d^2\eta}{d\xi^2} = \eta''(\xi)$ for curved characteristics. At point D, arrange/located on characteristic DE it is sufficiently close to point E, on the strength of continuity n, τ, $\eta'(\xi)$ and negativity $\eta''(0)$ the condition of motion "to the side of velocity" ($\tau \cdot n > 0$) it means that $|k(\xi)| > |\eta'(\xi)|$, where $\eta'(\xi) = \frac{d\eta}{d\xi}$.

If we designate moving coordinates of the rectilinear characteristic by ξ^0, η^0, then the equation of the family of rectilinear characteristics can be written in the form $\eta^0 - \eta(\xi) = k(\xi)(\xi^0 - \xi)$ whence it is easy to obtain by differentiation of the equation of the envelope of this family in the parametric form:

$$\xi^0 - \xi = \frac{k(\xi) - \eta'(\xi)}{k'(\xi)}; \quad \eta^0 - \eta(\xi) = \frac{k(\xi) - \eta'(\xi)}{k'(\xi)} \cdot k(\xi). \quad (2.95)$$

Page 98.

Let us show that $k'(0) \neq 0$. This will **indicate according** to (2.95) that when $\xi \longrightarrow 0$, then $\xi^0 \rightarrow 0$, $\eta^0 \rightarrow \eta(0) = \eta_1$, i.e., the envelope of rectilinear characteristics **is passed through** point E. Actually,

$$|k'(0)| = \lim_{D\to E}\left|\frac{k(\xi_D) - k(\xi_E)}{\xi_D - \xi_E}\right| = \lim_{D\to E}\left|\frac{k(\xi_D)}{\xi_D}\right| >$$
$$> \lim_{D\to E}\left|\frac{\eta'(\xi_D)}{\xi_D}\right| = \lim_{D\to E}\left|\frac{\eta'(\xi_D) - \eta'(\xi_E)}{\xi_D - \xi_E}\right| =$$
$$= |\eta''(\xi_E)| = |\eta''(0)| = 2(M_1^2 - 1)^{1/2} \neq 0.$$

(At point E, $\xi = k = \eta' = 0$.)

The conclusion that the envelope of rectilinear characteristics is passed through point E, contradicts the initial conclusion/derivation about the absence of the envelope in certain vicinity point E, therefore, case 2 is impossible, and proof of theorem is completed.

Consequence. Under conditions of theorem 1 during the motion, initiated "to the side of velocity" along the curved characteristics of simple wave, passing through the fixed/recorded rectilinear characteristic, the encountered parabolic points, if the same generally exist, cannot form continuous parabolic line.

On the strength of theorem 1 on each rectilinear characteristic, passing through this parabolic point of curved characteristics, acceleration it turns into zero. If parabolic points formed continuous line, then simple wave would degenerate into uniform flow, which contradicts the conditions of theorem 1.

Not dwelling on other properties of the simple conical waves which can be found in work [9], let us demonstrate one additional important theorem. It is well known that in two-dimensional problem of gas dynamics to uniform flow can adjoin (without shock wave) only simple wave, moreover the ranges, occupied with these flows, are divided by rectilinear characteristic (Mach line). In the case of conical flows, the uniform flow can be connected with another flow on plane $\xi\eta$ along rectilinear characteristic or Mach cone, which is here simultaneously parabolic line and characteristic (by envelope of the rectilinear characteristics of uniform flow).

Theorem 2. If the conical potential F of the flow, which adjoins the range of uniform flow along rectilinear characteristic (on plane $\xi\eta$), possesses continuous second derivatives in terms of ξ and η, then this flow is simple wave.

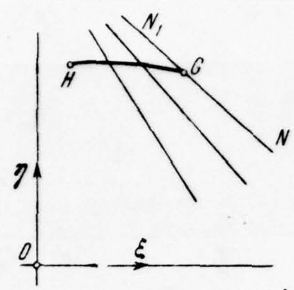


Fig. 39.

Page 99.

Let to uniform flow along the rectilinear characteristic NN_1 (Fig. 39) it adjoins certain conical flow, and the curve GH there is its curved characteristics, the point G not coinciding with parabolic point on characteristic NN_1. But to the values of the components of speed u, v, w in each point of the curve GH it is possible to find $d\eta/d\xi = k$ for the second characteristic direction of equation (1.29) and to construct simple wave, after requiring, so that u, v, w would retain their values on the appropriate straight lines with angular coefficients, equal k. The conical potential F of this simple wave possesses continuous second derivatives in terms of ξ and η and is the solution of the characteristic problem of Cauchy (Goursat's task) for equation (1.29) in which on characteristics HG, NG (Fig. 39) are

assigned the values $F = u\xi + v\eta + w,\ F_\xi = u,\ F_\eta = v,$ which satisfy the condition of strip dF = u dξ + v $d\eta$ and relaticnship/ratio $du + \left(\frac{B \mp \sqrt{B^2 - AC}}{A}\right)$ dv = 0, that is fulfilled on the characteristics of equation (1.29). The conical potential of the examined at first flow is also the solution of the formulated problem of Goursat, and on the strength of the uniqueness of the solution of this problem it coincides with F of simple wave (relative to Goursat's task see, for example, [13]).

2.18. Conical flows, which adjoin uniform flow along Mach cone. For the completion of the question concerning the conical flows which can adjoin the uniform flow, let us examine the flows, which adjoin the uniform flow along Mach cone [104]. If we direct axis O_1z along the speed of uniform flow V_1, then the Mach cone of this flow (with apex/vertex at point O_1) will be depicted on plane $\xi\eta$ as circumference $\xi^2 + \eta^2 = (M_1^2 - 1)^{-1}$, where $M_1 > 1$ is a Mach number of uniform flow. Subsequently it is convenient on plane $\xi\eta$ to introduce polar coordinates by the formulas

$$\left.\begin{aligned}\xi &= r\cos\theta,\\ \eta &= r\sin\theta.\end{aligned}\right\}$$

Then the equation of Mach cone he is written in the form $r = r_1 = (M^2_1 - 1)^{-1/2}$, and the components of speeds u, v, w along the axes of the Cartesian system will be determined from formulas (1.32):

$$u = \cos\theta \cdot F_r - \frac{\sin\theta}{r} F_\theta,\quad v = \sin\theta F_r + \frac{\cos\theta}{r} F_\theta,\ w = F - rF_r$$
$$\left(F_r = \frac{\partial F}{\partial r},\quad F_\theta = \frac{\partial F}{\partial \theta}\right).$$

Page 100.

The conical potential F satisfies equation (1.31)

$$L[F]=\{a^2(1+r^2)-[rF-(1+r^2)F_r]^2\}F_{rr}+ +2\left[F-\left(r+\frac{1}{r}\right)F_r\right]F_\theta\left(\frac{1}{r}F_{r\theta}-\frac{1}{r^2}F_\theta\right)+ +\left(a^2-\frac{1}{r^2}F_\theta^2\right)\left(\frac{1}{r^2}F_{\theta\theta}+\frac{1}{r}F_r\right)=0,$$

where the square of the speed of sound a^2 is expressed by the formula

$$a^2=a_1^2-\frac{\gamma-1}{2}\left[F_r^2+\frac{1}{r^2}F_\theta^2+(F-rF_r)^2-w_1^2\right] \quad (2.96)$$

(a_1, w_1 are speed of sound and the speed of uniform flow). Since axis O_1z is directed along the speed of uniform flow, for it u = v = 0, w = w_1, and from formula (1.32) follows that for the uniform flow

$$F_r=F_\theta=0, \quad F=F_1=w_1.$$

Thus, for the flows, which adjoin the uniform flow along Mach cone,

$$F_r=F_\theta=0,\ F=w_1 \text{ with } r=r_1. \quad (2.97)$$

Mach cone is the analog of direct/straight sonic line in two-dimensional problem of gas dynamics, since it is simultaneously the parabolic and characteristic line of equation (1.31). Let us assume that during, which adjoins the Mach cone, F has continuous third derivatives in terms of r and θ with r < r_1. Let us

differentiate equation (1.31) for r and fix r to r_1; within limit we will obtain equation for $(F_{rr})_{r=r_1}$:

$$\{(\gamma+1)\,r_1(1+r_1^2)\,w_1F_{rr}+2r_1(a_1^2-w_1^2)\}\,F_{rr}+\frac{a_1^2}{r_1}F_{rr}=0.$$

Hence it follows that

$$(F_{rr})_{r=r_1}=0 \text{ or } (F_{rr})_{r=r_1}=\frac{w_1}{\gamma+1}\frac{(M_1^2-1)^2}{M_1^4}. \quad (2.98)$$

Page 101.

The first value $(F_{rr})_{r=r_1}$ corresponds to uniform flow, the second - to solution of the elliptical type of equation (1.31), since with an accuracy to the positive factor of discriminant of equation (1.31) it is possible to present in the form

$$AC-B^2\sim F_r+\frac{2w_1}{\gamma+1}\frac{(M_1^2-1)^2}{M_1^4}(r_1-r)+\ldots$$

where in the dots are designated the terms of the higher order of smallness on to $r_1 - r$, than given. Hence, taking into account formula (2.98), it follows that

$$AC-B^2\sim\frac{w_1}{\gamma+1}\frac{(M_1^2-1)^2}{M_1^4}(r_1-r)+\ldots>0 \text{ for } r<r_1.$$

Since the solution to equation (1.31) in question belongs to elliptical type, it is analytical in the vicinity of Mach cone (see on this question, for example, [13]).

Repeated differentiation of equation (1.31) for r and passage to the limit r $\longrightarrow$ r_1 give:

$$\lim_{r\to r_1}[F_{rrr}P - a_1^2(M_1^2-1)^{1/2}F_{rrrr}(r-r_1)] + \alpha_1 = 0, \quad (2.99)$$

where

$$\alpha_1 = a_1^2 \cdot \frac{w_1}{(\gamma+1)^2}\frac{(M_1^2-1)^3}{M_1^8}[(\gamma+1)(2-M_1^2)-3(M_1^2-1)]\neq 0.$$

P = P(r, θ, F, $F_\theta, F_r, F_{rr}, F_{r\theta}, F_{\theta\theta}$) there is a polynomial from its variables; it it turns into zero, when r $\longrightarrow$ r_1.

Relationship/ratio (2.99) will be satisfied, if we assume that $\lim_{r\to r_1} F_{rrrr}$ (r_1 - r) = const; then F_{rrr} there will be order $\ln(r_1-r)$, and P - order (r_1 - r) ln (r_1 - r), and therefore $\lim_{r\to r_1} F_{rrr}$ P = 0. On the basis of these considerations, the solution to equation (1.31) we search for in the form

$$F = w_1 + \beta_1(r_1-r)^2 + \gamma_1(r_1-r)^3\ln(r_1-r) + R_1, \quad (2.100)$$

where letter R_1 designated the terms of the higher order of smallness relative to r_1 - r, than preceding/previous.

Page 102.

Substituting expression (2.100) in equation (1.31), we find β_1 and

γ_1:

$$\beta_1 = \frac{w_1}{2(\gamma+1)} \frac{(M_1^2-1)^2}{M_1^4}, \quad \gamma_1 = \frac{w_1}{6(\gamma+1)^2} \frac{(M_1^2-1)^2}{M_1^6} \times \times [3(M_1^2-1) - (\gamma+1)(2-M_1^2)]. \quad (2.101)$$

Let us now search for the dominant term of function R_1 in the form

$$R_1 = c(\theta)(r_1 - r)^3 + \ldots \quad (2.102)$$

With the substitution of expression (2.100), taking into account formula (2.101) and (2.102), into equation (1.31), we will obtain that by the first members of the lowest order on the left side of the equation (1.31) will be the members

$$O(r_1 - r), \quad O[(r_1 - r)^2 \ln (r_1 - r)], \quad O[(r_1 - r)^2],$$

appearing because of members, ertracted clearly in formulas (2.100) and (2.102). By cumbersome elementary calculation it is possible to show that terms $O(r_1 - r)$ are reduced as a result of the selection of coefficient β_1; terms $O[(r_1 - r)^2 \ln x (r_1 - r)]$ they are reduced at any values γ_1, $c(0)$; terms $O[(r_1 - r)^2]$, not containing $c(\theta)$, are reduced as a result of the selection of coefficient β_1; the remaining terms $O[(r_1 - r)^2]$ they are reduced during any selection of function $c(\theta)$, which indicates its arbitrariness. Finally we obtain, which in the vicinity of Mach cone $r = r_1$ F is represented by asymptotic expansion

$$F = w_1 + \beta_1 (r_1 - r)^2 + \gamma_1 (r_1 - r)^3 \ln (r_1 - r) + + c(\theta)(r_1 - r)^3 + \ldots, \quad (2.103)$$

where the coefficients β_1, γ_1 are determined from formulas (2.101), $c(\theta)$ - arbitrary analytic function θ. If we c by aid (2.103)

calculate with $r = r_1$ derivative in terms of r of the square of speed, then we will obtain the formula

$$(u^2+v^2+w^2)_r = -2\frac{w_1^2}{\gamma+1}\frac{(M_1^2-1)^{3/2}}{M_1^4}<0 \text{ with } r = r_1,$$

from which it follows that the flow in the vicinity of Mach cone $r = r_1$ is a flow of evacuation/rarefaction.

Page 103.

Since the solutions described by expansion (2.103), have functional arbitrariness, it is possible with large confidence to assume that other types of flows, which adjoin the uniform flow along Mach cone, do not exist. Let us note in conclusion that in the examined case the Mach cone with apex/vertex at point O_1 is constructed "downstream"; the case with the Mach cone, constructed "upstream", is analogous that which was examined, only flow in the vicinity of Mach cone will be the flow of compression, and this flow will be hyperbolic type flow for equation (1.31).

§3. Singular points of the solutions to the equations of conical flows.

3.1. Preliminary Remarks. The solutions to the equations of conical flows possess a series of the singular points whose existence it is necessary to consider both upon the formulation of the corresponding problems of flow and during the development of the methods of their solution (analytical and numerical). In present paragraph are examined the singular points of the exact solutions of the problems of flow; the special feature/peculiarities, which appear because of the use of appro imation methods, will be examined with the presentation of these methods.

3.2. Ferry's special feature/peculiarity. Greatest the important special feature/peculiarity of conical flows is the special feature/peculiarity of Ferri which was reveal/detected in connection with the solution of the problem of the flow about the round cone at an angle of attack [6]. As showed A. Ferri, on the lee side of cone, there is a point into which converge all lines constantly of entropy - flow line on single sphere or on plane $z = 1$ (plane $\xi\eta$).

Let us show that one or several Ferry's special feature/Peculiarities are in the solution of the problem of the nonseparated flow of almost any conical body. Let us assume that the streamlined body is such, that during the adequate/approaching

selection of the direction of axis O_1z in conical flow about body are satisfied the conditions

$$\rho(\xi, \eta) > \rho_e > 0, \quad w(\xi, \eta) > w_e > 0,$$

where ρ, w are density and the projection of velocity vector on axis O_1z; ρ_e, w_e — some constants.

Page 104.

Conditions $\rho_e \neq 0, w_e \neq 0$ mean that in flow there is no point where is reached maximum speed, and that the velocity vector not very strongly does change its direction. let us select now on plane z = 1 (plane $\xi\eta$) arbitrary locked duct, which partially lies on body surface. Through the points of this duct, let us conduct flow lines in space xyz. As a result we will obtain the tube of flow, limited by certain stream surface and by the surface of the streamlined body. Through each section of this tube with planes z = const per unit time, occur/flow/lasts one and the same mass of the gas

$$m = \iint\limits_{\Sigma(z)} \rho w \, dx \, dy,$$

where $\Sigma(z)$ — a sectional area of the tube of flow with plane z = const. Let us present m in the form $m = (\rho w)_{cp} \cdot \Sigma(z)$: the mark "sr" indicates average value in the section of the tube of flow with plane z = const. Hence it follows that

$$\Sigma(z) = \frac{m}{(\rho w)_{cp}} < \frac{m}{w_e \rho_e} < \infty.$$

On plane $\xi\eta$ the section of the tube of flow with plane z = const is depicted as domain with the area $\sigma(z)=z^{-2}\Sigma(z)$ which becomes how conveniently small with unlimitedly growing z. Let us designate the part of the boundary higher than named domain on plane $\xi\eta$, that consists of the points, which do not belong body surfaces, by curves $l_{(z)}$. begin and are terminated on body surface and the area, limited by these curves and body surface, $\sigma(z)$, it vanishes, when $z \rightarrow \infty$. On the other hand, the particle of gas in space xyz they move over conical stream surfaces, by sections of which plane z = 1 are flow lines on plane $\xi\eta$. from preceding/previous it follows that the flow lines on plane $\xi\eta$ intersect by the totality of the curves $l_{(z)}$, for which $\sigma(z) \longrightarrow 0$, $z \longrightarrow \infty$. This is possible only on the condition that the curves $l_{(z)}$ are confined with $z \longrightarrow \infty$ to point, either to curvilinear cut/section or to body surface. In the first case of flow line, they pass through this point, and here, consequently, is Ferry's special feature/peculiarity.

Page 105.

In the second case curvilinear cut/section consists of the points, which are traces from the intersection with plane z = 1 ray/beams,

which unlimitedly approach the flow lines in space xyz with z $\rightarrow \infty$. For such points x/u = y/v = z/w, i.e., $u-\xi w = v - \eta w = 0$. As it follows from point/item 1.3, on curvilinear cut/section $\frac{\partial p}{\partial \xi} = \frac{\partial p}{\partial \eta} = 0$ and it is isobar, if the components of velocity of the here continuously differentiated function, i.e., acceleration it is limited; the unlimitedness of acceleration on cut/section is not allow/assumed, since otherwise in flow must arise shock wave and flow it will be changed.

The conical flow in which there is a surface of constant pressure, which does not coincide with body surface, it is, if it generally exists, a very special case.

In the third case the flow must possess the piece-wise constant entropy S, since conical body surface is flow line and the line of constant entropy on plane $\xi\eta$; here S, piecewise constant function,. Since S is continuous in field of flow and the points of curves $l_{(z)}$ unlimitedly approach a body surface, S is piecewise constant function on flow lines, i.e., in field of flow.

Thus, Ferry's special feature/peculiarities are organically inherent in the solutions of the problems of the flow about the conical bodies when in flow form shock waves, i.e., entropy is different on different conical stream surfaces. In the irrotational

flows where S = const, there are points on the planes $\xi\eta$, which are junction/unit for flow lines, but they are not special for the solutions to equation (1.29).

Since the point where there is Ferry's special feature/peculiarity, is junction/unit for flow lines on plane $\xi\eta$ (to single sphere) and the equation of these lines is $\frac{d\xi}{u-\xi w}=\frac{d\eta}{v-\eta w}$, at this point must be fulfilled condition $u-\xi w=v-\eta w=0$, which can be written in the form x/u = y/v = z/w.

Page 106.

Thus, the necessary (but not sufficient) condition of the emergence of Ferry's special feature/peculiarity at point $(\xi_0; \eta_0)$ is the condition that the velocity vector on ray/beam in space xyz determined by values ξ_0, η_0, is directed along this ray/beam.

Let us study now the question concerning the structure of solution in the vicinity of the point where there is Ferry's special feature/peculiarity. Research let us conduct in spherical coordinates R, Θ, Φ (see point/item 1.2) with the aid of equations (1.1) - (1.5), (1.8), (1.15). Let us assume that on single sphere Ferry's special feature/peculiarity is determined by coordinates $\theta=\theta_0$, $\Phi=0$ [coefficients of equations (1.1)-(1.5) do not depend on Φ, therefore

the position of reference point Φ does not have a value]. According to equation (1.5) the differential equation of the lines of constant entropy takes the form

$$\frac{d\theta}{v}=\frac{\sin\theta\, d\Phi}{w}. \quad (3.1)$$

Point (θ_0, 0) must be junction/unit for the integral curves of equation (3.1); therefore here $v = w = 0$ ($u = u_0 \neq 0$). In order to move it is further, it is necessary to make supplementary assumptions about behavior v and w in the vicinity of point (θ_0, θ).

In work [105] the coordinate system is selected so that $\theta_0 = 0$, and it is assumed that the parameters of gas in the vicinity of the point (0; 0) are represented by the power series of the form

$$f(\theta,\Phi)=f_0(\Phi)+\theta f_1(\Phi)+\theta^2 f_2(\Phi)+\ldots$$

The lines of constant entropy in this case enter in point (0; 0), where there is Ferry's special feature/peculiarity, in different directions.

In book [29] it is assumed that $\theta = \theta_0 \neq 0$ and in the vicinity of point (θ_0; 0) v and $w/\sin\theta$ it is possible to present in the form

$$\left.\begin{aligned} v&=(\alpha^{\times}\vartheta+\beta^{\times}\Phi)\,g(\vartheta,\Phi)+g_1(\vartheta,\Phi),\\ \frac{w}{\sin\theta}&=(\gamma^{\times}\vartheta+\delta^{\times}\Phi)\,g(\vartheta,\Phi)+g_2(\vartheta,\Phi),\end{aligned}\right\} \quad (3.2)$$

where $\alpha^{\times}, \beta^{\times}, \gamma^{\times}, \delta^{\times}$ are some constants, $\vartheta = \theta - \theta_0$, g - the bounded function derivatives by which satisfy conditions $g_{\vartheta}=O(|\vartheta|^{-1}),\quad g_{\Phi}=O(|\Phi^{-1}|)$.

and functions g_i (i = 1, 2) such, that $g_i = O(\vartheta^2 + \Phi^2)$, $g_{i\vartheta} = O\left(\frac{\vartheta^2 + \Phi^2}{|\vartheta|}\right)$, $g_{i\Phi} = O\left(\frac{\vartheta^2 + \Phi^2}{|\Phi|}\right)$.

Page 107.

According to equation (3.1) the lines of constant entropy in vicinity of point (θ_0; 0) are in this case the integral curves of the equation

$$\frac{d\vartheta}{\alpha^{\times}\vartheta + \beta^{\times}\Phi} = \frac{d\Phi}{\gamma^{\times}\vartheta + \delta^{\times}\Phi}. \qquad (3.3)$$

Although the assumptions of the authors [29] bear more general character, than the assumptions, made in work [105], results for the dominant terms of solution in both works are actually identical.

For this reason, and also because the assumptions made in [29], all the same do not cover all possible cases, let us accept further precisely those assumptions which make it possible to obtain interesting us the type of solution for Ferry's special feature/peculiarity. Let us assume in formulas (3.2)

$$\alpha^{\times} = \delta^{\times} = 1, \quad \beta^{\times} = \gamma^{\times} = 0,$$

then

$$v = \vartheta \cdot g + g_1, \quad \frac{w}{\sin\theta} = \Phi g + g_2. \qquad (3.4)$$

From the equation of continuity (1.1), written in the form (1.15), follows the relationship/ratio

$$v_\theta + \frac{1}{\sin\theta} w_\Phi + 2u = O(\varkappa). \qquad (3.5)$$

where $\varkappa^2 = \Phi^2 + \vartheta^2$, or on the basis (3.4)

$$\vartheta g_\vartheta + \Phi g_\Phi + 2g + 2u = O(\varkappa). \tag{3.6}$$

Equation (1.2) will draw the equality

$$g(\vartheta u_\vartheta + \Phi u_\Phi) = O(\varkappa^2).$$

Page 108.

Therefore $u = f + O(\varkappa)$, where f > 0, satisfies the equation

$$\vartheta f_\vartheta + \Phi f_\Phi = 0,$$

general solution of which it is given by the formula

$$f = f(\zeta), \quad \zeta = \frac{\Phi}{\vartheta}, \tag{3.7}$$

where f(ζ) - the arbitrary function of its argument.

Since function g is determined with an accuracy to value $O(\varkappa)$, equation (3.6) will be satisfied, if we g subordinate to the condition

$$\vartheta g_\vartheta + \Phi g_\Phi + 2g + 2f = 0. \tag{3.8}$$

Equation (3.8) has the particular solution:

$$g = -f. \tag{3.9}$$

The solutions to the homogeneous equation

$$\vartheta g_\vartheta + \Phi g_\Phi + 2g = 0$$

are not limited with $\vartheta^2 +$ $\Phi^2 \to 0$.

Actually, along the characteristics of this equation, are fulfilled the relatio ship/ratios

$$\frac{d\vartheta}{\vartheta}=\frac{d\Phi}{\Phi}=-\frac{dg}{2g},$$

whence follows ϑ = const• Φ, g = const• Φ^{-2} and g $\longrightarrow$ ∞, when $\Phi\to 0$.

Since according to condition g - bounded function,

$$g=-f. \qquad (3.10)$$

From equations (1.3), (1.4) and formulas (3.4) it follows that

$$\frac{\partial p}{\partial\vartheta}=O(\varkappa), \quad \frac{\partial p}{\partial\Phi}=O(\varkappa),$$

i.e. the first derivatives of p are continuous at point (θ_0: 0). Let $p_0 = p_0$ (θ_0: 0); then $p=p_0+O(\varkappa^2)$. From (1.8) we obtain $i(p,\rho)=i_0-\frac{f^2}{2}+O(\varkappa^2)$, and if one assumes that i(p, ρ) sufficiently smooth function p and ρ,

$$\rho=\rho_0\left(i_0-\frac{f^2}{2}\right)+O(\varkappa^2). \qquad (3.11)$$

where ρ_0 is a sufficiently smooth function of its argument.

Page 109.

Utilizing (3.4), (3.10), we have

$$vv_\theta + \frac{w}{\sin\theta} v_\Phi = g^2\vartheta + O(\varkappa^2),$$
$$v\left(\frac{w}{\sin\theta}\right)_\theta + \frac{w}{\sin\theta}\left(\frac{w}{\sin\theta}\right)_\Phi = g^2\Phi + O(\varkappa^2),$$

therefore from equations (1.3), (1.4) taking into account (3.11) we will obtain

$$\frac{\partial p}{\partial\theta} = O(\varkappa^2), \quad \frac{\partial p}{\partial\Phi} = O(\varkappa^2),$$

since all terms $O(\varkappa)$ in (1.3), (1.4) average out. This fact indicates that function f(ξ) - is arbitrary. Furthermore, from the more precise estimation of derivatives of p we obtain

$$p = p_0 + O(\varkappa^3). \qquad (3.12)$$

Thus, in the vicinity of the point where there is Ferry's special feature/peculiarity, the components of velocity, pressure and density can be represented in the form

$$\left.\begin{aligned} u &= f(\zeta) + O(\varkappa), \\ v &= -(\theta-\theta_0) f(\zeta) + O(\varkappa^2), \\ w &= -\Phi\sin\theta_0 f(\zeta) + O(\varkappa^2), \\ p &= p_0 + O(\varkappa^3), \\ \rho &= \rho_0\left(i_0 - \frac{f^2}{2}\right) + O(\varkappa^2), \end{aligned}\right\} \qquad (3.13)$$

where

$$\zeta = \frac{\Phi}{\theta-\theta_0}, \quad \varkappa^2 = \Phi^2 + (\theta-\theta_0)^2, \quad p_0 = \text{const}.$$

From formulas (3.13) we find that in the vicinity of the point where there is Ferry's special feature/peculiarity,

$$\frac{\partial(v, w)}{\partial(\theta, \Phi)} = \sin\theta_0 f^2(\zeta) + O(\varkappa),$$

whence it follows that independent of the method of

approach/approximation to this point all the limiting values of jacobian are positive, i.e.,

$$\lim \frac{\partial (v, w)}{\partial (\theta, \Phi)} > 0, \quad \theta \to \theta_0, \Phi \to 0.$$

Page 110.

This condition, along with condition u = w = 0, $\theta = \theta_0$, $\Phi = 0$, is necessary and sufficient for the existence of Ferry's special feature/peculiarities (if, cf course, in the vicinity of point $\theta = \theta_0$, $\Phi = 0$ is realized type examined earlier of solution), since it is easy to show that at the point, which is saddle for a flow line on single sphere, is fulfilled the inequality

$$\lim \frac{\partial (v, w)}{\partial (\theta, \Phi)} < 0; \quad \theta \to \theta_0, \Phi \to 0,$$

if v, w are continuously differentiable.

3.3. special feature/peculiarity at point of attenuation of shock wave on Mach cone. If the streamlined body (for example, triangular plate at an angle of attack) is such, that from its one side the flow is expanded, and on the other hand - it is compressed, moreover the edge of body subsonic, then leading shock wave either wholly covers body or the domain, agitated by the streamlined body, is limited by partially shock wave and the Mach cone of undisturbed flow. The last/latter case is represented by more adequate/approaching from physical considerations, but for the

substantiation of the possibility of its realization, it is necessary to construct the solution in the vicinity of the ray/beam where are connected Mach cone and shock wave. Here must be formed singular point on plane $\xi\eta$ (special ray/beam in space xyz). Will be further constructed the solution of system of equations (1.17)-(1.20) in the vicinity of this singular point. With the presentation of question, we will follow [106], after correcting the available there inaccuracies.

It is oriented the axis of the system of coordinates O_1xyz so that the axis O_1z would be directed along speed V_1 of undisturbed flow, and plane O_1yz would contain the ray/beam along which are connected the Mach cone and shock wave. Then Mach cone on plane $\xi\eta$ is depicted as the circumference whose equation in polar plane coordinates $\xi\eta$ is $r = r_1 = (M_1^2 - 1)^{-1/2}$, where M_1 is a Mach number of uniform flow, but ray/beam mentioned above - by point with coordinates $\xi = 0$, $\eta = \eta_1 = r_1$. In Fig. 40 circular arc 2 - 1 - 2' depicts Mach cone, curved 1 - 3 - shock wave; at point 1, occurs their connection.

For the determination of solution in the vicinity of point 1, we will use asymptotic expansion (2-103) for the conical potential F in the vicinity of Mach cone 1 - 2:

$$F = w_1 + \beta_1(r_1 - r)^2 + \gamma_1(r_1 - r)^3 \ln (r_1 - r) + c\,(0)(r_1 - r)^3 + \dots$$

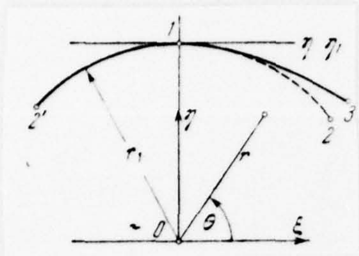


Fig. 40.

Page 111.

Since searches for the solution with special feature/peculiarity at point 1, let us assume that $c(\theta)$ has when $\theta = \theta_1 = \pi/2$ a pole of multiplicity p, i.e.,

$$c(\theta) = \frac{\nu}{\vartheta^p} + \frac{\mu}{\vartheta^k} + \ldots, \qquad (3.14)$$

where $\vartheta = \theta - \theta_1$, $p > k > 0$, ν, μ - constant. In the vicinity of point 1, let us introduce new coordinates ρ and ϑ by the formulas

$$\rho = (r_1 - r)\,\vartheta^{\,k},\ \vartheta = \theta - \theta_1. \qquad (3.15)$$

From (2.103), taking into account (3.14), it follows that

$$F = w_1 + (\nu\rho^3 + \ldots)\,\vartheta^{3k-p} + (\beta_1\rho^2 + \mu\rho^3 + \ldots)\,\vartheta^{2k} + \ldots, \qquad (3.16)$$

i.e. the expansion for F in the vicinity of point 1 takes the form

$$F = w_1 + R(\rho)\,\vartheta^{3k-p} + F(\rho)\,\vartheta^{2k} + \ldots, \qquad (3.17)$$

moreover with $\rho \longrightarrow 0$ $R(\rho) \sim \nu\rho^3$, $F(\rho) \sim \beta_1\rho^2 + \mu\rho^3$. Substituting expansion (3.17) in equation (1.31), equalizing the coefficients with the smallest degree ϑ, we will obtain that the only possible value for p with p > k is p = 2, and then 1 < k < 2. If we instead of $R(\rho)$ introduce the new unknown

$$\Omega(\rho) = (1 + r_1^2) r_1^3 (\gamma + 1) w_1 a_1^{-2} R(\rho),$$

from $\Omega(\rho)$ satisfies the equation

$$(k^2\rho^2 - \Omega')\,\Omega'' - 5k(k-1)\,\rho\Omega' + 3\,(k-1)(3k-2)\,\Omega = 0 \quad (3.18)$$

($\Omega' = d\Omega/d\rho$, $\Omega'' = d^2\Omega/d\rho^2$), moreover with $\rho \longrightarrow 0$ $\Omega \sim \rho^3$. The unknown solutions to equation (3.18) with small ρ are represented by the expansion

$$\Omega(\rho) = \frac{1}{3}\rho^3 + \text{const}\cdot\rho^{3+\frac{1}{k-1}} + \ldots \quad (3.19)$$

These solutions correspond to the flow about the bent walls, which intersect Mach cone. For example, with k = 3/2 is obtained solution for a wall with final curvature at point 1; it takes the form

$$\Omega(\rho) = c^{-3} v(t), \qquad t = c\rho,$$

$$v(t) = -\frac{9}{10}\left[\left(\sqrt{1+t^3}+t\right)^{1/3} - \left(\sqrt{1+t^3}-t\right)^{1/3}\right]^3 + \frac{27}{20}\,t\left[\left(\sqrt{1+t^3}+t\right)^{1/3} - \left(\sqrt{1+t^3}-t\right)^{1/3}\right]^2,$$

where c is an arbitrary constant.

Page 112.

Solution of the type p > k for the construction of singular point 1 is not approached therefore let us assume that $\nu = 0$, i.e.

$R(\rho) \equiv 0$. Substituting (3.17) in (1.31) we will obtain under this condition that $k \geqslant 2$. Let us examine case of $k > 2$, which will be used further. Since $c(\theta)$ - arbitrary function, let us select expansion for $c(\theta)$ in the form

$$c(\theta)=\frac{\mu}{\vartheta^{k}}+\frac{\lambda}{\vartheta^{2}}+\varepsilon \ln \vartheta+\nu_{1}+\ldots, \tag{3.20}$$

μ, λ, $\varepsilon, \nu_1, \ldots$ - arbitrary constants. After passage to variables ρ and ϑ from (2.103), taking into account (3.20), we will obtain

$$F=w_{1}+F(\rho) \vartheta^{2k}+\Phi(\rho) \vartheta^{3k-2}+T(\rho) \vartheta^{3k} \ln \vartheta+R_{1}(\rho) \vartheta^{3k}+\ldots, \tag{3.21}$$

moreover with small ρ

$$\begin{aligned} & F(\rho)=\beta_{1} \rho^{2}+\mu \rho^{3}+\ldots, \quad \Phi(\rho)=\lambda \rho^{3}+\ldots, \\ & T(\rho)=(\gamma_{1} k+\varepsilon) \rho^{3}+\ldots, \quad R_{1}(\rho)=\gamma_{1} \rho^{3} \ln \rho+\nu_{1} \rho^{3}+\ldots \end{aligned} \tag{3.22}$$

After substitution (3.21) in (1.31) and the equating of the coefficients with identical degrees ϑ and $\ln\vartheta$, we obtain equations for $F(\rho)$ so forth:

$$F''\left[2 r_{1}^{2}\left(w_{1}^{2}-a_{1}^{2}\right) \rho-\left(1+r_{1}^{2}\right) w_{1} r_{1}^{2}(\gamma+1) F'\right]-a_{1}^{2} F'=0, \tag{3.23}$$

$$\begin{aligned} \Phi''\left[2 r_{1}^{2}\left(w_{1}^{2}-a_{1}^{2}\right) \rho-\left(1+r_{1}^{2}\right) w_{1} r_{1}^{2}(\gamma+1) F'\right] & - \\ -\left(1+r_{1}^{2}\right) w_{1} r_{1}^{2}(\gamma+1) F'' \Phi'-a_{1}^{2} \Phi' & = \\ =-\frac{a_{1}^{2}}{r_{1}}\left[2 k(k-1) F-k(3 k-1) \rho F'+k^{2} \rho^{2} F''\right], & \end{aligned} \tag{3.24}$$

so forth.

All these equations can be integrated in the locked form, and their solutions satisfy conditions (3.22), for example,

$$F(\rho)=\frac{2 \beta_{1}}{3 c^{2}}\left[(1+2 c \rho)^{3/2}-3 c \rho-1\right], \tag{3.25}$$

where c is an arbitrary constant.

For future reference will be required only asymptotic formulas for these functions with large ρ. They, as it is easy to establish/install from equations (3.23) and so forth, take the form

$$F(\rho) = \frac{8\beta_1}{3\sqrt{2c}}\rho^{3/2} + \ldots,\quad \Phi(\rho) = -\frac{2\beta_1}{15\sqrt{2c}}\rho^{5/2}\frac{k(k-2)}{r_1} + \ldots,$$
$$T(\rho) \sim \rho^{3/2},\ R_1 \sim \rho^{5/2} \text{ (or } \rho^3 \text{ with } k = 4\text{, and so forth).} \tag{3.26}$$

Page 113.

Let us assume now that the values $\rho = (r_1 - r)\vartheta^{-k}$ are great, but values $\zeta = (r_1 - r)r_1^{-1}\vartheta^{-2}$ are final, and let **us pass from** variables ρ, ϑ to **variables** ζ, ϑ. Under the indicated conditions subsequent members of expansion (3.21) stop the same order, as preceding/previous; therefore it is necessary to re-group the terms of this series, utilizing formulas (3.26). **As a result we will obtain**

$$F = w_1 + g\left[\zeta^{3/2} - \frac{k(k-2)}{20}\zeta^{5/2} + \ldots\right]\vartheta^{\frac{k}{2}+3} + \ldots, \tag{3.27}$$

where $g = \dfrac{8\beta_1 r_1^{3/2}}{3\sqrt{2c}} > 0$. **From** (3.27) it follows that with $\rho \longrightarrow \infty$ and final ζ **F is represented** by the expansion

$$F = w_1 + \chi(\zeta)\vartheta^{\frac{k}{2}+3} + \ldots, \tag{3.28}$$

moreover with small ζ

$$\chi(\zeta) = g\left[\zeta^{3/2} + \frac{k(k-2)}{20}\zeta^{5/2} + \ldots\right]. \tag{3.29}$$

Substituting (3.28) in (1.31), we will obtain for $\chi(\zeta)$ the equation

$$2\zeta(1+2\zeta)\chi''+[1+2\zeta(k+3)]\chi'+\left(\frac{k}{2}+2\right)\left(\frac{k}{2}+3\right)\chi=0. \quad (3.30)$$

(This equation is reduced to hypergeometric by replacement $2\zeta = -t$).

Substituting (3.29) in (3.30), we are convinced that (3.29) is the solution of (3.30) with small ζ, where for k = 0, 2, 4, 6 series (3.29) breaks itself and solution is obtained in the locked form:

$$\chi(\zeta)=g\left[\zeta^{k/4}-\frac{k(k-2)}{20}\zeta^{k/4}\right]. \quad (3.31)$$

For other k, on the basis of (3.30), is easy to find asymptotic expression for χ at large ζ:

$$\chi(\zeta)\sim\zeta^{\frac{k}{4}+\frac{3}{2}} \text{ or } \chi(\zeta)\sim\zeta^{\frac{k}{4}+1}. \quad (3.32)$$

Let us examine case of k = 6, which we utilize for the construction of solution in the vicinity of point 1. For k = 6, expansion (3.28) takes the form

$$F=w_1+\chi(\zeta)\theta^6+\ldots \quad (3.33)$$

Substituting (3.33) in (1.31), it is easy to perceive, that the further terms of a series (3.33) they must be record/written as follows:

$$F=w_1+\chi(\zeta)\theta^6+\Gamma(\zeta)\theta^8+\Psi(\zeta)\theta^{10}+\ldots \quad (3.34)$$

Page 114.

Functions $\Gamma(\zeta)$, $\Psi(\zeta)$, ... satisfy the equations

$$2\zeta(2\zeta+1)\Gamma''-(1+26\zeta)\Gamma'+56\Gamma=5\zeta^2\chi''-\zeta\chi'+\frac{1}{2\beta_1 r_1^2}\chi'\chi'', \quad (3.35)$$

$$\begin{aligned}2\zeta(2\zeta+1)\Psi''-(1+34\zeta)\Psi'+90\Psi=\\=\chi''\left\{\left[2\frac{w_1}{a_1^2}r_1^2+(1+r_1^2)\frac{w_1}{a_1^2}(\gamma+1)\right]\chi-\frac{2w_1}{a_1^2}(\gamma+1)r_1^2\zeta\chi'+\right.\\\left.+\frac{w_1}{a_1^2}(1+r_1^2)(\gamma+1)(\Gamma'-\zeta\chi')\right\}+\Gamma''\left[\zeta^2+(\gamma+1)(1+r_1^2)\frac{w_1}{a_1^2}\chi'\right]-\\-2\zeta(56\Gamma-26\zeta\Gamma'+4\zeta^2\Gamma'')+\left[(\gamma-1)\frac{w_1}{a_1^2}\chi'-3\zeta^2\right](\chi'-2\zeta\chi'')+\\+\zeta\Gamma'+\zeta^2\chi'-\frac{8w_1}{a_1^2}(3\chi-\zeta\chi')(\zeta\chi''-2\chi'), \quad (3.36)\end{aligned}$$

so forth.

Since series (3.34) represents F with final ζ and ϑ, but series (3.21) - with final ρ and ϑ, after the regrouping of the terms of series (3.21) for $\rho \longrightarrow \infty$, $\vartheta < \infty$ from it must be obtained a series (3.34). Simple computations show that the first terms of series (3.34) are written correctly and that with small ϑ

$$\left.\begin{aligned}\chi(\zeta)=\frac{8\beta_1 r_0^{3/2}}{3\sqrt{2c}}\zeta^{3/2}+\ldots,\quad \Gamma(\zeta)=-\frac{2\beta_1 r_0}{c}\zeta+\ldots,\\ \Psi(\zeta)=\frac{\beta_1\sqrt{2}r_1^{1/2}}{c^{3/2}}\zeta^{1/2}+\ldots\end{aligned}\right\} \quad (3.37)$$

Equation (3.35) after substitution χ will take this form:

$$\begin{aligned}2\zeta(2\zeta+1)\Gamma''-(1+26\zeta)\Gamma'+56\Gamma=\\=\frac{9}{16}\frac{g^2}{r_1^2\beta_1}(1-8\zeta+12\zeta^2)+\frac{3}{4}g(3\zeta^{3/2}+26\zeta^{5/2}). \quad (3.38)\end{aligned}$$

One Solution to homogeneous equation for Γ easily is located in the locked form

$$\Gamma_{(1)}(\zeta)=F\left(-\frac{7}{2},-4,-\frac{1}{2},-2\zeta\right)= \\ =1+56\zeta-840\zeta^2+1120\zeta^3-112\zeta^4, \quad (3.39)$$

where symbol F(a, b, c, z) indicates hyper-geometric series; another solution can be written in the form

$$\Gamma_{(2)}=\Gamma_{(1)}\int\limits_0^\zeta (2\zeta)^{1/2}(1+2\zeta)^6\,\Gamma_{(1)}^{-2}\,d\zeta. \quad (3.40)$$

Page 115.

From (3.40) it follows that $\Gamma_{(2)}\sim\zeta^{3/2}$ with $\zeta\longrightarrow 0$ and $\Gamma_{(2)}\sim\zeta^{1/2}$ when $\zeta\longrightarrow\infty$. It is easy to also find the particular solution to nonhomogeneous equation (3.38). It takes the form

$$\Gamma_H=\frac{3}{448}\frac{g^2}{r_1^2\beta_1}(1-28\zeta+84\zeta^2)+\frac{g}{4}(74\zeta^{3/2}-13\zeta^{5/2}). \quad (3.41)$$

Since Γ with $\zeta\longrightarrow 0$ must satisfy condition (3.37), combining (3.41) with (3.39) and adding (3.40), we obtain necessary type solution in the form

$$\Gamma=\frac{3}{448}\frac{g^2}{r_1^2\beta_1}(112\zeta^4-1120\zeta^3+924\zeta^2-84\zeta)+ \\ +\frac{g}{4}(74\zeta^{3/2}-13\zeta^{5/2})+\mathrm{const}\cdot\Gamma_{(2)}(\zeta). \quad (3.42)$$

As it will be shown further, for the determination of the dominant term Γ when $\zeta\longrightarrow\infty$ it is not required to know all the solution $\Psi(\zeta)$; therefore in equation (3.36) in right side let us leave only greatest member: this will be:

$$\zeta^2\Gamma''-2\zeta(56\Gamma-26\zeta\Gamma'+4\zeta^2\Gamma'')+\zeta\Gamma'-\frac{8w_1}{a_1^2}(3\chi-\zeta\chi')(\zeta\chi''-2\chi')= \\ =g^2\left(\frac{36}{5}\frac{w_1}{a_1^2}+\frac{42}{r_1^2\beta_1}\right)\zeta^4+\ldots \quad (3.43)$$

From formula (3.43) it follows that any solution to equation (3.36) when $\zeta \longrightarrow \infty$ will contain the term

$$g^2\left(\frac{18}{5}\frac{w_1}{a_1^2}+\frac{21}{r_1^2\beta_1}\right)\zeta^4. \quad (3.44)$$

The solution to homogeneous equation for Ψ when $\zeta \longrightarrow \infty$ takes the form

$$\Psi \sim \tilde{l}_2\zeta^5+\tilde{l}_3\zeta^{9/2},$$

therefore the solution to equation (3.36) when $\zeta \longrightarrow \infty$ takes the form

$$\Psi(\zeta)=\tilde{l}_2\zeta^5+\tilde{l}_3\zeta^{9/2}+g^2\left(\frac{18}{5}\frac{w_1}{a_1^2}+\frac{21}{r_1^2\beta_1}\right)\zeta^4+\ldots, \quad (3.45)$$

where $\tilde{l}_2, \tilde{l}_3$ are some constants. From (3.42) it follows that when $\zeta \longrightarrow \infty$

$$\Gamma(\zeta)=\frac{3}{4}\frac{g^2}{r_0^2\beta_1}(\zeta^4-10\zeta^3)+\tilde{l}_1\zeta^{7/2}+\ldots, \quad (3.46)$$

where $\tilde{l}_1$ - is constant.

Page 116.

Transfer/converting when $\zeta \longrightarrow \infty$ in expansion (3.34) to alternating/variable r and ϑ, taking into account formulas (3.31), (3.45), (3.46), after the regrouping of terms we will obtain

$$F = w_1 + \left[\frac{3}{4}\frac{h^2}{\beta_1 r_1^3}(r_1 - r)^4 + l_2(r_1 - r)^5 + \ldots\right] +$$
$$+ \vartheta\left[-\frac{6h}{5r_1}(r_1 - r)^{5/2} + l_1(r_1 - r)^{7/2} + l_3(r_1 - r)^{9/2} + \ldots\right] +$$
$$+ \vartheta^2\left[-\frac{15}{2}\frac{h^2}{r_1^2\beta_1}(r_1 - r)^3 + \frac{h^2}{r_1}\left(\frac{18w_1}{5a_1^2} + \frac{21}{r_1^2\beta_1}\right)(r_1 - r)^4 + \ldots\right] +$$
$$+ \vartheta^3[h(r_1 - r)^{3/2} + \ldots] + \ldots, \quad (3.47)$$

where $h = gr_1^{-3/2}$, l_j are constant. From formula (3.47) it is evident that the terms with l_j do not enter in dominant terms for F, F_r, F_θ. Subsequently let us be interested only by dominant terms of solution; therefore let us extract only those terms which determine the dominant terms of potential F and of its first and second derivatives in terms of r and 0 in the vicinity of point 1:

$$F = w_1 + \frac{3}{4}\frac{h^2}{\beta_1 r_1}(r_1 - r)^4 - \frac{6h}{5r_1}(r_1 - r)^{5/2}\cdot\vartheta +$$
$$+ h^2\left[-\frac{15}{2}\frac{1}{r_1^2\beta_1}(r_1 - r)^3 + \frac{1}{r_1}\left(\frac{18w_1}{5a_1^2} + \frac{21}{r_1^2\beta_1}\right)(r_1 - r)^4 + \ldots\right]\vartheta^2 + \ldots \quad (3.48)$$

Expression (3.48) is determined F with $\vartheta > 0$, $r_1 - r > 0$ and the condition that with $\vartheta \longrightarrow 0$ $\vartheta^2(r_1 - r)^{-1} \longrightarrow 0$.

Let us check final the correctness of formulas (3.47), (3.48). According to (3.47) F in the vicinity of the straight line ϑ = of 0, it is represented by the expansion

$$F = F_0(r) + F_1(r)\vartheta + F_2(r)\vartheta^2 + \ldots, \quad (3.49)$$

moreover with small $r_1 - r$

$$F_0(r) = \frac{3}{4}\frac{h^2}{\beta_1 r_1}(r_1 - r)^4 + \ldots, \qquad F_1(r) = -\frac{6h}{5r_1}(r_1 - r)^{5/2} + \ldots, \quad (3.50)$$

$$F_2(r) = -\frac{15}{2}\frac{h^2}{r_1^2\beta}(r_1 - r)^3 + \frac{h^2}{r_1}\left(\frac{18w_1}{5a_1^2} + \frac{21}{r_1^2\beta_1}\right)(r_1 - r)^4 + \ldots \quad (3.51)$$

Page 117.

Page 117.

Since the straight line $\vartheta = 0$ is not the characteristic of equation (1.31), solution in its vicinity will be determined, if with $\vartheta = 0$ they will be assigned F and F_ϑ, i.e., $F_0(r)$, $F_1(r)$; specifically, unambiguously will be determined

$$F_2(r) = \left(\frac{1}{2} F_{rr}\right)_{\vartheta=0}.$$

Substituting expansion (3.49) in equation (1.31) and equalizing to zero members, who do not contain ϑ, we will obtain formula for $F_2(r)$, from which it follows that if with small $r_1 - r$ $F_0(r)$, $F_1(r)$ they are assigned by formulas (3.50), then $F_2(r)$ really/actually it is determined from formula (3.51). Now it is necessary to construct the solution in the vicinity of point 1 with $\vartheta < 0$, and then to produce the coupling of solutions, valid with $\vartheta > 0$ and with $\vartheta < 0$ in the vicinity of the straight line $\vartheta = 0$.

Let us assume that the equation of shock wave 1 - 3 (see Fig. 39) in the vicinity of point 1, it is possible to write in the form

$$r = r_*(\vartheta) = r_1 - \rho_1\vartheta^6 + \ldots,$$

where $\rho_1 < 0$ is unknown constant. If condition on shock wave (1.47), (1.48) to convert to polar coordinates, then of (1.48) it follows:

$$s_2 - s_1 = d_1\vartheta^{18} + \ldots, \qquad d_1 = -\frac{16}{3}\frac{\rho_1^3}{(\gamma+1)^3}\frac{(M_1^2-1)^{9/2}}{M_1^6}, \quad (3.52)$$

i.e. entropy in the vicinity of point 1 is changed little. For this

reason let us search for the irrotational solution, which satisfies conditions on Mach cone and conditions (1.47) on shock wave. The dominant term of this solution, as it will be shown further, is the dominant term of solution for the speeds of vortex/eddy problem. Conditions on shock wave (1.47) in vicinity $\vartheta = 0$ can be represented in the form (see point/item 1.9)

$$F_2 = F_1 = w_1; \quad (F_r)_2 = -\frac{4w_1}{\gamma+1} \cdot \frac{(M_1^2-1)^2}{M_1^4} \rho_1 \vartheta^6 + \ldots \quad (3.53)$$

(w_1 - the speed of uniform flow, $F_r = \frac{\partial F}{\partial r}$). We search for the conical potential F in the vicinity of shock wave 1 - 3 (see Fig. 40) as expansion

$$F = w_1 + F_1(\rho)\vartheta^{2k} + \Phi_1(\rho)\vartheta^{3k-2} + \ldots, \quad (3.54)$$

where k = 6, $\rho = (r_1 - r)\vartheta^{-k}$, i.e., the form of expansion with $\vartheta < 0$ the same, as in the vicinity of Mach cone. Functions $F_1(\rho)$, $\Phi_1(\rho)$, ... satisfy equations (3.23), (3.24), but initial conditions for $F_1(\rho)$ one should obtain from relationship/ratios (3.53). On shock wave 1 - 3

$$\rho = [r_1 - r_s(\vartheta)]\vartheta^{-6} = \rho_1 + \ldots,$$
$$F_2 = w_1 + F_1(\rho_1)\vartheta^{12} + \ldots = w_1,$$
$$(F_r)_2 = -F_1'(\rho_1)\vartheta^6 + \ldots = -\frac{4w_1}{\gamma+1}\frac{(M_1^2-1)^2}{M_1^4}\rho_1\vartheta^6 + \ldots,$$

whence it follows

$$F_1(\rho_1) = 0, \quad F_1'(\rho_1) = \frac{4w_1}{\gamma+1}\frac{(M_1^2-1)^2}{M_1^4}\rho_1. \quad (3.55)$$

Page 118.

The solution to equation (3.23), that satisfies conditions (3.55), is

$$F_1(\rho) = -\frac{2\beta_1}{c_1}\left[\frac{(1+2c_1\rho)^{3/2}-(1+2c_1\rho_1)^{3/2}}{3c_1}+\rho-\rho_1\right], \quad (3.56)$$

where $c_1 = \frac{3}{8|\rho_1|} > 0$. With large ρ

$$F_1(\rho) = -\frac{8\beta_1}{3\sqrt{2c_1}}\rho^{3/2}+\ldots, \quad \text{a} \quad \Phi_1(\rho) = \frac{2\beta_1}{15\sqrt{2c_1}}\,\frac{k(k-2)}{r_1}\times \times\rho^{5/2}+\ldots$$

Transfer/converting with large ρ to ζ in (3.54), we will obtain

$$F = w_1 + \chi_1(\zeta)\,\vartheta^{\frac{k}{2}+3}+\ldots, \quad (3.57)$$

where

$$k=6,\quad \chi_1(\zeta) = g_1\left[\zeta^{3/2} - \frac{k(k-2)}{20}\zeta^{5/2}\right],\quad g_1 = h_1 r_1^{3/2} < 0,$$
$$h_1 = -\frac{8\beta_1}{3\sqrt{2c_1}} < 0.$$

Further terms of expansion (3.57) are record/written in the form

$$F = w_1 + \chi_1(\zeta)\,\vartheta^6 + \Gamma_1(\zeta)\,\vartheta^8 + \Psi_1(\zeta)\,\vartheta^{10}+\ldots \quad (3.58)$$

Functions $\Gamma_1(\zeta)$, $\Psi_1(\zeta)$ satisfy equations (3.35), (3.36), where instead of $\chi(\zeta), \Gamma(\zeta)$ it is necessary to write $\chi_1(\zeta), \Gamma_1(\zeta)$. With small ζ

$$\left.\begin{aligned} &\chi_1(\zeta) = -\frac{8\beta_1 r_1^{3/2}}{3\sqrt{2c_1}}\zeta^{3/2}+\ldots, \quad \Gamma_1(\zeta) = -\frac{2\beta_1 r_1}{c_1}\zeta+\ldots,\\ &\Psi_1(\zeta) = -\frac{\sqrt{2}\,\beta_1 r_1^{1/2}}{c_1^{3/2}}\zeta^{1/2}+\ldots \end{aligned}\right\} \quad (3.59)$$

Since the equations and boundary conditions for $\Gamma_1(\zeta)$ coincide with the same for $\Gamma(\zeta)$, $\Gamma_1(\zeta)$ is determined by formula (3.42), with the only difference, that instead of g it is necessary to substitute g_1. It is easy to see that remains valid the formula (3.44) and, consequently, also (3.45), then (3.46). During passage in expansion

(3.58) to to r_1 - r and ϑ for $\zeta \longrightarrow \infty$ it is necessary to consider that for the vicinity of shock wave 1 - 3 $\vartheta < 0$; therefore $|\vartheta| = -\vartheta$ and, for example, $\zeta^{5/2}\vartheta^{6} = (r_1 - r)^{5/2} r_1^{-5/2} |\vartheta| = -(r_1 - r)^{5/2} r_1^{-5/2}\vartheta$.

Page 119.

The formula, analogous (3.48) will take the form

$$F = w_1 + \frac{3}{4}\frac{h_1^2}{\beta_1 r_1}(r_1 - r)^4 + \frac{6h_1}{5r_1}(r_1 - r)^{5/2}\vartheta +$$
$$+ h_1^2\left[-\frac{15}{2}\frac{1}{r_1^2\beta_1}(r_1 - r)^3 + \frac{1}{r_1}\left(\frac{18w_1}{5a_1^2} + \frac{21}{r_1^2\beta_1}\right)(r_1 - r)^4 + \right.$$
$$\left. + \ldots\right]\vartheta^2 + \ldots \quad (3.60)$$

Comparing formulas (3.48) and (3.60), we come to the conclusion that expansion for F (3.60) it is the analytical continuation of expansion (3.48) for $\vartheta < 0$, if $c = c_1$, since

$$h = \frac{8\beta_1}{3\sqrt{2c}},$$

and

$$h_1 = -\frac{8\beta_1}{3\sqrt{2c_1}}.$$

Thereby is constructed the dominant term of irrotational solution, who satisfies conditions on Mach cone and shock wave, and the containing parameter ρ_1, which characterizes the curvature of jump in the vicinity of point 1. Let us show that the dominant term for the speeds of irrotational problem is the same, also, for vortex problem. System of equations (1.17)-(1.20), that describes vortex flows, contains equations (1.19), (1.20), where enters S; equation (1.17),

(1.18) S do not contain. Let us assume that the obtained dominant terms for speeds are dominant terms for a vortex/eddy problem; then from (1.20) it is possible to determine dominant term for S. Then we are convinced that the members with S in equation (1.19) are small in comparison with members, who appear because of the solution of irrotational problem. This means that the dominant terms for speeds coincide in vortex/eddy and irrotational problems. Omitting the details of calculations, let us give only formulas for the dominant term s:

in variables ρ, ϑ

$$s = d_1\vartheta^{18} + O(\vartheta^{34}),$$

in variables ζ, ϑ (when $\rho \longrightarrow \infty$, $\zeta < \infty$)

$$s = d_1\vartheta^{18} + O(\vartheta^{24}),$$

in alternating/variable $r_1 - r$ and ϑ, on the condition that

$$\vartheta^2 (r_1 - r)^{-1} \to 0, \quad \vartheta \to 0,$$

$$s = d_1\left[\vartheta + \frac{32}{35}\,\frac{\beta_1}{r_1^4 w_1 \sqrt{2c_1}}\,(r_1 - r)^{7/2}\right]^{18} + \ldots, \qquad (3.61)$$

where d_1 is determined by formula (3.52).

End Section.

Page 120.

Checking formula (3.61) can be produced thus. The equation of flow line s = 0 is obtained from equation (3.61)

$$\vartheta = -\frac{32}{35}\frac{\beta_1}{r_1^4 w_1 \sqrt{2c_1}}(r_1 - r)^{7/2} + \ldots \quad (3.62)$$

But this equation can be found directly by the integration of equation for flow lines; in its polar coordinates it is possible to write after simplification in the form

$$\frac{d\vartheta}{dr} = -\frac{1}{r_1^3 w_1} F_\vartheta + \ldots,$$

where the dominant term for F_ϑ, as can be seen from (3.60), there is

$$F_\vartheta = \frac{6h_1}{5r_1}(r_1 - r)^{5/2} + \ldots;$$

after the integration of this equation actually is obtained formula (3.62).

Let us note in conclusion that point 1 in Fig. 40 in certain measure is analogous to the point of the termination of shock wave in the local supersonic zone, which appears during the flow of nearsonic flow about the airfoil/profile of gas, in vicinity of which, however, construct the corresponding solution could not.

3.4. Special feature/peculiarity at junction of simple wave and

AD-A066 205 FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO F/G 20/4

NONLINEAR CONICAL FLOWS OF GAS, (U)

MAR 78 B M BULAKH

UNCLASSIFIED FTD-ID(RS)T-0266-78 NL

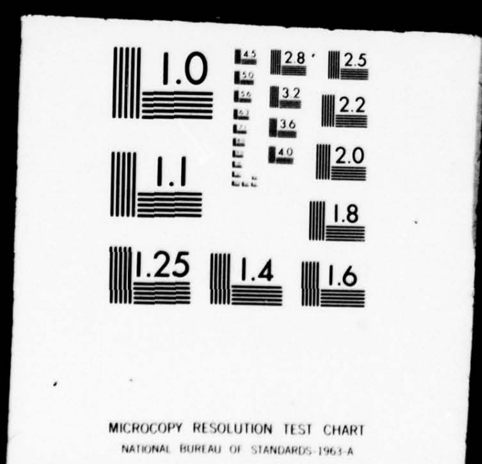
3 OF 7

AD

A066205

3 OF [illegible]

AD
A066205



flow after Mach cone. During the solution of the problems of the flow about the triangular plate with supersonic edges, the edge of the rectangular plate and other problems appears the need for the study of the possibility of the joining of flows after the Mach cone and flow of Prandtlya - Mayer (Fig. 41).

Uniform flow at the Mach number M_1, speed w_1, the speed of sound a_1, is expanded partially during after Mach cone 1-2, partially - during Prandtlya - Mayer, who begins on rectilinear characteristic 2-3. Curve 2-4 there is curvilinear characteristic of the flow of Prandtlya-Mayer, passing through parabolic point 2. Curve 2-5 there is the shock wave, which damps at point 2. The picture, depicted on Fig. as 41, corresponds to the system of coordinates O_1xyz with axis O_1z, directed along the speed of flow on Mach cone 1-2, and plane yO_1z, which contains the ray/beam along which are connected cone of Mach 1-2 and characteristic plane 2-3. At point 2, is a special feature/peculiarity.

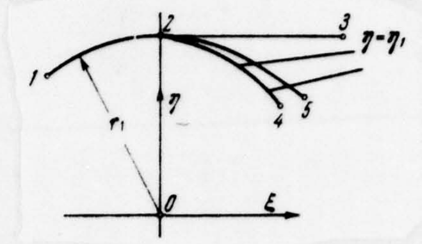


Fig. 41.

Page 121.

To construct the appropriate solution in the vicinity of point 2 at present to end/lead did not manage. Following [106], let us examine the possible way of this construction.

We will return to expansion (3.21), representing the conical potential F in the vicinity of Mach cone, after placing k = 2 and after designating

$$\zeta = \rho r_1^{-1} = (r_1 - r) r_1^{-1}\vartheta^{-2},\ \vartheta = \theta - \frac{\pi}{2},\quad Y(\zeta) = F(\rho)(2\beta_1 r_1)^{-1}.$$

Then Y (ζ) satisfies the equation

$$(2\zeta + 4\zeta^2 - Y')\,Y'' - (10\zeta + 1)\,Y' + 12Y = 0, \qquad (3.63)$$

moreover with small ζ

$$Y(\zeta) = \frac{1}{2}\zeta^2 + \lambda\zeta^3 + \ldots,$$

where λ is an arbitrary constant. Although equation (3.63) in the

locked form it is impossible to integrate, nature of asymptotic behavior Y (ζ) when ζ ⟶ ∞ to establish/install is not lifficult. Are possible two types of asymptotic representations when ζ ⟶ ∞:

$$Y(\zeta) \sim c_1\zeta^2 + c_2\zeta^{5/2} \quad \text{and} \quad Y(\zeta) \sim \frac{\zeta^3}{3} + \frac{\zeta^2}{2} + c_3\zeta$$

(C_1, C_2, C_3 are constant). Furthermore, there are such λ, that with certain $\zeta < \infty,\ |Y| < \infty,\ |Y'| < \infty,\ |Y''| = \infty.$ When λ - 0 equation (3.63) has exact solution Y (ζ) =(1/2)ζ^2. It is logical to assume that with small λ the asymptotic representation Y (ζ) when ζ ⟶ ∞ is Y (ζ) ~ $\underbrace{c_1\zeta^2 +}$ $c_2\zeta^{3/2}$. Transfer/converting when ζ ⟶ ∞ in expansion (3.21) then r_1 - r and ϑ, we will obtain

$$F = w_1 + 2\beta_1 r_1^2\left[\frac{c_1}{r_1^2}(r_1 - r)^2 + \frac{c_2}{r_1^{5/2}}(r_1 - r)^{5/2}\vartheta\right] + \ldots,$$

Whence it follows that in vicinity ϑ = 0

$$\left.\begin{aligned} u &= \cos\theta\cdot F_r - \sin\theta\,\frac{1}{r}\cdot F_\theta = -2\beta_1 r_1^{-1/2} c_2 (r_1 - r)^{3/2} + \ldots,\\ v &= \sin\theta\cdot F_r + \cos\theta\,\frac{1}{r}\cdot F_\theta = -4\beta_1 c_1 (r_1 - r) + \ldots,\\ w &= w_1 - r_1 v + \ldots \end{aligned}\right\} \quad (3.64)$$

Let us examine flow with ξ > 0.

Page 122.

Since point 2 (see Fig. 41) is the parabolic point of simple wave, in it (see (2.83))

$$F_{\eta\eta} = \frac{2w_1}{\gamma + 1}\,\frac{(M_1^2 - 1)^2}{M_1^4}. \qquad (F_\xi = F_\eta = F_{\xi\xi} = F_{\xi\eta} = 0).$$

and potential F it is known here with an accuracy to a small second order; therefore in the vicinity of point 2 in Fig. 0 components of velocity vector in simple wave they will be written in the form

$$\left.\begin{aligned} &u = O\left[(\eta-\eta_1)^2\right],\\ &v = \frac{2w_1}{\gamma+1}\,\frac{(M_1^2-1)^2}{M_1^4}\,(\eta-\eta_1)+\ldots,\\ &w = w_1 - \frac{2w_1}{\gamma+1}\,\frac{(M_1^2-1)^{3/2}}{M_1^4}\,(\eta-\eta_1)+\ldots \qquad (\eta_1=r_1). \end{aligned}\right\} \quad (3.65)$$

The equation of characteristic 2-4 according to (2.87) is

$$\eta_1-\eta=(M_1^2-1)^{1/2}\,\xi^2+\ldots$$

For generality let us examine the case when through the point of 2 figures 40 passes shock wave 2-5; its equation we will write in the form

$$\eta_1-\eta=l\xi^2+\ldots,$$

where l, is constant,.

If $l=(M_1^2-1)^{1/2}$, then shock wave degenerates into the wave of Mach. From (3.65) it follows that the speeds to 2-5 before the transition of the particles of the gas through wave front of shock are

$$\begin{aligned} &u = O(\xi^4),\\ &v = -l\cdot\frac{2w_1}{\gamma+1}\,\frac{(M_1^2-1)^2}{M_1^4}\cdot\xi^2+O(\xi^4),\\ &w = w_1+l\,\frac{2w_1}{\gamma+1}\,\frac{(M_1^2-1)^{3/2}}{M_1^4}\,\xi^2+O(\xi^4), \end{aligned}$$

and immediately after shock wave according to (1.47) they are determined from the formulas

$$\left.\begin{aligned} u &= -l^2 \frac{16w_1}{\gamma+1} \frac{(M_1^2-1)^{3/2}}{M_1^4} (l-\sqrt{M_1^2-1})\,\xi^3+\ldots, \\ v &= -l \frac{2w_1}{\gamma+1} \frac{(M_1^2-1)^{5/2}}{M_1^4} (4l-3\sqrt{M_1^2-1})\,\xi^2+\ldots, \\ w &= w_1-\eta_1 v+\ldots \end{aligned}\right\} \quad (3.66)$$

Let us search for solution after shock wave 2-5 in the form

$$\left.\begin{aligned} &u = u(\sigma)\,\xi^3+\ldots,\quad v = v(\sigma)\,\xi^2+\ldots, \\ &w = w_1+w(\sigma)\,\xi^2+\ldots,\quad s = s(\sigma)\,\xi^6+\ldots, \\ &\sigma = \eta_1(\eta_1-\eta)\,\xi^{-2}. \end{aligned}\right\} \quad (3.67)$$

Page 123.

Substituting (3.67) into system of equations (1.17)-(1.20), we will obtain that the dominant term of solution for speeds will be irrotational and function u(σ), v(σ), w(σ) they are expressed as one function X(σ) by the formulas

$$\left.\begin{aligned} u(\sigma) &= \frac{2w_1}{\gamma+1} \frac{(M_1^2-1)^3}{M_1^4} (2X-\sigma X'), \\ v(\sigma) &= -\frac{w_1}{\gamma+1} \frac{(M_1^2-1)^{5/2}}{M_1^4} X', \\ w(\sigma) &= -\eta_1 v(\sigma) \left(X' = \frac{dX}{d\sigma}\right). \end{aligned}\right\} \quad (3.68)$$

Function X(σ) it satisfies the equation

$$(X'+2\sigma-4\sigma^2)X''+(10\sigma-4)X'-12X=0 \quad (3.69)$$

and the initial conditions

$$\left.\begin{aligned} &X(\sigma_1)=\sigma_1^2,\quad X'(\sigma_1)=2\sigma_1(4\sigma_1-3), \\ &\sigma_1 = l\eta_1,\quad 0<\sigma_1<1, \end{aligned}\right\} \quad (3.70)$$

which follow from formulas (3.66)-(3.68).

For the solutions to equation (3.69) when $\sigma \longrightarrow \infty$ are possible two types of the asymptotic expressions:

$$X(\sigma) \sim b_1\sigma^2 + b_2\sigma^{5/2} \quad \text{and} \quad X(\sigma) \sim \frac{\sigma^3}{3} + b_3\sigma$$

(b_1, b_2, b_3 - constant).

For the problem in question they are of interest of first type solution. Specifically, equation (3.69) has a family of the exact solutions:

$$X(\sigma) = \sigma^2 + b_2\left(\sigma^{5/2} + \frac{3}{32}b_2\right),$$

which with $b_2 = -(32/27)\sqrt{3}$, $\sigma_1 = 1/3$ satisfies conditions (3.70).

Apparently, there are solutions with $b_1 \neq 1$. When $\sigma \longrightarrow \infty$ these solutions give:

$$\left.\begin{aligned} u &= b_1\frac{w_1}{\gamma+1}\frac{(M_1^2-1)^{5/4}}{M_1^4}(\eta_1-\eta)^{1/2}+\ldots, \\ v &= -b_1\frac{2w_1}{\gamma+1}\frac{(M_1^2-1)^2}{M_1^4}(\eta_1-\eta)+\ldots, \\ w &= w_1-\eta_1 v+\ldots \end{aligned}\right\} \quad (3.71)$$

Page 124.

Comparing formulas (3.64) and (3.71), it is possible to draw a

conclusion that for the joining of flows it is necessary that constant c_1, c_2, the depending on λ, and constants b_1, b_2, that depend on σ_1, would give the equal coefficients with the appropriate degrees of $r_1 - r$ and $\eta_1 - \eta$. (Difference r and η in the vicinity of point 2 is unessential). This is reduced to the system of two equations

$$c_1 = b_1, \; c_2 = -b_2$$

for determining λ and σ_1, **which, apparently,** can be composed and solved by numerical methods.

If shock wave 2-5 (see Fig. 41), it degenerates into characteristic 2-4, then $\sigma_1 = 1$, $X(1) = 1$, $X'(1) = 2$ and equation (3.69) has singular point when $\sigma = 1$. In this case there is exact solution $X(\sigma) = -2 + 4\sigma - \sigma^2$. Another family of solutions with σ, close to one, **is the expansion**

$$X(\sigma) = 1 + 2(\sigma - 1) + (\sigma - 1)^2 + b(\sigma - 1)^3 + \\ + \frac{b(3b-1)}{2}(\sigma - 1)^4 + \ldots,$$

where b is an arbitrary constant. The question concerning the joining of flows in this case is analogous to the case with jump 2-5, only role σ_1 here plays constant b.

3.5. Other types of singular points. The singular points, examined in point/items 3.2-3.4, do not exhaust all types of the singular points of the solutions to the equations of conical flows

(for example, is not examined the special feature/peculiarity, which appears at the parabolic point through which passes the characteristic, carrying the discontinuity/interruption of acceleration). But already the results, presented in point/items 3.2-3.4, show that the equations of conical flows possess the supply of the solutions, which describe fairly complicated physical phenomena.

This conclusion is related in essence to the problems of the nonseparated flow of conical bodies. But if flow does blow away from the surface of the streamlined body, then scarcely on possible the basis precise equations of nonviscous gas with the aid of singular points to describe the flow of gas. For this reason the singular points, which can refer to detached flows, here are not examined. Let us note also that most recently appeared R. Melnik's work [233], from whom it follows that, apparently, there exist also such solutions to the equations of the conical flows of gas, in which the lines of constant entropy enter in the point where there is Ferry's special feature/peculiarity, concerning one direction. Page 125.

Chapter 2.

Chapter 2.
SUPERSONIC CONICAL FLOWS OF GAS.

A. Flow conical of bodies, completely included within the Mach cones of undisturbed flow.

§4. Classification and the diagrams of the flow about the conical bodies.

4.1. Preliminary observations. The different cases of the flow of supersonic uniform flows about the conical bodies of gas with the moderate values of the Mach numbers M_1 of the incident for bodies flows it is logical to divide into three groups. First group, A, will compose those cases in which the streamlined bodies wholly lying within the Mach cones of the undisturbed flows, constructed for the apex/vertexes of bodies. To second group, B, let us relate the cases when the streamlined bodies are completely arrange/located outside Mach cones mentioned above; to third group, B, let us relate the cases in which the streamlined bodies partially are located outside the appropriate Mach cones, i.e., group C they compose the cases, which are intermediates between the cases of groups A and B. For conical bodies it is convenient to preserve classification to fine/thin and extended.

Slender body is called such body at whose significant dimension in one direction is much less than significant dimensions in other directions (for example, the triangular plate). The extended body is called the body at whose significant dimension in one direction is much be greater than significant dimensions in other directions (for example, round cone with the small angle of half-aperture). Strictly speaking, the classification of bodies into fine/thin and extended is valid only within the framework of the theory of small disturbance. We will it utilize not in literal sense, but only for characteristics of the type of the streamlined body (wing-shaped or shell-like).

Page 126.

4.2. Extended bodies. For the extended bodies with the rounded cross sections of flow pattern, are completely clear, if angle of attack is small. Body is completely surrounded by the bow shock, connected only to its apex/vertex, flow nonseparable. Figures 49 depicts the flow pattern of cone with cross section in the form of ellipse at small angle of attack 6.

Here the cross section of body is shaded, bow shock is depicted as solid line, Mach cone is dash, flow lines on plane $\xi\eta$ - dash-line

(they are passed through two points where there are Ferry's special feature/peculiarities). If the curvature of the duct of the cross section of body with plane z = 1 changes smoothly from one point to the next, then flow everywhere conically subsonic (see §1, point/item 1.8). With an increase of angle of attack, the picture becomes complicated, since in flow appear the regions, where the flow becomes conical-supersonic. Is most well studied the flow pattern of round cone. For it according to work [107] conical-supersonic regions appear first about shock wave, from the lee side, then, increasing with an increase δ, they reach body surface. With large δ is realized the mode/conditions with "closing", when flow from the lee side of cone does not affect flow from windward face, since here, the analogous with the flow about circular cylinder in two-dimensional problem, appear the maximum characteristics, after which the flow is conical-supersonic.

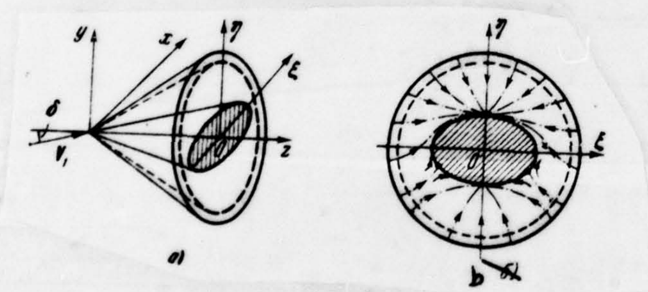


Fig. 42.

Page 127.

With δ, close to the half-angle of cone ε the flow blows away from the surface of cone and is formed complex vortex flow (see [108]). However, when $\delta > \varepsilon$ the flow about the cone occurs with "closing", a conical flow about the windward face of cone is retained with large (see [109]).

If the curvature of the duct of the cross section of body with plane z = 1 changes substantially from one point to the next, for example, the mentioned section is the elongated ellipse, then conical-supersonic regions appear at body surface, and there are formed shock waves. Flow breakaway and its "closing" they occur at smaller angles of attack how this occurs for bodies with the smooth duct of cross section.

If the duct of the cross section of body has points of inflection, for example, it is rhomb, then flow blows away from body surface already at low angles of attack δ, and flow parameters substantially are changed with change δ (see [110]).

4.3. Slender bodies. The most important representative of slender bodies is the triangular plate in example of which further are examined the characteristic features of the flow about such

bodies. If apex angle of plate is not small, then the flow pattern of the sharp leading edges of plate has much in common with flow pattern of sharp edge in two-dimensional problem. With moderate angles of attack δ near edges are small separation zones above which appear the regions where the flows conical-supersonic; in these regions is a system of weak shock waves. With an increase δ, the effect of separation zones grow/rises, and they become determining in the formation of flow about plate up to such value δ, at which occurs the "closing" of flow about the windward face of plate. With large δ by this flow does not affect the flow about the lee side of plate. Figures 43 schematically depicts, according to [111-113], the flow pattern of triangular plate with such δ, when the determining phenomenon becomes flow breakaway from the leading edges of plate. On the strength of symmetry here is depicted the half of the region where the flow is agitated.

Page 128.

In Fig. 43b is given the picture of flow in plane $\xi\eta$. Section 1-2' depicts plate, curved 3-4.4-12-13-5 respectively leading shock waves and the Mach cone of undisturbed flow; curves 1-3, 1-11, 6-7, 8-9, 1-12, 1-13, 1-5 depict flow lines on plane $\xi\eta$. At points 2, 10 flow blows away, forming complex vortex/eddy systems. Points 7, 9, 10 are the tracks for flow lines; in point 1 is formed Ferry's special feature/peculiarity.

4.4. Methods of solution of problems of section A. In recent years considerable development received the numerical methods of the solution of problems with application/use of ETsVM [ЭЦВМ. - digital computer]. These methods possess those by the irrefutable advantage in comparison with analytical methods, that they make it possible to obtain the solution of problem with the assigned accuracy. However, not all problems of group A can be solved by the existing at present torque/moment numerical methods. The numerical methods, among which the most important are finite-difference method of establishment (see [29]), method of integral relationship/ratios (see [107]), the method of straight lines (see [109]) the methods of the solution of the reverse problem (see [114, 115]), successfully are applied when flow nonseparable and in flow there are no swallowed shocks (i.e. there is only leading shock wave), or the flow about the body occurs with the "closing" of flow.

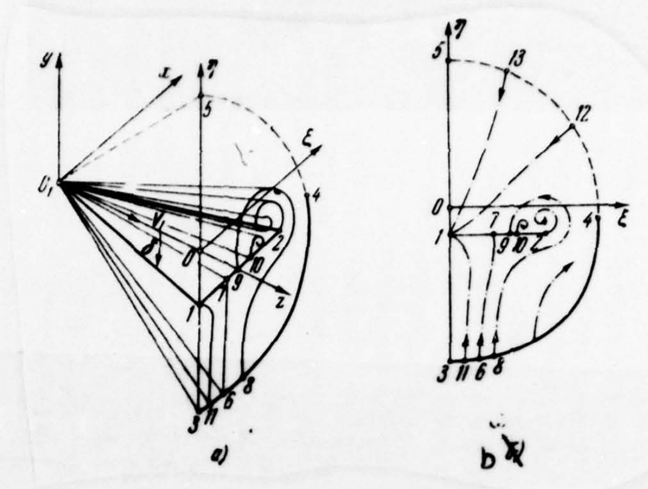


Fig. 43.

Page 129.

Analytical methods during the solution of such problems play at present secondary role, if we speak about quantitative results, but they they make it possible to determine singular points, the lines where the derivatives of the unknown functions go to infinity, and for which must be directed special attention during the use of numerical methods. A characteristic example in this direction gives the problem of cone at an angle of attack. Long time is analytical solution. A. Stone [116, 117] it was the sole source of quantitative data in the problem of the flow about the cone. Is at present

numerical solution of problem, [19] which is more accurately than the solution of Stone. However, precisely the improvement of the solution of Stone made it possible to reveal the special feature/peculiarity of the Ferry, to reveal/detect the vorticity layer where solution by finite-difference methods without special precautions not always does give good results (for example, when $M_1 = 7$, $\varepsilon = 30°$, $\delta = 5°$), moreover these cases they can be revealed on the basis of the analytical first-order solution in the entropy case; see [118].

When flow blows away from the surface of the streamlined body or about body are formed local conical-supersonic zones with shock waves, a precise setting of the corresponding boundary-value problems is represented difficult (or by simply impossible on the basis of the equations of nonviscous gas.

Here the dominant role play the approximate analytical methods, since they make it possible to obtain the missing conditions, for example, in breakaway zone, with the aid of the adequate/approaching schematization of phenomena or with the aid of the supplementary assumptions which can be checked experimentally (see for example, [119]). One of the main methods of the solution of these problems is small parameter method (perturbation method).

Subsequently first is set forth the small parameter method, then - numerical methods.

Page 130.

§ 5. Conditions on head impact wave and Mach cone in small parameter method.

5.1. Preliminary observations. As is known, small parameter method is applied when the unknown function F of independant variables $\xi, \eta, \ldots$ and the parameter μ can be easily defined with $\mu = 0$, for example, as solution to quasi-linear differential equation; then with $\mu \neq 0$ F it searches for as expansion, in the simplest case according to degrees μ,

$$F(\xi, \eta, \ldots, \mu) = F_0(\xi, \eta, \ldots) + \mu F_1(\xi, \eta, \ldots) + \\ + \mu^2 F_2(\xi, \eta, \ldots) + \ldots, \quad (5.1)$$

where $F_0 = F|_{\mu=0}$, a F_k $(k = 1, 2, \ldots)$ - the "disturbance/perturbations", to be determined already from linear equations.

If we as F take conical potential, after F_0 - the conical potential of the axisymmetric flow about the round cone, after μ -

the geometric parameter, which characterizes the difference between the streamlined body and the round cone, then with the aid of expansion (5.1) it is possible to find the solution of the problem of the flow about the conical bodies, not which very strongly differ from round cone (see on this question work [120], where is taken into account also the eddying of flow).

However there is the greatest interest in the case when there is a family of bodies, which depend on geometric parameter μ (where μ, for example, the thickness ratio of body or angle of attack), thus, that when $\mu = 0$ body does not agitate the incident to it flow.

In this case F_0, there is simply conical potential of uniform flow, and determination F_1 is the problem of the linear conical flow theory, which detailed (see [3, 4]).

As it will be shown further, a change of the entropy in the flows of section A is O (δ^6) for the slender bodies where δ is an angle of attack either another analogous parameter or $O(\varepsilon^{12})$ where ε is the thickness ratio of extended body; therefore these flows with an accuracy to the indicated values can be considered as irrotational.

Page 131.

The problem of determination F_2 in expansion (5.1) is a problem of the determination of the second approach/approximation for slender bodies. In the case of the extended bodies, expansion (5.1) is replaced by expansion according to degrees μ and ln μ; see, for example, [121].

Expansions of type (5.1) are valid only within the Mach cone of the undisturbed flow whose equation in polar plane coordinates $\xi\eta$ is $r = r_1 = (M^2_1 - 1)^{-1/2}$ where M_1 is a Mach number of undisturbed flow. The flow of gas in the vicinity of the Mach cone of undisturbed flow and vicinity of leading shock wave is not described by expansion of the type (5.1), since line $r = r_1$ is person for the equations, which define F_k in (5.1) (for example, see [10], which used that developed by it the method of the "strain of independent variables" (Puankaret - Layytkhilla - Go's method, briefly, method PLG) to considered and it showed, as it is necessary to modify expansion of the type (5.1) so that it would be valid in the region where the flow was agitated. Specifically, you obtained formula for determining position of leading shock wave (and of its force) according to the results of linear theory.

Another path, proposed by B. Bulakh [122] lies in the fact that,

besides expansion of the type (5.1), is examined one additional expansion for F which it is correct in the vicinity of the Mach cone of undisturbed flow or bow shock; the solution of problem is obtained by means of the coupling of these expansions in the vicinity of line $r = r_1$. Although expansion (5.1) is correct with $r < r_0 < r_1$, its coefficients F_k can be formally determined within circle $r \leq r_1$, so that the operation of coupling of above asymptotic expansions for F is reduced to the determination of boundary conditions for F_k with $r \to r_1$. Since there is the basic interest in the determination of expansion (5.1), the described method is, apparently, more natural in the problem in question, than the method of the "strain of independent variables".

5.2. Expansion in series in low parameter of conical potential F in vicinity of bow shock or Mach cone of undisturbed flow.

Page 132.

The conical potential F satisfies equation (1.31)

$$\{a^2(1+r^2)-[rF-(1+r^2)F_r]^2\}F_{rr}+2\left[F-\left(r+\frac{1}{r}\right)\right]F_\theta\times$$
$$\times\left(\frac{1}{r}F_{r\theta}-\frac{1}{r^2}F_\theta\right)+\left(a^2-\frac{1}{r^2}F_\theta^2\right)\left(\frac{1}{r^2}F_{\theta\theta}+\frac{1}{r}F_r\right)=0,$$

where r, θ are polar plane coordinates $\xi\eta$ $(r\cos\theta=\xi,\ r\sin\theta=\eta)$:

$$a^2=a_1^2-\frac{\gamma-1}{2}\left[F_r^2+\frac{1}{r^2}F_\theta^2+(F-rF_r)^2-W_1^2\right];\ a_1,\ W_1,\ M_1>1$$

- respectively the speed of sound, the velocity modulus and the Mach number of undisturbed flow. The projections of velocity vector on the axis of the Cartesian system of coordinates u, v, w are expressed by formulas (1.32). The Mach cone of undisturbed flow in the coordinate system, in which axis O_1z is oriented in the direction of the speed of undisturbed flow, on plane $\xi\eta$ is depicted as circumference $r=r_1=\frac{1}{m_1}, m_1=\sqrt{M_1^2-1}$. In uniform flow according to (1.32)

$$F=W_1 \quad (F_r=F_\theta=0). \tag{5.2}$$

Equation of leading shock wave let us write in the form

$$r=r_s(\theta)=r_1+\lambda\varphi(\theta,\lambda), \tag{5.3}$$

where $\lambda > 0$ is certain low parameter, $\phi(\theta, \lambda) > 0$, $\phi(\theta, 0) = \phi_0(\theta) < \infty$.

Conditions on shock wave for F according to formulas (1.47) after transition to polar coordinates can be written in the form

$$F=W_1, F_r=-\frac{2W_1}{(\gamma+1)M_1^2}\cdot\frac{r_s^2(1-r_s^2m_1^2)+r_s'^2}{r_s[r_s^2(1+r_s^2)+r_s'^2]} \tag{5.4}$$

with $r=r_s(\theta)$,

where $r_s'=\frac{dr_s}{d\theta}$. From (5.4) it follows that when $r=r_s=r_1+\lambda\phi(\theta, \lambda)$ $F_r=\frac{4W_1}{\gamma+1}\frac{m_1^4}{M_1^4}\cdot\lambda\varphi+O(\lambda^2)$, while from (1.31) by differentiation we find

$$F_{rr}=-\frac{2W_1}{\gamma+1}\cdot\frac{m_1^4}{M_1^4}+O(\lambda), F_{rrr}=O(\lambda^{-1}) \text{ so forth.} \tag{5.5}$$

Page 133.

Expressions (5.5) clearly show that expansion (5.1) not is useful in the vicinity of shock wave. If flow is expanded in the vicinity of Mach cone, then F is here represented by expansion (2.103) from which it follows that

$$F_{rr} = 2\beta_1 + O[(r_1 - r)\ln(r_1 - r)]$$

in vicinity $r = r_1$, i.e., expansion (5.1) also not is useful near Mach cone, since the acceleration with $r = r_1$ remains final, as small not were disturbance/perturbations.

Let us search for F in the vicinity of shock wave in the form of the series

$$F = W_1 + \lambda^2 F_1(\theta, t, \lambda) + \lambda^4 F_2(\theta, t, \lambda) + \ldots, \quad (5.6)$$

where $t = (r - r_1)[\lambda\phi(\theta, \lambda)]^{-1}$. Value $t = 1$ corresponds to shock wave, value $t = 0$ - to the Mach cone of undisturbed flow.

Substituting (5.6) in (1.31), equalizing the coefficients with identical degrees λ, we will obtain equations for F_1, F_2, ... If we introduce function $y(t)$ with the aid of the relationship/ratio

$$F_1(\theta, t, \lambda) = \varphi^2(\theta, \lambda)\frac{W_1}{\gamma+1}\cdot\frac{m_1^4}{M_1^4}y(t),$$

by y (t) satisfies the equation

$$y''(2t-y')-y'=0 \qquad \left(y'=\frac{dy}{dt}\right). \tag{5.7}$$

Equations for $F_k(\theta, t, \lambda), k>1$ are reduced to the form

$$\frac{\partial^2 F_k}{\partial t^2}(y'-2t)+\frac{\partial F_k}{\partial t}(y''+1)=P_k, \tag{5.8}$$

where P_k depend on $F_1, F_2, \ldots F_{k-1}$. Conditions on shock wave (5.4) after substitution $\underbrace{\text{there}}_{r_s = r_1+\lambda\varphi}$ and the decomposition of result in terms of exponents λ can be presented in the form

$$F=W_1,\quad F_r=\frac{4W_1}{\gamma+1}\cdot\frac{m_1^4}{M_1^4}\varphi(\theta,\lambda)\cdot\lambda-$$
$$-\frac{2W_1}{\gamma+1}\cdot\frac{m_1^5}{M_1^4}\left[\varphi'^2(\theta,\lambda)+\frac{M_1^4+4}{M_1^2}\varphi^2(\theta,\lambda)\right]\lambda^2+O(\lambda^3),$$

whence taking into account the fact that

$$F_r=\frac{1}{\varphi(\theta,\lambda)}\cdot\frac{\partial F_1}{\partial t}\lambda+\frac{1}{\varphi(\theta,\lambda)}\frac{\partial F_2}{\partial t}\lambda^2+\ldots,$$

we obtain, that on shock wave, i.e., with t = 1,

$$F_1=0,\ F_2=0,\ldots$$
$$\frac{\partial F_1}{\partial t}=\frac{4W_1}{\gamma+1}\frac{m_1^4}{M_1^4}\varphi^2(\theta,\lambda),\ \frac{\partial F_2}{\partial t}=-\frac{2W_1}{\gamma+1}\frac{m_1^5}{M_1^4}\left[\varphi'^2(\theta,\lambda)+\right.$$
$$\left.+\frac{M_1^2+4}{M_1^2}\varphi^2(\theta,\lambda)\right]\varphi(\theta,\lambda),\ldots\left(\varphi'=\frac{d\varphi}{d\theta}\right). \tag{5.9}$$

Page 134.

Solving equation (5.7) under the conditions (5.9) we will find

$$y(t)=\frac{8}{3}\left[t-\frac{8}{9}\left(1-\frac{3}{4}t\right)^{3/2}-\frac{8}{9}\right]. \tag{5.10}$$

Functions P_K are sums the product of functions only of t and only of θ; therefore the solution to equations (5.8) is reduced to the integration of ordinary linear differential first-order equations. Specifically, for F_2 we obtain

$$F_2=\frac{W_1 m_1^5}{(\gamma+1)M_1^4}\left[\varphi^3(\theta,\lambda)z_1(t)+\varphi(\theta,\lambda)\varphi'^2(\theta,\lambda)z_2(t)+\right.$$
$$\left.+\varphi^2(\theta,\lambda)\varphi''(\theta,\lambda)z_3(t)\right], \qquad (5.11)$$

where functions $z_1(t)$, $z_2(t)$, $z_3(t)$ satisfy the equations

$$\left.\begin{aligned}
&z_1''(y'-2t)+z_1'(y''+1)=y''\left[t^2-ty'+\frac{m_1^2}{2(\gamma+1)M_1^2}y'^2-\right.\\
&\left.-\frac{2}{M_1^2}ty'+\frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}y\right]+ty'-\frac{\gamma-1}{\gamma+1}\frac{m_1^2}{M_1^2}y,\\
&z_2''(y'-2t)+z_2'(y''+1)=-t^2y''+2ty'-2y,\\
&z_3''(y'-2t)+z_3'(y''+1)=ty'-2y,
\end{aligned}\right\} \qquad (5.12)$$

and, according to (5.9), to the initial conditions

$$\left.\begin{aligned}
&z_1(1)=z_2(1)=z_3(1)=0,\\
&z_1'(1)=-2\frac{M_1^2+4}{M_1^2},\quad z_2'(1)=-2,\quad z_3'(1)=0.
\end{aligned}\right\} \qquad (5.13)$$

From equations (5.12) and formulas (5.10), (5.11) it is easy to obtain asymptotic expression for F_2 with $t \to -\infty$:

$$F_2=-\frac{W_1 m_1^5}{(\gamma+1)M_1^4}\frac{2\sqrt{3}}{45}(9\varphi^3+\varphi\varphi'^2-2\varphi^2\varphi'')(-t)^{3/2}+\ldots \qquad (5.14)$$

and to find subsequent members of this expression

$$F_2=b_1(-t)^{3/2}+b_2(-t)^2+b_3(-t)^{3/2}\ln(-t)+\ldots, \qquad (5.15)$$

where b_K are some functions θ and λ whose explicit form subsequently will not be required.

Page 135.

Let us find now expansion for F in the vicinity of the Mach cone of undisturbed flow $r = r_1 = 1/m_1$. Let us as before search for F in vicinity $r = r_1$ in the form (5.6), only here $\phi(\theta, \lambda)$ there will be the indefinite thus far function; λ is the parameter, which characterizes the order of speeds at $t \longrightarrow -\infty$. Equations (5.7), (5.8), (5.11), (5.12) retain their form, are changed only initial conditions. On the Mach cone of perturbation rate equal to zero, therefore, $F = W_1$, $F_r = 0$, i.e.

$$F_k = \frac{\partial F_k}{\partial t} = 0 \text{ with } t = 0;$$

hence

$$\left.\begin{array}{l} y(0) = y'(0) = 0, \quad z_1(0) = z_2(0) = z_3(0) = 0, \\ z_1'(0) = z_2'(0) = z_3'(0) = 0. \end{array}\right\} \qquad (5.16)$$

The solution to equation (5.7) with initial conditions (5.16) it is

$$y(t) = \frac{1}{c}\left\{t + \frac{1}{3c}\left[(1-2ct)^{3/2} - 1\right]\right\}, \qquad (5.17)$$

where c is arbitrary positive constant.

With small t, as can be seen from (5.17)

$$y = \frac{t^2}{2} + \ldots,$$

and the solution to the homogeneous equation

$$z_0''(y' - 2t) + z_0'(y'' + 1) = 0$$

takes the form

$$x_0 = t^3 + \ldots$$

From (5.12) we will obtain that with small t

$$x_1(t) = \frac{(\gamma+4)(m_1^2-1)}{6(\gamma+1)M_1^2} t^3 \ln(-t) + \ldots;$$
$$x_2(t) = O(t^3); \quad x_3(t) = O(t^3).$$

From these expressions it follows that if we with small t in expansion (5.6) pass to alternating/variable $r_1 -$ r and θ, then as a result we will obtain in accuracy expansion (2.103), which confirms correctness (5.6).

Page 136.

With large negative t the solution behaves in perfect analogy with the case with shock wave, in particular, is retained the formula (5.15). If we now with large negative t, but small r_1 - r pass in expansion (5.6) to r and θ, then it appears that all terms $\lambda^k F_k(\theta, t, \lambda)$ will be $O(\lambda^{1/2})$, and a series (5.6) must be transformed putting together terms $O(\lambda^{1/2})$, $O(\lambda)$ etc.; such a transformation of a series let us call regrouping. Decompose/expanding in (5.6) function $F_k(\theta, t, \lambda)$ according to the decreasing degrees of t [and ln (-t)], transfer/converting from t to r_1-r, after the regrouping of a series we will obtain

$$F = W_1 - \lambda^{1/2}\varphi^{1/2}(\theta,\lambda)\frac{W_1}{\gamma+1}\frac{m_1^4}{M_1^4}\frac{8\sqrt{3}}{9}\Big\{(r_1-r)^{1/2} +$$
$$+ \frac{9}{20}m_1\Big[1+\frac{\varphi'^2(\theta,\lambda)-2\varphi(\theta,\lambda)\varphi''(\theta,\lambda)}{9\varphi^2(\theta,\lambda)}\Big](r_1-r)^{3/2}+\ldots\Big\} -$$
$$-\lambda\varphi(\theta,\lambda)\frac{W_1}{\gamma+1}\frac{8m_1^4}{3M_1^4}\{(r_1-r)+\tilde{b}_2(r_1-r)^2+\ldots\}+O(\lambda^{3/2}\ln\lambda). \tag{5.18}$$

where $\tilde{b}_2$ is certain function θ and λ, and

$$w = F - rF_r = W_1 - \lambda^{1/2}\varphi^{1/2}\frac{W_1}{\gamma+1}\cdot\frac{m_1^{5/2}}{M_1^4}\cdot\frac{4\sqrt{3}}{3}\left\{\left(\frac{r_1-r}{r_1}\right)^{1/2} + \right.$$
$$\left. + \frac{5\varphi^2+\varphi'^2-2\varphi\varphi''}{12\varphi^2}\left(\frac{r_1-r}{r_1}\right)^{3/2}+\ldots\right\} -$$
$$-\lambda\varphi\frac{W_1}{\gamma+1}\cdot\frac{m_1^3}{M_1^4}\cdot\frac{8}{3}\{1+\cdots\} + O(\lambda^{3/2}\ln\lambda). \qquad (5.19)$$

Dots in the curly braces replace the terms, which have the higher order of smallness on to $r_1 - r$, than given.

Expressions (5.18), (5.19) are related to the case when there is a bow shock. For solution in the vicinity of the Mach cone of the undisturbed flow about which the flow is expanded, we will obtain

$$F = W_1 + \lambda^{1/2}\psi(\theta,\lambda)^{1/2}\frac{W_1}{\gamma+1}\cdot\frac{m_1^4}{M_1^4}\cdot\frac{8\sqrt{3}}{9}\left\{(r_1-r)^{3/2} + \right.$$
$$\left. + \frac{9}{20}m_1\left[1+\frac{\psi'^2(\theta,\lambda)-2\psi(\theta,\lambda)\psi''(\theta,\lambda)}{9\psi^2(\theta,\lambda)}\right](r_1-r)^{5/2}+\cdots\right\} -$$
$$-\lambda\psi(\theta,\lambda)\frac{W_1}{\gamma+1}\cdot\frac{m_1^4}{M_1^4}\cdot\frac{8}{3}\{(r_1-r)+\tilde{\tilde{b}}_2(r_1-r)^2+\cdots\} + O(\lambda^{3/2}\ln\lambda). \qquad (5.20)$$

$$w = F - rF_r = W_1 + \lambda^{1/2}\psi^{1/2}\frac{W_1}{\gamma+1}\frac{m_1^{5/2}}{M_1^4}\frac{4\sqrt{3}}{3}\left\{\left(\frac{r_1-r}{r_1}\right)^{1/2} + \right.$$
$$\left. + \frac{5\psi^2+\psi'^2-2\psi\psi''}{12\psi^2}\left(\frac{r_1-r}{r_1}\right)^{3/2}+\cdots\right\} -$$
$$-\lambda\psi\frac{W_1}{\gamma+1}\cdot\frac{m_1^3}{M_1^4}\frac{8}{3}\{1+\cdots\} + O(\lambda^{3/2}\ln\lambda). \qquad (5.21)$$

moreover $\psi^{1/2}(\theta,\lambda) = \frac{3}{2\sqrt{6c}}\varphi^{1/2}(\theta,\lambda)$.

Expressions (5.18) and (5.20) differ only in terms of the sign with $\lambda^{1/2}$ and in the facts that ϕ is replaced by $\psi, \tilde{b}_2$ - by $\tilde{\tilde{b}}_2$. Let us show now that the regions, de are valid expansion (5.6) and (5.1), that represent conical potential within Mach cone, i.e., with $r < r_1$, they overlap. If this overlap occurs, then terms from $\lambda^{1/2}$ in expansions (5.18)-(5.21) must be the solution, given by usual linear theory, moreover on Mach cone $r = r_1$ potential and perturbation rates are must in the linear solution to be equal to zero, as this follows from (5.18)-(5.21).

Expansions (5.18)-(5.21) do not represent solution in direct vicinity of shock wave or Mach cone, but each term of these expansions it is possible analytically to continue to $r = r_1$ and to obtain for it the corresponding boundary condition with $r \longrightarrow r_1$]. From linear conical flow theory, it follows that $w - W_1$ the harmonic function of polar coordinates $\alpha = \frac{1-\sqrt{1-m_1^2 r^2}}{m_1 r}$ and θ (for example, see [3]). But any harmonic function, which turns into zero when $\alpha = 1$ ($r = r_1$), can be represented in the form

$$w - W_1 = c_0 \ln \alpha + \sum_{n=1}^{\infty} \left(\frac{1}{\alpha^n} - \alpha^n\right)(c_{n1} \cos n\theta + c_{n2} \sin n\theta), \quad (5.22)$$

where c_0, c_{n1}, c_{n2} are constant. In the vicinity of Mach cone $r = r_1$ $w - W_1$, it is decompose/expanded in a series according to degrees $\left(\frac{r_1 - r}{r_1}\right)^{1/2}$

$$w - W_1 = \\ = \left[-c_0\sqrt{2} + \sum_{n=1} 2\sqrt{2}\, n\,(c_{n1}\cos n\theta + c_{n2}\sin n\theta)\right]\left\{\left(\frac{r_1 - r}{r_1}\right)^{1/2} + \right. \\ \left. + \frac{-5c_0 + \sum_{n=1}^{\infty} 2n\,(5 + 4n^2)\,(c_{n1}\cos n\theta + c_{n2}\sin n\theta)}{12\left[-c_0 + \sum_{n=1}^{\infty} 2n\,(c_{n1}\cos n\theta + c_{n2}\sin n\theta)\right]}\cdot\left(\frac{r_1 - r}{r}\right)^{3/2} + \dots\right\}. \quad (5.23)$$

Page 138.

Equalizing coefficients with identical degrees $\left(\frac{r_1-r}{r_1}\right)^{1/2}$ $\left(\frac{r_1-r}{r_1}\right)^{3/2},\dots$ in (5.23) and the expression, which contains $\lambda^{1/2}$ in (5.19), we will obtain

$$-\lambda^{1/2}\varphi^{1/2}\frac{W_1}{\gamma+1}\frac{m_1^{3/2}}{M_1^4}\frac{4\sqrt{3}}{3} = -c_0\sqrt{2} + \sum_{n=1}^{\infty} 2\sqrt{2}\, n\,(c_{n1}\cos n\theta + c_{n2}\sin n\theta), \quad (5.24)$$

$$\frac{5\varphi^2 + \varphi'^2 - 2\varphi\varphi''}{12\varphi^2} = \frac{-5c_0 + \sum_{n=1}^{\infty} 2n\,(5 + 4n^2)\,(c_{n1}\cos n\theta + c_{n2}\sin n\theta)}{12\left[-c_0 + \sum_{n=1}^{\infty} 2n\,(c_{n1}\cos n\theta + c_{n2}\sin n\theta)\right]}, \quad (5.25)$$

so forth. Analogous equalities must be fulfilled, when there is flow of expansion in the vicinity of Mach cone $r = r_1$. Relationship/ratio (5.24) is determined ϕ; (5.25) so forth they are consequences (5.24), if term from $\lambda^{1/2}$ in (5.19) it is the solution of linear problem. After writing expression $\frac{5\varphi^2 + \varphi'^2 - 2\varphi\varphi''}{\varphi^2}$ in the form $5 - \left(\frac{\varphi'}{\varphi}\right)^2 - 2\left(\frac{\varphi'}{\varphi}\right)'$. it is possible to easily ascertain that (5.25) really/actually there is a consequence (5.24). (Furthermore, for a check, this fact was checked

in special cases of the triangular plate and round cone for which $w - W_1$ are determined in the locked form).

All this gives grounds to consider that the regions where are valid expansions (5.1) and (5.6), overlap, moreover in the region of overlap are valid formulas (5.18)-(5.21), that make it possible to determine the behavior of the coefficients of expansion (5.1) with $r \to r_1$.

With large negative t and small $r_1 - r$ expansion of (5.6) taking into account (5.10), (5.17), (5.15) can be written in the form

$$
\begin{aligned}
F = {} & W_1 + \lambda^2 F_1 + \lambda^3 F_2 + \ldots = \\
= {} & W_1 + \lambda^2 \left[a_1 (-t)^{3/2} + a_2 (-t) + a_3 (-t)^{1/2} + a_4 + a_5 (-t)^{-1/2} + \ldots\right] + \\
& + \lambda^3 \left[b_1 (-t)^{5/2} + b_2 (-t)^2 + b_{21} (-t)^{3/2} \ln(-t) + \ldots\right] + \ldots, \qquad (5.26)
\end{aligned}
$$

where $a_k = a_k(\theta, \lambda)$ is coefficients of expansion F according to the decreasing degrees (-t). After transition in (5.26) to to $r_1 - r$ and the regrouping of a series, we obtain

$$
\begin{aligned}
F = {} & W_1 + \lambda^{1/2} \left[a_1 \varphi^{-3/2} (r_1 - r)^{3/2} + b_1 \varphi^{-5/2} (r_1 - r)^{5/2} + \cdots\right] + \\
& + \lambda \left[a_2 \varphi^{-1} (r_1 - r) + \cdots\right] + \lambda^{3/2} \ln \lambda \left[b_{21} \varphi^{-3/2} (r_1 - r)^{3/2} + \cdots\right] + \\
& + \lambda^{3/2} \left[a_3 \varphi^{-1/2} (r_1 - r)^{1/2} + \cdots\right] + \lambda^2 \left[a_4 + \cdots\right] + \\
& + \lambda^{5/2} \left[a_5 \varphi^{1/2} (r_1 - r)^{-1/2} + \cdots\right] + \cdots \qquad (5.27)
\end{aligned}
$$

Page 139.

Series in the brackets are constructed according to the increasing degrees $(r_1 - r)^{1/2}$; beginning with term $O(\lambda^{3/2})$ in them they appear

additionally the product of form $(r_1 - r)^{k/2} [\ln (r_1 - r)]^n;\ k, n > 0$. Coefficients $a_k(\theta, \lambda), b_k(\theta, \lambda)$ in turn, can be represented in the form of series according to the degrees $\lambda^{1/2}$ and $\ln \lambda$.

Formula (5.27) gives the detailed representation of expansion (5.1) with small, but final $r_1 - r$. Hence it follows that the coefficients of the expansion of the velocity potential of disturbance/perturbation $F - W_1$ in a series in the low parameter within Mach cone, i.e., at $r < r_1$, during analytical continuation them to value of $r = r_1$, they turn there into zero up to terms O (λ^2). (Coefficient with λ^2 is final with $r = r_1$, the coefficient when $\lambda^{5/2}$ and the subsequent coefficients unlimitedly grow/rise with $r \to r_1$). The parameter λ is determined by the order of disturbance/perturbations in linear solution. For slender bodies $\lambda^{1/2} = \delta$, where δ is an angle of attack (or another analogous parameter), for extended bodies $\lambda^{1/2} = \varepsilon^2$, where ε - the thickness ratio of body (for example, see [3]).

Thus, if we search for the conical velocity potential of perturbance, $F - W_1$ within the Mach cone of undisturbed flow in the form of a series in the low geometric parameter, first term of which is the solution of usual linear problem, the coefficients of this expansion turn into zero on the Mach cone of undisturbed flow up to terms O (δ^4) for slender bodies and $O(\varepsilon^8)$ for the extended bodies.

5.3. Position of leading shock wave. With the aid of the results of point/item 5.2, it is not difficult to obtain the formula, which determines the position of the leading shock wave, produced by the streamlined body. Within the Mach cone of undisturbed flow, i.e., with $r < r_1$, the projection of velocity vector on axis O_1z, w can be represented by expansion $w = W_1 (1 + \lambda^{1/2} w_1 + \lambda w_2 + \ldots)$, where w_1 there is solution of linear problem, i.e., w_1 is an harmonic function of the polar coordinates

$$\alpha = \frac{1-\sqrt{1-m_1^2 r^2}}{m_1 r} \text{ and } \theta \left(m_1 r = \frac{2\alpha}{1+\alpha^2}\right).$$

In the vicinity of Mach cone $\alpha = 1$ $(r = r_1 = 1/m_1)$ w_1 it is possible to present in the form

$$w_1 = -\left(\frac{\partial w_1}{\partial \alpha}\right)_{\alpha=1} (1-\alpha) + O[(1-\alpha)^2]. \tag{5.28}$$

On the other hand, accordingly formulas (5.19), (5.21)

$$w_1 = \begin{Bmatrix} -\varphi_0^{1/2}(\theta) \\ +\psi_0^{1/2}(\theta) \end{Bmatrix} \frac{4\sqrt{3}}{3} \frac{m_1^{3/2}}{(\gamma+1) M_1^4} \left\{ \left(\frac{r_1 - r}{r_1}\right)^{1/2} + O\left[\left(\frac{r_1 - r}{r_1}\right)^{3/2}\right]\right\}, \tag{5.29}$$

in vicinity $r = r_1$ $(\alpha = 1)$.

Page 140.

After transition in (5.29) from alternating/variable r to variable α from formula $m_1 r = 2\alpha/(1 + \alpha^2)$, we will obtain

$$w_1 = \begin{Bmatrix} -\varphi_0^{1/2}(\theta) \\ +\psi_0^{1/2}(\theta) \end{Bmatrix} \frac{2\sqrt{6}}{3} \frac{m_1^{5/2}}{(\gamma+1) M_1^4}(1-\alpha) + O[(1-\alpha)^2]. \quad (5.30)$$

We equate the coefficients with $1 - \alpha$ in formulas (5.28), (5.30):

$$\left(\frac{\partial w_1}{\partial \alpha}\right)_{\alpha=1} = \begin{Bmatrix} \varphi_0^{1/2}(\theta) \\ -\psi_0^{1/2}(\theta) \end{Bmatrix} \frac{2\sqrt{6}}{3} \frac{m_1^{5/2}}{(\gamma+1) M_1^4}. \quad (5.31)$$

Recall that $\varphi_0(\theta) = \varphi(\theta, \lambda)|_{\lambda=0}; \psi_0(\theta) = \psi(\theta, \lambda)|_{\lambda=0}$. From (5.31) it follows that for those θ with which $\left(\frac{\partial w_1}{\partial \alpha}\right)_{\alpha=1} > 0$, i.e., the flow in the vicinity of Mach cone is compressed (in the linear approach/approximation),

$$\left(\frac{\partial w_1}{\partial \alpha}\right)_{\alpha=1} = \varphi_0^{1/2}(\theta) \frac{2\sqrt{6}}{3} \cdot \frac{m_1^{5/2}}{(\gamma+1) M_1^4}$$

and

$$\varphi_0(\theta) = \frac{3(\gamma+1)^2}{8} \frac{M_1^8}{m_1^5} \left[\left(\frac{\partial w_1}{\partial \alpha}\right)_{\alpha=1}\right]^2. \quad (5.32)$$

The position of leading shock wave is determined from the formula

$$m_1 r_0(\theta) = m_1[r_1 + \lambda\varphi(\theta, \lambda)] = m_1[r_1 + \lambda\varphi_0(\theta) + o(\lambda)] = \\ = 1 + \lambda \frac{3(\gamma+1)^2}{8} \frac{M_1^8}{m_1^4} \left[\left(\frac{\partial w_1}{\partial \alpha}\right)_{\alpha=1}\right]^2 + o(\lambda), \quad (5.33)$$

which was obtained, several in another form, by M. Lighthill [10]. Knowing the position of leading shock wave, it is not difficult to obtain estimation for the gallop of entropy on shock wave, O (λ^3), whence follows estimation indicated earlier for the vortex/eddy terms of solution.

For those θ, where $\left(\frac{\partial w_1}{\partial \alpha}\right)_{\alpha=1}<0$, method PLG makes it possible to only establish that the shock wave of the same force, as in the case $\left(\frac{\partial w_1}{\partial \alpha}\right)_{\alpha=1}>0$, it will not be.

The method of the "union of asymptotic expansions", as this follows from formula (5.31), gives that for those θ, with which $\left(\frac{\partial w_1}{\partial \alpha}\right)_{\alpha=1}<0$, shock wave not at all appears and flow in the vicinity of the Mach cone of undisturbed flow is a flow of the evacuation/rarefaction whose speed is characterized by function $\psi_0(\theta)$ where

$$\psi_0(\theta)=\frac{3(\gamma+1)^2}{8}\frac{M_1^8}{m_1^5}\left[\left(\frac{\partial w_1}{\partial \alpha}\right)_{\alpha=1}\right]^2. \quad (5.34)$$

An example of such situation when derivative $\left(\frac{\partial w_1}{\partial \alpha}\right)_{\alpha=1}$ on the half of circumference α = 1 is positive, on another half it is negative, gives the case of triangular plate from subsonic edges, streamlined at an angle of attack δ. According to the obtained in point/item 5.3 results the region where the flow is agitated, limited from the windward face of plate by leading shock wave, from leeward - by the Mach cone of undisturbed flow; in the junction of shock wave and Mach cone, is formed singular point described in p. 3.3.

The details of relatively triangular plate and extended body with arbitrary cross section can be found in work [10].

§6. Methods of the determination of the second approach/approximation for slender bodies.

6.1. Preliminary observations. In work [123] F. Moore gave the formulation of the problem of the determination of the second approach/approximation for slender bodies. Potential and perturbation rates were decompose/expanded in a series according to the degrees of geometric parameter; boundary conditions on the Mach cone of undisturbed flow were located with the aid of M. Lighthill's method [10]; the perturbation rates of the first and second order satisfied with respect to the two-dimensional equations of Laplace and Poisson.

Further the formulation of the problem of second approximation for slender bodies will be carried out more simply with the aid of canonical variables [14], and results of §5. It is directed axis O_1z of the Cartesian system along speed V_1 of undisturbed flow. The surface of the streamlined body on plane ξ, η let us assign by the

equation

$$\eta = \delta\Psi_n(\xi), \quad \xi_1 \leqslant \xi \leqslant \xi_2,$$

where δ are the low parameter, n = 1, 2 (Fig. 44).

Page 141.

Conical potential within the Mach cone of undisturbed flow (i.e. with $r < r_1$, where $r = 1/m_1$, $m_1 = (M^2_1 - 1)^{-1/2}$, M_1 - the Mach number of undisturbed flow) we search for in the form

$$F = W_1 (1 + \delta f_1 + \delta^2 f_2 + \ldots) \qquad (6.1)$$

(W_1 is modulus of velocity of uniform flow). The determination of term O (δ) in expansion (6.1) composes the problem of linear theory; the determination of term O (δ^2) - a problem of second-order theory.

Recall that the flow can be considered as irrotational with an accuracy to values O (δ^6).

6.2. Equations of theories of first and second approach/approximations for slender bodies. Conical potential F satisfies equation (1.29)

$$L[F] = AF_{\xi\xi} + 2BF_{\xi\eta} + CF_{\eta\eta} = 0;$$

here

$$
\begin{aligned}
&A = a^2(1+\xi^2) - (u - \xi w)^2, \\
&B = a^2\xi\eta - (u - \xi w)(v - \eta w), \\
&C = a^2(1+\eta^2) - (v - \eta w)^2, \\
&a^2 = a_1^2 - \\
&\quad - \frac{\gamma - 1}{2}(u^2 + v^2 + w^2 - W_1^2), \\
&\quad u = F_\xi, \quad v = F_\eta, \\
&\quad w = F - \xi F_\xi - \eta F_\eta.
\end{aligned}
$$

Since F searches for (6.1), the velocity components along the axes of the Cartesian system of coordinates O_1xyz are represented in the following form:

$$
\left.
\begin{aligned}
u &= W_1(\delta f_{1\xi} + \delta^2 f_{2\xi} + \ldots) = W_1(\delta u_1 + \delta^2 u_2 + \ldots), \\
v &= W_1(\delta f_{1\eta} + \delta^2 f_{2\eta} + \ldots) = W_1(\delta v_1 + \delta^2 v_2 + \ldots), \\
w &= W_1[1 + \delta(f_1 - \xi f_{1\xi} - \eta f_{1\eta}) + \\
&\quad + \delta^2(f_2 - \xi f_{2\xi} - \eta f_{2\eta}) + \ldots] = \\
&\quad = W_1(1 + \delta w_1 + \delta^2 w_2 + \ldots).
\end{aligned}
\right\} \quad (6.2)
$$

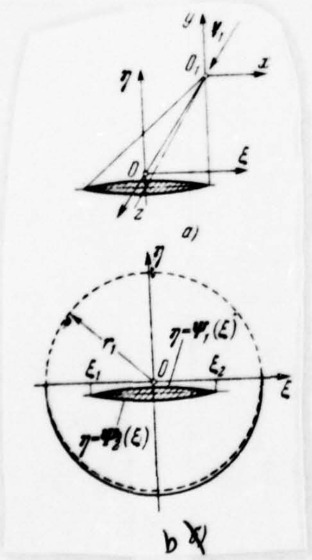


Fig. 44.

Substituting (6.1) in L [F] = 0 and equating terms with identical degrees δ, we obtain equations for f_1, f_2, ...:

$$(1 - m_1^2\xi^2) f_{1\xi\xi} - 2m_1^2\xi\eta f_{1\xi\eta} + (1 - m_1^2\eta^2) f_{1\eta\eta} = 0, \quad (6.3)$$

$$\begin{aligned}(1 - m_1^2\xi^2) f_{2\xi\xi} - 2m_1^2\xi\eta f_{2\xi\eta} + (1 - m_1^2\eta^2) f_{2\eta\eta} = \\ = - M_1^2 \{[2\xi (u_1 - \xi w_1) - (\gamma - 1) w_1 (1 + \xi^2)] f_{1\xi\xi} + \\ + 2 [u_1\eta + v_1\xi - (\gamma + 1) w_1\xi\eta] f_{1\xi\eta} + \\ + [2\eta (v_1 - \eta w_1) - (\gamma - 1) w_1 (1 + \eta^2)] f_{1\eta\eta}\}, \quad (6.4)\end{aligned}$$

where M_1 is a Mach number of undisturbed flow, $m_1 = \sqrt{M^2_1 - 1}$. Let us write equations (6.3), (6.4) in the form

$$A_0 f_{k\xi\xi} + 2B_0 f_{k\xi\eta} + C_0 f_{k\eta\eta} = D_k, \quad (6.5)$$

where $A_0 = 1 - m_1^2\xi^2$, $B_0 = - m_1^2\xi\eta$, $C_0 = 1 - m_1^2\eta^2$; $k = 1, 2$; $D_1 \equiv 0$; D_2 is equal to the right side of the equation (6.4). Inside the Mach cone of the undisturbed flow $\left(r = \sqrt{\xi^2 + \eta^2} < \frac{1}{m_1}\right)$

$$B_0^2 - A_0C_0 = m_1^2 (\xi^2 + \eta^2) - 1 < 0.$$

Let us now move on in equations (6.5) to canonical variables ρ, σ point/item 1.7. Equation (6.5) is equivalent to canonical system of equations:

$$\left.\begin{aligned}\mathcal{P} = A_0u_{k\rho} + B_0v_{k\rho} + \sqrt{A_0C_0 - B_0^2}v_{k\sigma} - D_k\xi_\rho = 0, \\ Q = A_0u_{k\sigma} + B_0v_{k\sigma} - \sqrt{A_0C_0 - B_0^2}v_{k\rho} - D_k\xi_\sigma = 0, \\ N = A_0\eta_\rho - B_0\xi_\rho + \sqrt{A_0C_0 - B_0^2}\cdot\xi_\sigma = 0, \\ R = A_0\eta_\sigma - B_0\xi_\sigma - \sqrt{A_0C_0 - B_0^2}\xi_\rho = 0, \\ S = \Delta w_k + \eta\Delta v_k + \xi\Delta u_k + \\ + \frac{D_k}{\sqrt{A_0C_0 - B_0^2}} (\xi_\rho\eta_\sigma - \xi_\sigma\eta_\rho) = 0.\end{aligned}\right\} \quad (6.6)$$

Here

$$u_k = f_{k\xi},\ v_k = f_{k\eta},\ w_k = f_k - \xi f_{k\xi} - \eta f_{k\eta},\ \Delta = \frac{\partial^2}{\partial\rho^2} + \frac{\partial^2}{\partial\sigma^2} -$$

the operator of Laplace. Furthermore, on boundary of the region on the plane $\rho\sigma$ in which searches for the solution of system (6.6) must be fulfilled the condition

$$dw_k + \eta dv_k + \xi du_k = 0. \qquad (6.7)$$

Page 144.

After transition to polar plane coordinates $\xi\eta$ and $\rho\sigma$ by the formulas

$$\left.\begin{aligned} m_1\xi &= \tilde{r}\cos\theta, \\ m_1\eta &= \tilde{r}\sin\theta, \end{aligned}\right\} \quad \left.\begin{aligned} \rho &= \alpha\cos\beta, \\ \sigma &= \alpha\sin\beta, \end{aligned}\right\} \qquad (6.8)$$

the solution of equations R = N = 0 in system of equations (6.6) are record/written in the form

$$\tilde{r} = \frac{2|\Phi_1(\tau)|}{1+|\Phi_1(\tau)|^2},\quad \theta = \arg\Phi_1(\tau), \qquad (6.9)$$

where $\Phi_1(\tau)$ - arbitrary analytic function $\tau = \alpha e^{i\theta}$; sign "arg" designates functions". For the solution to the remaining equations of system (6.6) let us compose the combinations:

$$\begin{aligned}\mathcal{P}_\rho + Q_\sigma = A_0\Delta u_k + B_0\Delta v_k + A_{0\rho}u_{k\rho} + A_{0\sigma}u_{k\sigma} + B_{0\rho}v_{k\rho} + \\ + B_{0\sigma}v_{k\sigma} + (\sqrt{A_0C_0 - B_0^2})_\rho v_{k\sigma} - (\sqrt{A_0C_0 - B_0^2})_\sigma v_{k\rho} - \\ - (D_k\xi_\rho)_\rho - (D_k\xi_\sigma)_\sigma = 0, \quad (6.10)\end{aligned}$$

$$\begin{aligned}\mathcal{P}_\sigma - Q_\rho = \sqrt{A_0C_0 - B_0^2}\Delta v_k + A_{0\sigma}u_{k\rho} - A_{0\rho}u_{k\sigma} + \\ + B_{0\sigma}v_{k\rho} - B_{0\rho}v_{k\sigma} + (\sqrt{A_0C_0 - B_0^2})_\sigma v_{k\sigma} + \\ + (\sqrt{A_0C_0 - B_0^2})_\rho v_{k\rho} - (D_k\xi_\rho)_\sigma + (D_k\xi_\sigma)_\rho = 0. \quad (6.11)\end{aligned}$$

Let us show that the terms with first-order derivatives, which do not contain D_k in (6.10), (6.11) are the linear combinations of the same in $\mathcal{P} = 0,\ Q = 0$. For example, for equation (6.10) the following equation should be satisfied

$$\begin{aligned}A_{0\rho}u_{k\rho} + A_{0\sigma}u_{k\sigma} + B_{0\rho}v_{k\rho} + B_{0\sigma}u_{k\sigma} + (\sqrt{A_0C_0 - B_0^2})_\rho v_{k\sigma} - \\ - (\sqrt{A_0C_0 - B_0^2})_\sigma v_{k\rho} = \\ = l_1[A_0u_{k\rho} + B_0v_{k\rho} + \sqrt{A_0C_0 - B_0^2}v_{k\sigma}] + \\ + l_2[A_0u_{k\sigma} + B_0v_{k\sigma} - \sqrt{A_0C_0 - B_0^2}v_{k\rho}] \quad (6.12)\end{aligned}$$

for any solution u_k, v_k, w_k of system (6.6), where l_1 and l_2 are certain functions ρ, σ. Equating the coefficients for the corresponding derivatives of u, v, to ρ, σ in (6.12), we will obtain four conditions for l_1 and l_2:

$$\left.\begin{aligned}&A_{0\rho} = l_1A_0, \quad A_{0\sigma} = l_2A_0,\\ &B_{0\rho} - (\sqrt{A_0C_0 - B_0^2})_\sigma = l_1B_0 - l_2\sqrt{A_0C_0 - B_0^2},\\ &B_{0\sigma} + (\sqrt{A_0C_0 - B_0^2})_\rho = l_2B_0 + l_1\sqrt{A_0C_0 - B_0^2}.\end{aligned}\right\} \quad (6.13)$$

For the compatibility of system (6.13), the equations obtained by the substitution of $l_1=\frac{A_{0\rho}}{A_0}, l_2=\frac{A_{0\sigma}}{A_0}$ in the last two equation of the system should be satisfied. After multiplying them by A_0^{-1} they can be reduced to

$$\left(\frac{B_0}{A_0}\right)_\rho=\left(\frac{\sqrt{A_0C_0-B_0^2}}{A_0}\right)_\sigma, \quad \left(\frac{B_0}{A_0}\right)_\sigma=-\left(\frac{\sqrt{A_0C_0-B_0^2}}{A_0}\right)_\rho. \tag{6.14}$$

The same conditions (6.14) are obtained from equations (6.11).

Conditions (6.14) are satisfied, if

$$\frac{B_0}{A_0}+i\frac{\sqrt{A_0C_0-B_0^2}}{A_0}=\varphi_1(\tau),$$

where $\phi_1(\tau)$ is certain analytic function $\tau=\alpha e^{i\theta}$.

Since $A_0=1-m_1^2\xi^2=1-\tilde{r}^2\cos^2\theta$, $B_0=-m_1^2\xi\eta=-\tilde{r}^2\cos\theta\sin\theta$, $C_0=1-m_1^2\eta^2=1-\tilde{r}^2\sin^2\theta$, $A_0C_0-B_0^2=1-\tilde{r}^2$, that

$$\frac{B_0}{A_0}+i\frac{\sqrt{A_0C_0-B_0^2}}{A_0}=\frac{-\tilde{r}^2\cos\theta\sin\theta+i\sqrt{1-\tilde{r}^2}}{1-\tilde{r}^2\cos^2\theta}=$$

$$=i\frac{1+\Phi_1^2(\tau)}{1-\Phi_1^2(\tau)}=\varphi_1(\tau),$$

if one takes into account, that

$$\tilde{r}=\frac{2|\Phi_1(\tau)|}{1+|\Phi_1(\tau)|^2}, \quad |\Phi_1(\tau)|=\Phi_1(r)e^{-i\theta},$$

$$\cos\theta=\frac{e^{i\theta}+e^{-i\theta}}{2}, \qquad \sin\theta=\frac{e^{i\theta}-e^{-i\theta}}{2i}.$$

SUBJECT CODE 142D

Page 146.

Replacing terms with first-order derivatives, not containing D_k, by terms with D_k from equations $\mathcal{P} = Q = 0$, let us give equations (6.10), (6.11) to the form

$$A_0\Delta u_k + B_0\Delta v_k + \frac{D_k}{A_0}(A_{0\rho}\xi_\rho + A_{0\sigma}\xi_\sigma) - (D_k\xi_\rho)_\rho - (D_k\xi_\sigma)_\sigma = 0, \quad (6.15)$$

$$\sqrt{A_0C_0 - B_0^2}\cdot\Delta v_k + \frac{D_k}{A_0}(A_{0\sigma}\xi_\rho - A_{0\rho}\xi_\sigma) + (D_k\xi_\sigma)_\rho - (D_k\xi_\rho)_\sigma = 0. \quad (6.16)$$

Furthermore,

$$\Delta w_k + \eta\Delta v_k + \xi\Delta u_k + \frac{D_k}{\sqrt{A_0C_0 - B_0^2}}(\xi_\rho\eta_\sigma - \xi_\sigma\eta_\rho) = 0. \quad (6.17)$$

From equations (6.15)-(6.17) it follows that for k = 1, when $D_1 \equiv 0$, $\Delta u_1 = \Delta v_1 = \Delta w_1 = 0$, i.e., u_1, v_1, w_1 there are harmonic functions of variables ρ and σ. In this case

$$w_1 + is_1 = \frac{1}{m_1}\mathcal{F}_1(\tau), \quad (6.18)$$

where $\mathcal{F}_1(\tau)$ there is arbitrary analytical function $\tau = \alpha e^{i\beta} = \rho + i\sigma$. Components u_1, v_1 are determined from the formula

$$d(u_1 + iv_1) = -\frac{1}{2}\left[\Phi_1(\tau)d\mathcal{F}_1(\tau) + \frac{1}{\Phi_1(\tau)}\overline{d\mathcal{F}_1(\tau)}\right]. \quad (6.19)$$

In fact, multiplying (6.19) by $e^{-i\theta}$ and separate/liberating the real part of the product, we will obtain

$$dw_1 + \eta dv_1 + \xi du_1 = 0.$$

Hence follow the equalities

$$w_{1\rho} + \eta v_{1\rho} + \xi u_{1\rho} = 0, \quad w_{1\sigma} + \eta v_{1\sigma} + \xi u_{1\sigma} = 0,$$

from which by differentiation with respect to ρ and σ, taking into account equations N = R = 0, $\Delta u_1 = \Delta v_1 = \Delta w_1 = 0$, we obtain $\mathcal{P} = 0$, $Q = 0$.

Thus, solution of system of equations (6.6) with k = 1 is given by the formulas of linear theory [2].

Page 147.

If we place $\Phi_1(\tau) = \tau$, then $\tilde{r} = 2\alpha/1 + \alpha^2$, $\theta = \beta$ and plane τ is the plane, introduced by A. Busemann [2].

Subsequently will be required only the equation for w_2. It is obtained from relationship/ratios (6.15)-(6.17) by means of exception/elimination Δu_2, Δv_2 and has the form

$$\Delta w_2 = \frac{1}{\sqrt{A_0 C_0 - B_0^2}} [\eta (D_{2\rho}\xi_\sigma - D_{2\sigma}\xi_\rho) - \xi (D_{2\rho}\eta_\sigma - D_{2\sigma}\eta_\rho) - (\eta_\sigma\xi_\rho - \eta_\rho\xi_\sigma) D_2]. \quad (6.20)$$

For case of $\tilde{r} = 2\alpha/(1 + \alpha^2)$, $\theta = \beta$, which will be examined further, equation (6.20) can be written as follows:

$$\left.\begin{aligned} &\Delta w_2 = \frac{4}{m_1^2(1+\alpha^2)}\left(\frac{\alpha D_{2\alpha}}{1-\alpha^2} - \frac{D_2}{1+\alpha^2}\right), \\ &\left(\Delta = \frac{\partial^2}{\partial\rho^2} + \frac{\partial^2}{\partial\sigma^2} = \frac{\partial^2}{\partial\alpha^2} + \frac{1}{\alpha}\frac{\partial}{\partial\alpha} + \frac{1}{\alpha^2}\frac{\partial^2}{\partial\beta^2}\right). \end{aligned}\right\} \quad (6.21)$$

In polar coordinates D_2, it is determined from the formula

$$D_2 = M_1^2\{f_{1rr}[f_1(\gamma-1+(\gamma+1)r^2)- -(\gamma+1)(1+r^2)rf_{1r}]\}-2f_{1\theta}\left(\frac{1}{r}f_{1r\theta}-\frac{1}{r^2}f_{1\theta}\right)+ +(\gamma-1)\left(\frac{1}{r^2}f_{1\theta\theta}+\frac{1}{r}f_{1r}\right)(f_1-rf_{1r}). \quad (6.22)$$

The polar radii r and $\tilde{r}$ are connected by the relationship $m_1 r = \tilde{r}$, so that to the Mach cone of undisturbed flow $r = r_1 = 1/m_1$ correspond values $\tilde{r} = \alpha = 1$. From (6.22) it follows that in vicinity $r = r_1 D_2$ is represented by expansion $D_2 = D_{(0)}(\theta) + D_{(1)}(\theta)(r_1 - r) + \ldots$, from which, after conversion to α and β we obtain

$$D_2 = D_{(0)}(\beta) + D_1(\beta)\frac{(1-\alpha)^2}{m_1(1+\alpha^2)} + \ldots \quad (6.23)$$

From (6.23) it follows that the right side of equation (6.21) is limited when $\alpha \rightarrow 1$.

Page 148.

6.3. Boundary conditions in theories of first and second approach/approximation. For the determination of boundary conditions when $\alpha \rightarrow 1$ we will use expansions (5.19), (5.21). In the case of slender bodies, it is necessary to assume $\lambda^{1/2} = \delta$ and to present function $\varphi(\theta, \lambda)$, $\psi(\theta, \lambda)$ in the form

$$\left.\begin{aligned}\varphi(\theta,\lambda) &= \varphi_0(\theta) + \lambda^{1/2}\varphi_{1/2}(\theta) + \ldots\\ \psi(\theta,\lambda) &= \psi_0(\theta) + \lambda^{1/2}\psi_{1/2}(\theta) + \ldots\end{aligned}\right\} \quad (6.24)$$

Substituting expansions (6.24) in (5.19), (5.20) and set/assuming $\lambda^{1/2} = \delta$, we will obtain in the vicinity of the Mach cone, $r = r_1$,

$$w = W_1 + \delta\begin{Bmatrix}-\varphi_0^{1/2}(\theta)\\ +\psi_0^{1/2}(\theta)\end{Bmatrix}\frac{W_1}{\gamma+1}\frac{m_1^3}{M_1^4}\frac{4\sqrt{3}}{3}[(r_1-r)^{1/2}+\ldots] - -\delta^2\begin{Bmatrix}\varphi_0(\theta)\\ \psi_0(\theta)\end{Bmatrix}\frac{W_1}{\gamma+1}\frac{m_1^3}{M_1^4}\frac{8}{3}[1+\ldots]+o(\delta^2).$$

Hence it follows that in vicinity $r = r_1 (\alpha = 1)$

$$w_1 = \begin{Bmatrix} -\varphi_0^{1/2}(\theta) \\ +\psi_0^{1/2}(\theta) \end{Bmatrix} \frac{1}{\gamma+1} \frac{m_1^3}{M_1^4} \frac{4\sqrt{3}}{3} (r_1 - r)^{1/2} + \ldots =$$

$$= \begin{Bmatrix} -\varphi_0^{1/2}(\beta) \\ +\psi_0^{1/2}(\beta) \end{Bmatrix} \frac{1}{\gamma+1} \frac{m_1^{5/2}}{M_1^4} \frac{2\sqrt{6}}{3} (1 - \alpha) + \ldots, \quad (6.25)$$

$$w_2 = -\begin{Bmatrix} \varphi_0(\theta) \\ \psi_0(\theta) \end{Bmatrix} \frac{1}{\gamma+1} \frac{m_1^3}{M_1^4} \frac{8}{3} + \ldots = -\begin{Bmatrix} \varphi_0(\beta) \\ \psi_0(\beta) \end{Bmatrix} \frac{1}{\gamma+1} \frac{m_1^3}{M_1^4} \frac{8}{3} + \ldots \quad (6.26)$$

Expression (6.25) shows that $w_1 \rightarrow 0$, $\alpha \rightarrow 1$, as this is accepted in linear theory. After determination of w_1 functions $\varphi_0(\beta)$, $\psi_0(\beta)$ they are determined from the formula

$$\begin{Bmatrix} \varphi_0(\beta) \\ \psi_0(\beta) \end{Bmatrix} = \frac{(\gamma+1)^2 M_1^8}{m_1^5} \frac{3}{8} \left[\left(\frac{\partial w_1}{\partial \alpha} \right)_{\alpha=1} \right]^2,$$

and, according to (6.26),

$$w_2 = -(\gamma+1) \frac{M_1^4}{m_1^2} \left[\left(\frac{\partial w_1}{\partial \alpha} \right)_{\alpha=1} \right]^2 \text{ with } \cancel{\text{при}}\ \alpha = 1. \quad (6.27)$$

(Analogously are obtained formulas for u_2, v_2 when $\alpha = 1$).

Page 149.

Let us derive boundary condition on the surface of the streamlined body whose equation is accepted in the form

$$\eta = \delta\psi_n(\xi); \quad n = 1, 2; \ \xi_1 \leqslant \xi \leqslant \xi_2. \quad (6.28)$$

Body surface is stream surface; therefore along it must be fulfilled the relationship/ratio

$$\frac{d\xi}{u-\xi w}=\frac{d\eta}{v-\eta w}.$$

Substituting here (6.28), expanding functions in series according to degrees δ and equalizing the coefficients with identical degrees δ, we will obtain the following boundary conditions:

$$\left.\begin{aligned}&v_1(\xi,\pm 0)=\psi_n(\xi)-\xi\psi_n'(\xi),\\&v_2(\xi,\pm 0)=u_1(\xi,\pm 0)\psi_n'(\xi)+w_1(\xi,\pm 0)\times\\&\quad\times[\psi_n(\xi)-\xi\psi_n'(\xi)]-v_1(\xi,\pm 0)\psi_n(\xi)\end{aligned}\right\}\quad(6.29)$$

and so forth.

Let us note now that

$$\frac{dv_k(\xi,0)}{d\xi}=\left(\frac{\partial^2 f_k}{\partial\xi\,\partial\eta}\right)_{\eta=0},$$

$$\left(\frac{\partial w_k}{\partial\eta}\right)_{\eta=0}=\left[\frac{\partial}{\partial\eta}(f_k-\xi f_{k\xi}-\eta f_{k\eta})\right]_{\eta=0}=$$
$$=\left(\frac{\partial^2 f_k}{\partial\xi\,\partial\eta}\right)_{\eta=0}(-\xi)=-\xi\frac{dv_k(\xi,0)}{d\xi},$$

therefore according to (6.29) the boundary conditions of the nonseparated flow of body can be written in the form

$$\left.\begin{aligned}&\left(\frac{\partial w_1}{\partial\eta}\right)_{\eta=\pm 0}=\xi^2\psi_n''(\xi)\ (n=1,2;\ \xi_1\leqslant\xi\leqslant\xi_2),\\&\left(\frac{\partial w_2}{\partial\eta}\right)_{\eta=\pm 0}=-\xi\frac{d}{d\xi}[u_1\psi_n'+w_1(\psi_n-\xi\psi_n')-v_{1\eta}\psi_n]_{\eta=\pm 0}.\end{aligned}\right\}\quad(6.30)$$

After transition to variables ρ, σ conditions (6.30) take this form:

$$\left(\frac{\partial w_k}{\partial\sigma}\right)_{\sigma=\pm 0}=H_{kn}(\rho)\qquad(\rho_1\leqslant\rho\leqslant\rho_2)\qquad(6.31)$$
$$k=1,2;\quad n=1,2,$$

where $H_{kn}(\rho)$ - known functions of ρ (ρ_1 and ρ_2 correspond to ξ_1 and

ξ_2; n = 1 for σ = +0, n = 2 for σ = -0).

Page 150.

6.4. Leading edges in slender-body theory. Thus, the problem of the determination of first approximation for slender bodies came to the task of the solution to the equation of Laplace $\Delta w_1 = 0$ in the range, depicted on Fig. 45, which is unit circle $\alpha \leqslant 1$ with cut/section along the diameter $\sigma = 0$, at which the boundary conditions are assigned by formula (6.31), furthermore, $w_1 = 0$ when $\alpha = 1$. Let us note that for the bodies whose cross section by plane z = 1 is limited by the line segments, by the methods of the theory of conformal mappings in a number of cases it is possible to obtain the solution of problem in the locked form (see on this question [3, 4]).

For the determination of the second approach/approximation, it is necessary to determine only w_2, since potential f_2 is expressed as w_2 by the formula

$$f_2 = -r \int_{r_1}^{r} w_2 r^{-2} dr.$$

The task of determination w_2 is reduced to the solution to the equation of Poisson (6.21) in the range where was determined function w_1, with boundary conditions (6.27), (6.31). Pressure coefficient

C_p is calculated from formula $C_p = -2w_1 - [2w_2 - m^2{}_1 w_1^2 + v_1^2 + u_1^2]$. The solution to the formulated boundary-value problems for w_1 and w_2 not is singular. Actually, a difference in two solutions satisfies the equation of Laplace and uniform boundary conditions. It is not difficult to construct harmonic function with special feature/peculiarity at point $\rho = \rho_2$, $\sigma = 0$, that satisfies these requirements. Let us examine, for example, harmonic function $w^{(1)} = (\alpha^*)^{-n} \cos n\beta^*$, where α^*, β^* are polar coordinates with center at point $\rho = \rho_2$, $\sigma = 0$ in Fig. 45, n - positive integer number. When $\sigma = \pm 0$ $\frac{\partial w^{(1)}}{\partial \sigma} = 0$ everywhere, besides point $\rho = \rho_2$, and on circumference $\alpha = 1$, $w^{(1)}$ takes the values equal to at the points, symmetrical relative to axis $O\rho$. If we designate by $w^{(2)}$ the harmonic in circle $\alpha < 1$ function, which accepts with the circumference $\alpha = 1$ of values, reciprocal in terms of sign values $w^{(1)}$ then $\frac{\partial w^{(2)}}{\partial \sigma} = 0$ when $\sigma = \pm 0$ and the harmonic function

$$w = w^{(1)} + w^{(2)}$$

is the unknown "its own" solution of problem.

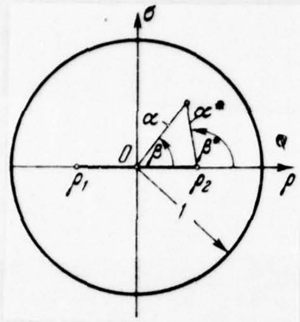


Fig. 45.

Page 151.

Analogously is matter also with point $\rho = \rho_1$, $\sigma = 0$. [Furthermore, the expressions for w_1 and w_2 contain on one arbitrary constant which appear because of transition from boundary conditions (6.29) to conditions (6.30). After determination w_1, w_2, v_1, v_2, these constants are determined by the satisfaction of conditions (6.29) at one point (for example, see [3])].

The nonuniqueness of the solution to boundary-value problems for w_1, w_2 is connected with those by the fact that expansion (6.1) does not represent the conical potential F in the vicinity of the leading edges of body, since near leading edges the flow is strongly agitated even with small δ. The appearing here difficulties are analogous to the difficulties, available in the theory of the fine/thin

airfoil/profile, streamlined with the subsonic flow of gas (see [53]), and the existing methods of the overcoming of these difficulties are also analogous to the methods of two-dimensional problem of gas dynamics.

In linear theory the nonuniqueness of solution is removed, actually, with the aid of the requirement so that in the solution would be contained the special feature/peculiarities of the lowest order of all possible. It is possible also to enter also in second-order theory. But this approach is formal, and for obtaining correct solution of outside the vicinities leading edges it is necessary to examine flows in the vicinity of these edges. Thus, appear two tasks: first, it is necessary to give the rule of selection of solution, valid of outside the small vicinities leading edges, and, in the second place, to indicate, as one should correct the results, given by this "formal" solution, with those in order to consider the effect of edges.

If we follow analogy with the task of the flow of subsonic flow about the fine/thin airfoil/profile of gas, then it is necessary to distinguish the cases when the edge the rounded, tapered symmetrical or tapered of general view (during flow about which gas overflows from one "side" of edge to another). Most powerful effect on flow has chamfered edge, since in its vicinity is deceleration point for flow

in the plane, perpendicular to the edge. However, since this point is not deceleration point for three-dimensional/space flow, the effect of edge will be smaller than in two-dimensional gas-dynamic problem.

As shown in M. Van Dyke's work [124], for an elliptical cone at zero angle of attack correct first approximation for pressure on body surface is obtained from the "formal" solution, which satisfies the requirements for the "smallest order of special feature/peculiarities on leading edges", with the aid of the procedure, described in [125]. The idea of this method consists in that, then is examined flow of the incompressible fluid about the parabola, which has the same radius of curvature, which it has a section of body as the plane, perpendicular to edge. The solution of problem for a parabola takes the simple form. Then "formal" solution is multiplied by the relation of solution for a parabola and its expansions in a series in the low parameter. For the second approach/approximation this procedure makes it possible to exclude in solution algebraic special feature/peculiarities on leading edges, on logarithmic special feature/peculiarities they remain in expression for pressure coefficient.

Page 152.

During the stricter solution of problem, it is necessary to introduce

various expansion in terms of the small parameter in the vicinity of leading edge and by means of its coupling with expansion (6.1) to obtain the complete solution of the problem of the second approach/approximation.

In the case of the symmetrical wedge-shaped edge, streamlined at zero angle of attack, in the theories of the first and second approach/approximation appear only integrated (logarithmic) special feature/peculiarities on leading edges (as this it follows from the results for two-dimensional problem); therefore leading edge effect here can be disregarded and used "formal" solutions in the first and second approach/approximations.

If the leading edges of body are close to the Mach cone of undisturbed flow, then it is necessary to consider their effect in all cases. To this question are dedicated works of E. Sun [126], L. Fraenkel and R. Watson [127], which showed that because of the transonic character of streamlining of leading wing edges in the coefficient of the wave impedance of body appear terms $O(\delta^{7/6})$, where δ is a low geometric parameter, for example, the thickness ratio of body, while C_x according to the linear theory $O(\delta^2)$. Thus, the task of determining the second approach/approximation in these cases considerably becomes complicated (see on this question also [128]). Most important is the case when flow blows away from leading edges.

This occurs for the tapered edges, streamlined at an angle of attack, and chamfered edges at sizable angles of attack. If angle of attack is small and the wingspan small, then separating phenomena can be disregarded (see [128]). At not low angles of attack or considerable sweepback of wing, separating phenomena play the significant role in the formation of flow about body.

Since precise formulation of the problem on the basis of the equations of nonviscous gas is here impossible, it is necessary for determining solution (6.1) outside separating zone to make the supplementary assumptions, which rest on the results of experiment. Detached flows are examined in §9.

6.5. Results of second-order theory it composes determination of particular integral of equation of Poisson (6.21). If we pass from variables ρ, σ to variables $\tau = \rho + i\sigma$ and $\bar{\tau} = \rho - i\sigma$, then the unknown particular solution it is possible formally to write in the form of twofold integral in terms of τ and $\bar{\tau}$ of the first part of the equation (6.21) whose calculation, however, requires large work. Furthermore, remain difficulties with special feature/peculiarities on leading edges and the determination of the harmonic function with the aid of which are satisfied the boundary conditions. (Here are not examined the approximate particular integrals of the type, similar found in work [129]). F. Moore [123] found the second

approach/approximation for the conical body, cross section of which is rhomb, at zero angle of attack.

Page 153.

Formulas giving the solution of the problem of the second approach/approximation, were so/such bulky that the author did not give them in work [123]. By these formulas was designed one specific case. H. Tan [130] produced the calculations of pressure on body surface for many cases. The results of the calculations of F. Moore and H. Tan are given partially in M. Lighthill's article [131] in which also is given their detailed analysis. (Let us note that there are doubts as to the correctness of these calculations; see [124, 132]).

M. Van Dyke in work [124] with the method of iterations found in the locked form the second approach/approximation in the theory of the extended bodies for an elliptical cone at the zero angle of attack, from which was also obtained the second approach/approximation of slender-body theory.

At present flows about smooth bodies can be designed more accurately, for example, by finite-difference method [29], called the method of establishment.

For those cases when flow blows away from leading wing edges (and for which there are no numerical methods of calculation), the task of the second approach/approximation is of large interest, but in no way developed at present. For wings with powerful sweepback, approximate solutions are examined in §9.

§7. Higher approach/approximations for the extended bodies and method of linearized characteristics.

7.1. Higher approach/approximations for extended bodies. At present supersonic flows about the extended conical bodies can be designed with preassigned accuracy, for example, with the aid of the method of establishment, if the flow about the bodies is nonseparable. Analytical theories retain their value in essence for the very elongated bodies which are used as components of different measuring meters and flow about which is difficult to calculate by finite-difference methods.

Basic results on higher approach/approximations for the extended bodies are obtained to 1955 and are examined in detail in M.

Lighthill's work [131]. For this reason here is given only short enumeration of the most important results, which relate to the extended conical bodies.

Page 154.

D. Broderick [133] found pressure coefficient C_p on the surface of cone with half-angle ε, at zero angle of attack, in the form

$$C_p = -t^2 + 2t^2 \ln \frac{2}{(M_1^2-1)^{1/2} t} + \\ + 3(M_1^2-1) t^4 \left[\ln \frac{2}{(M_1^2-1)^{1/2} t}\right]^2 - (5M_1^2-1) t^4 \ln \times \\ \times \frac{2}{(M_1^2-1)^{1/2} t} + \left[\frac{13}{4} M_1^2 + \frac{1}{2} + \frac{(\gamma+1) M_1^4}{M_1^2-1}\right] t^4 + o(t^4),$$

where t = tg. ε, γ - adiabatic index, M_1 - the Mach number of undisturbed flow. The method of the solution of problem of Broderick consisted in the fact that the velocity potential of disturbance/perturbation was decompose/expanded in a series according to degrees of t and ln t. M. Van Dyke [54] by the method of iterations obtained in the locked form the solution this same problem, which proved to be more precise than the solution of Broderick. M. Van Dyke obtained also the second approach/approximation for a cone at an angle of attack, which, however, appears by very bulky and is not given in work [54]. The solution of the problem of the second approach/approximation for bodies of revolution at an angle of attack (including for a round

cone) is found by M. Lighthill in work [135].

The elegant theory of first approximation for the extended bodies with arbitrary cross sections belongs to G. Ward and is presented in detail in its book [121].

The principles of obtaining the second approach/approximation for the extended bodies of general view by the method of iterations are developed by M. Lighthill in work [131]. M. Van Dyke in work [124] with the method of iterations found in the locked form the solution of the problem of the second approach/approximation for an elliptical cone at zero angle of attack. This solution will agree well with experiment and serves as standard for the different approximate theories.

7.2. Method of linearized characteristics. In the theories of the fine/thin and extended bodies main stream on which are superimposed the disturbance/perturbations, is uniform flow. It is possible as basic to examine any flow.

Page 155.

If we as basic take axisymmetric flow about round cone and to be restricted to the disturbance/perturbations of the first order, then

as a result we will obtain the method of linearized characteristics, proposed by A Ferri in work [136].

The essence of this method consists in the fact that if for the determination of main stream it is necessary to solve nonlinear differential equations, then during the determination of disturbance/perturbations it is necessary to deal with linear equations, which considerably simplifies task. In work [120] by the method of linearized characteristics are examined numerous examples of the flow about the conical bodies with cross section in the form of ellipse, triangle with the rounded edges, etc. In works [137, 138], is given certain improvement of method. The accuracy of method of linearized characteristics, on the whole, is not high (see [137]), but it it is universal and simple in appendices, what is valuable quality for estimate calculations; at present this method is applied mainly at the hypersonic speeds where it possesses larger accuracy.

Further are set forth the main points of method of linearized characteristics. Let us examine conical shock (see Fig. 46) whose equation in spherical coordinates takes this form:

$$\vartheta_s = \vartheta_{0s} + \sum_1^\infty \vartheta_{ms} \cos m\varphi + \sum_1^\infty \vartheta_{ns} \sin n\varphi, \qquad (7.1)$$

where $\vartheta_{0s}, \vartheta_{ms}, \vartheta_{ns}$ are constant, moreover

$$\vartheta_{ms}/\vartheta_{0s} < 1, \quad \vartheta_{ns}/\vartheta_{0s} < 1,$$

so that their squares $\vartheta_{ms}, \vartheta_{ns}$ and their products can be disregarded in comparison with ϑ_{0s}.

Thus, shock wave is close about form to round cone. This occurs for the conical bodies, which possess the moderate relative thickness. Let us assume for simplification that the streamlined body has a plane of symmetry, then the second sum in (7.1) turns into zero.

Page 156.

From usual laws of conservation on oblique shock wave, immediately behind wave, the components of the velocity in the direction of increase R, ϑ, φ they are record/written respectively in the form

$$\left.\begin{aligned} u &= W_1 \cos\vartheta_s, \\ v &= -\frac{\gamma-1}{\gamma+1}\,\frac{V_{np}^2 - W_1^2\cos\vartheta_s - W_1^2\sin^2\vartheta_s\cdot\sin^2\beta}{W_1\sin\vartheta_s} - \\ &\qquad - W_1\sin\vartheta_s\cdot\sin^2\beta, \\ w &= -W_1\sin\vartheta_s\sin\beta\cos\beta + \\ &+ \frac{\gamma-1}{\gamma+1}\left(V_{np}^2 - W_1^2\cos\vartheta_s - W_1^2\sin^2\vartheta_s\sin^2\beta\right)\frac{\operatorname{tg}\beta}{W_1\sin\vartheta_s}, \end{aligned}\right\} \tag{7.2}$$

where W_1, V_{np} are velocity and the maximum speed of undisturbed flow,

$$\operatorname{tg}\beta = \frac{1}{\sin\vartheta_s}\,\frac{d\vartheta_s}{d\varphi} = \sum_1^\infty \frac{m\vartheta_{ms}\sin m\varphi}{\sin\vartheta_s}.$$

Disregarding in expressions (7.2) the terms, which have the higher order of smallness, than ϑ_{ms}, we will obtain

$$
\left.\begin{aligned}
u &= W_1\cos\vartheta_{0s} - W_1\sin\vartheta_{0s}\sum_1^\infty \vartheta_{ms}\cos m\varphi,\\
v &\doteq -\frac{\gamma-1}{\gamma+1}\,\frac{V_{\text{пр}}^2 - W_1^2\cos^2\vartheta_{0s}}{W_1\sin\vartheta_{0s}} - \frac{\gamma-1}{\gamma+1}\cos\vartheta_{0s}\times\\
&\quad\times\left(2W_1 - \frac{V_{\text{пр}}^2 - W_1^2\cos^2\vartheta_{0s}}{W_1\sin^2\vartheta_{0s}}\right)\sum_1^\infty \vartheta_{ms}\cos m\varphi,\\
w &= \left[W_1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V_{\text{пр}}^2 - W_1^2\cos\vartheta_{0s}}{W_1\sin^2\vartheta_{0s}}\right)\right]\sum_1^\infty m\vartheta_{ms}\sin m\varphi
\end{aligned}\right\} \quad (7.3)
$$

when $\vartheta = \vartheta_s$.

The components of the velocity during after the shock wave, determined by equation (7.1), search for in the form of the series

$$
\left.\begin{aligned}
u &= u_0 + \sum_1^\infty \vartheta_{ms}u_m\cos m\varphi,\\
v &= v_0 + \sum_1^\infty \vartheta_{ms}v_m\cos m\varphi,\\
w &= \sum_1^\infty \vartheta_{ms}w_m\, m\sin m\varphi.
\end{aligned}\right\} \quad (7.4)
$$

Page 157.

Here u_0, v_0 - the components of the velocity in the axisymmetric flow, which corresponds to the shock wave, defined by equation $\vartheta = \vartheta_{0s}$; u_m, v_m, w_m - some functions only ϑ (just as u_0, v_0). Expressions (7.4) satisfy conditions on shock wave with an accuracy to small ϑ_{ms} inclusively, if when $\vartheta = \vartheta_{0s}$ are carried out the equalities

$$\left.\begin{aligned}
u_m &= -W_1 \sin\vartheta_{0s} - v_0,\\
v_m &= -\frac{\gamma-1}{\gamma+1}\cos\vartheta_{0s}\left(2W_1 - \frac{V_{\text{пр}}^2 - W_1^2\cos\vartheta_{0s}}{W_1\sin^2\vartheta_{0s}}\right) - \\
&\qquad - \left(\frac{dv_0}{d\vartheta}\right)_{\vartheta=\vartheta_{0s}},\\
w_m &= W_1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V_{\text{пр}}^2 - W_1^2\cos^2\vartheta_{0s}}{W_1\sin^2\vartheta_{0s}}\right) = W_1 + \frac{v_0}{\sin\vartheta_{0s}}.
\end{aligned}\right\} \quad (7.5)$$

(It is here taken into account, that in axisymmetric flow $du_0/d\vartheta = v_0$; furthermore,

$$\frac{dv_0}{d\vartheta} = \left[-u_0 - \frac{u_0 + v_0\operatorname{ctg}\vartheta}{1-\frac{v_0^2}{a_0^2}}\right];$$

See formulas (2.29), (2.30)).

Substituting expansions (7.4) in the equation of continuity (1.15) and retaining only first-order terms relatively ϑ_{ms}, we will obtain for the ideal gas of equation for determining u_m, v_m, w_m:

$$\begin{aligned}
&\left(u_m + \frac{dv_m}{d\vartheta}\right)\left[1-\left(\frac{v}{a}\right)_0^2\right] = \qquad (7.6)\\
&= -v_m\left\{\operatorname{ctg}\vartheta + \left[2\left(\frac{v}{a}\right)_0 + (\gamma-1)\left(\frac{v}{a}\right)_0^3\right]\frac{\left(\frac{u}{a}\right)_0 + \left(\frac{v}{a}\right)_0\operatorname{ctg}\vartheta}{1-\left(\frac{v}{a}\right)_0^2}\right\} - \\
&\quad - u_m\left[1 + (\gamma-1)\frac{\left(\frac{u}{a}\right)_0 + \left(\frac{v}{a}\right)_0\operatorname{ctg}\vartheta}{1-\left(\frac{v}{a}\right)_0^2}\left(\frac{u}{a}\right)_0\left(\frac{v}{a}\right)_0^2\right] - \frac{m^2 w_m}{\sin\vartheta}.
\end{aligned}$$

Page 158.

Here the subscript zero designates values that relate to axisymmetric flow.

From equations (1.16), (1.16a) ensues the equality

$$u \sin \vartheta \frac{\partial S}{\partial \vartheta} = -w \frac{\partial S}{\partial \varphi},$$

from which it follows that the derivative $\frac{\partial S}{\partial \vartheta}$ there will be order ϑ_{ms}^2 and it can be disregarded, since $\frac{\partial S}{\partial \varphi}$ it is of the order ϑ_{ms}, which is evident from equation (1.16).

For these reasons expression for entropy S can be written in the form

$$S = S_0 + S_1 \sum_0^\infty \vartheta_{ms} \cos m\varphi, \qquad (7.7)$$

where the constant S_1 is determined with the aid of relationship/ratios on shock wave from the formula

$$S_1 = \left(\frac{dS}{d\vartheta}\right)_{\vartheta=\vartheta_{0s}}.$$

Expression (7.7) is correct everywhere during after shock wave, with the exception of thin "vorticity layer" near the surface of the streamlined body (see §8).

After the substitution of expansions (7.4), (7.7) into equation (1.16a) we will obtain

$$v_m = \frac{du_m}{d\vartheta}, \qquad (7.8)$$

while equation (1.16) gives

$$-T_0S_1 = v_0 \sin\vartheta \frac{dw_m}{d\vartheta} + u_0' u_m + v_0 v_m + u_0 w_m \sin\vartheta + $$
$$+ v_0 w_m \cos\vartheta = u_0(u_m + w_m \sin\vartheta) + v_0 \frac{d}{d\vartheta}(u_m + w_m \sin\vartheta). \quad (7.9)$$

From linear differential equation (7.9) and initial condition $u_m + w_m \sin\vartheta = 0$ when $\vartheta = \vartheta_{s0}$ [see (7.5)] is located combination $u_m + w_m \sin\vartheta$ as known function ϑ.

Page 159.

In the case of ideal gas, we will obtain

$$u_m + w_m \sin\vartheta =$$
$$= -\left(\frac{S_1}{\gamma R}\right) \cdot a_0^{\frac{1}{\gamma-1}} (-v_0 \sin\vartheta)^{1/2} \int_{\vartheta_{0s}}^{\vartheta} \frac{a_0^{\frac{2\gamma-3}{\gamma-1}}}{v_0(-v_0 \sin\vartheta)^{1/2}} d\vartheta. \quad (7.9a)$$

Thus, the task of the determination of flow after shock wave came to the solution to the ordinary differential equations of the second order for u_m with initial conditions (7.5). Let us note that u_m, v_m, w_m they depend only on $M_1, \gamma, \vartheta_{0s}$ and do not depend on ϑ_{ms}, i.e., on the concrete/specific/actual form of disturbance/perturbations in the equation of shock wave (7.1).

The equation of the surface of the conical body during flow about which is formed the shock wave, determined by equation (7.1), he is record/written in the form

$$\vartheta_b = \vartheta_{0b} + \sum_1^\infty \vartheta_{mb} \cos m\varphi. \quad (7.10)$$

During nonseparated flow body surface must be conical stream surface, i.e., on it must be fulfilled the condition

$$\frac{v}{w}=\frac{1}{\sin\vartheta_b}\frac{d\vartheta_b}{d\varphi}=\frac{1}{\sin\vartheta_b}\left(-\sum_1^\infty m\vartheta_{mb}\sin m\varphi\right). \quad (7.11)$$

But on body surface $(\vartheta=\vartheta_b)$ v, w it is possible to present in the form of Taylor series according to degrees $\vartheta_b-\vartheta_{0b}$, i.e.,

$$\begin{aligned} v = {} & v_0|_{\vartheta_{0b}}+\frac{dv_0}{d\vartheta}\Big|_{\vartheta_{0b}}(\vartheta_b-\vartheta_{0b})+\\ & +\frac{d^2v_0}{d\vartheta^2}\Big|_{\vartheta_{0b}}\frac{(\vartheta_b-\vartheta_{0b})^2}{2}+\sum_1^\infty\vartheta_{ms}\cdot v_m|_{\vartheta_{0b}}\cos m\varphi+\\ & +\sum_1^\infty\vartheta_{ms}\frac{dv_m}{d\vartheta}\Big|_{\vartheta_{0b}}(\vartheta_b-\vartheta_{0b})\cos m\varphi+\ldots, \quad (7.12)\\ w = {} & \sum_1^\infty\vartheta_{ms}w_m|_{\vartheta_{0b}}m\sin m\varphi+\\ & +\sum_1^\infty\vartheta_{ms}\frac{dw_m}{d\vartheta}\Big|_{\vartheta_{0b}}(\vartheta_b-\vartheta_{0b})m\sin m\varphi+\ldots \quad (7.13)\end{aligned}$$

Page 160.

Substituting expansions (7.12), (7.13), (7.10) under boundary condition (7.11), it is possible to obtain the relationship/ratios, which determine coefficients ϑ_{ms} through ϑ_{mb}. The given relationships will be different depending on assumptions about the relative order of coefficients ϑ_{mb} and ϑ_{ms}. For large M_1 and large ϑ_{0b} the form of shock wave is close to the form of body and coefficients ϑ_{mb} and ϑ_{ms} they will be one order. retaining under these conditions only terms of order ϑ_{ms}, ϑ_{mb} in (7.11), we will obtain

$$v|_{\theta_b} = 0$$

or

$$v_0|_{\theta_{0b}} + \frac{dv_0}{d\theta}\Big|_{\theta_{0b}} \sum_1^\infty \theta_{mb} \cos m\varphi + \sum_1^\infty \theta_{ms} v_m|_{\theta_{0b}} \cos m\varphi = 0. \tag{7.14}$$

If with given M_1 value θ_{0s} is the angle of the slope of the jump, caused by the flow about the round cone with half-angle θ_{0b}, then $v_0|_{\theta_{0b}} = 0$ and

$$\frac{dv_0}{d\theta}\Big|_{\theta_{0b}} = -2u_0|_{\theta_{0b}},$$

therefore the boundary conditions, which escape/ensue from equation (7.14), take the form

$$\theta_{ms} = 2\theta_{mb}\left(\frac{u_0}{v_m}\right)\Big|_{\theta_{0b}}. \tag{7.15}$$

Equations (7.15) establish/install communication/connection between the coefficients of the equation of shock wave θ_{ms} and the coefficients of the equation of the body surface θ_{mb} and of dates possibility to solve the direct problem, if are preliminarily determined u_0, v_m. (From the value of velocity is determined enthalpy i from the integral of Bernoulli, p, ρ - from equations of state for known S and i). If body does not have a plane of symmetry, then is the procedure of determination u_n, v_n, w_n, that correspond to the second sum of equation (7.1), it appears in perfect analogy examined (see [120]).

Page 161.

§8. Flow about the round cone at an angle of attack (Stone's theory).

8.1. Lead-in observations. The task of determining the supersonic flow about the round cone, streamlined at an angle of attack, is one of the most important tasks of gas dynamics. It served as the object/subject of many theoretical and experimental studies. In §8 are examined the works, in which the problem of cone is solved by the perturbation method, suitable both with those who were moderated and at high values of the Mach number M_1 of incoming flow. The methods intended for the solution of problem only with large M_1, and also numerical methods are examined separately. Without being stopped on early works (for example, see [~~1334~~ 134]), in which either the half-angle of cone and angle of attack were considered small or flow about cone was assumed to be irrotational, let us turn to A. Stone's works [116, 117]. In these works with basic, is the flow about cone at zero angle of attack. The effect of angle of attack δ is considered by the imposition of the disturbance/perturbations of the first and second order on main flow. By other owls, flow parameters

about cone are decompose/expanded in series according to degrees δ, and in these expansions are held the terms, which contain by factors δ and δ^2, i.e., any parameter f is represented in the form

$$f = f_0 + \delta f_1 + \delta^2 f_2 + o(\delta^2). \quad (8.1)$$

Boundary conditions on leading shock wave and the surface of cone with the aid of the Taylor theorem are carried to the front of the shock wave and body surface with $\delta = 0$. Stone utilizes in essence a system of spherical coordinates with the axis, directed along the speed of undisturbed flow (Fig. 46).

More convenient is the coordinate system, connected with body (which is also utilized by Stone) (Fig. 47).

The components of vector of the particle speed of the gas in the direction of increase R, θ, Φ (R, ϑ, ϕ) let us subsequently designate u, V, W (u, v, w). In expression (8.1) function f_0 depends only on θ (or ϑ), function f_1, f_2 - from θ and Φ; they can be decomposed in Fourier series in cos $n\Phi$ or sin $n\Phi$ ($n = 1, 2, \ldots$).

Page 162.

As showed Stone, these series contain one or two members, and the solution of the problem of round cone with accuracy $O(\delta^2)$ inclusively takes the following form:

$$\left.\begin{aligned} u &= U_0 + \delta U_1 \cos\Phi + \delta^2 (U_2 + U_3 \cos 2\Phi), \\ V &= V_0 + \delta V_1 \cos\Phi + \delta^2 (V_2 + V_3 \cos 2\Phi), \\ W &= \delta W_1 \sin\Phi + \delta^2 W_2 \sin 2\Phi, \\ \frac{p}{p_0} &= 1 + \delta P_1 \cos\Phi + \delta^2 (P_2 + P_3 \cos 2\Phi), \\ \frac{\rho}{\rho_0} &= 1 + \delta R_1 \cos\Phi + \delta^2 (R_2 + R_3 \cos 2\Phi), \\ S &= S_0 + \delta S_1 \cos\Phi + \delta^2 (S_2 + S_3 \cos 2\Phi), \end{aligned}\right\} \quad (8.2)$$

where function U_0, V_0, ρ_0, p_0, S_0 essence flow parameters at $\delta = 0$, and U_1, U_2, ..., S_3 depend only on θ (S - specific enthropy).

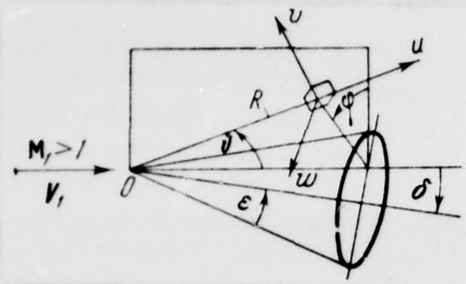


Fig. 46.

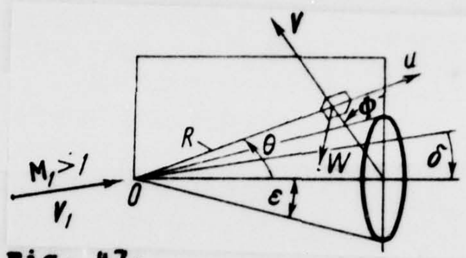


Fig. 47.

Page 163.

In exactly the same manner appears solution in variables ϑ, ϕ, and, knowing coefficients in formulas (8.2), it is easy to determine the same in the appropriate formulas for variables ϑ, ϕ, [116].

During the determination of terms $O(\delta)$ in (8.2) no difficulties were met (specifically, it turned out that S_1 = const). Analyzing equations for U_2, ..., R_3, Stone noted that some terms in these equations go to infinity when $\theta = \varepsilon$, therefore, of expansion (8.2) are unsuitable in the vicinity of the surface of cone. Stone, however, considered that formulas (8.2) determine the flow parameters when $\theta > \theta^*$, where $\theta^* - \varepsilon$ is a very low value. The boundary condition on the surface of cone, $v = 0$, was expressed in the form $V_1 = V_2 = V_3 =$ [redacted] 0 when $\theta = \theta^* \approx \varepsilon$. Z. Kopal [200, 201] it make table

for flow parameters about cone in the first and second approach/approximations. In these tables with the aid of extrapolation are obtained also values R_k, U_k (k = 2, 3) when $\theta = \varepsilon$. Which theoretically turn here into logarithmic infinity.

In works [139-141] are made some technical improvements, which facilitate the use of tables of Copal, and are also examined the limits of the applicability of the solution of Stone. The experiments of M. Holt and D. Blackie [142] showed that the theory of Stone of second order very well predicts the distribution of pressure on the surface of cone, if $\delta < \frac{\varepsilon}{2}$. (When $\frac{\varepsilon}{2} < \delta < \varepsilon$ the results are obtained also fair). A. Ferri [143], being based on physical considerations, it showed that on the surface of cone, the entropy must be constant. Other lines of constant entropy (on single sphere) converge into point $\Phi = 0, \theta = \varepsilon$, where is formed special feature/peculiarity called now Ferry's special feature/peculiarity (Fig. 48). Since the entropy on the surface of cone in the first approximation, in Stone's theory is changed according to the law $S = S_0 + \delta S_1 \cos \Phi$, it hence immediately followed that Stone's theory does not befit in vicinity $\theta = \varepsilon$. A. Ferri introduced the concept "vorticity layer", by thickness $O(\delta)$, adjacent to the surface of the cone in which flow parameters, besides pressure and the normal component of velocity, can differ significantly from the values, given by Stone's theory, and it gave correcting formulas for the components of velocity on the surface of

cone with accuracy $O(\delta)$.

Page 164.

D. Willett [144, 145] it found the components of velocity on the surface of cone with accuracy $O(\delta^2)$, after assuming that Stone's theory correctly determines pressure on the surface of cone and the jump in entropy during transition through the shock wave in the plane of the symmetry of flow.

Since Stone's theory corresponded well to experimental data, was the natural wish to correct it in "vorticity layer" and to obtain thus the complete solution of problem. The formulas, which determine entropy in "vorticity layer" with accuracy $O(\delta)$, were obtained by B. Woods [146] and by H. Cheng [147, 148], formulas for the remaining flow parameters are B. Woods [149, 150]. However, since expansions (8.2) do not represent the solution of problem in "vorticity layer", but the boundary conditions $V_1 = V_2 = V_3 = 0$ are satisfied, actually, when $\theta = \varepsilon$, were doubts of the validity of the solution of Stone (for example, see [149]). The substantiation of Stone's theory (first and second orders) was given by B. Bulakh in work [118], who showed that the solution of Stone with the boundary conditions $V_1 = V_2 = V_3 = 0$ when $\theta = \varepsilon$ represents the solution of the problem of the cone of the outside "vorticity layer", by thickness $O(\delta)$ (in more detail about

the thickness "vorticity layer" it will be said later than). Was constructed the solution of the first order [with accuracy $O(\delta)$] in "vorticity layer", which outside it transfer/converted to the solution of Stone of first order. This solution made it possible to base the theory of Stone of second order and explained the reason for the appearance of logarithmic special feature/peculiarities in formulas for U_2, J_3, R_2, R_3, S_2, S_3 when $\theta = \varepsilon$.

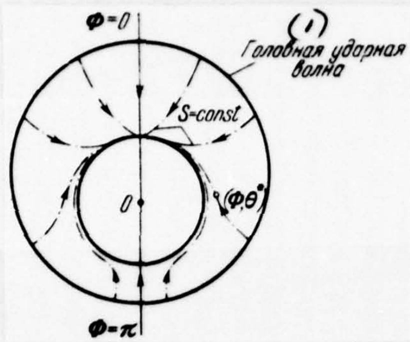


Fig. 48. Key: (1). Leading shock wave.

Page 165.

It was also shown, that the pressure on the surface of cone is given by Stone's theory correctly, i.e., with accuracy $O(\delta^2)$ inclusively. Let us note that the substantiation of Stone's theory of the first order was partially carried out in H. Cheng's work [148]. Subsequently the solution of problem in "vorticity layer" with accuracy $O(\delta^2)$ was obtained by A. Manson [151], B. Woods [152] by the method of the coupling of asymptotic expansions and Ya. Sapunkov [153] the same method which was by it developed for hypersonic speeds.

In all previously named solutions for "vorticity layer" of the line of constant entropy they enter in point $\Phi = 0,\ \theta = \varepsilon,$

concerning one direction, but, apparently, these solutions they do not befit in small vicinity of point $\Phi = 0,\ \theta = \varepsilon,$ so that it is here necessary to introduce its expansion in terms of δ.

It should be noted that the theory of Stone of second order is very bulky and not always does make it possible to conduct calculation at any point of flow. Fortunately, at present K. Babenko with colleagues [29] by establishment it calculated the tables of the flow about the cone at an angle of attack exceeding the tables of copal on accuracy and over range of change of $M_1,\ \delta,\ \varepsilon.$ Stone's theory retains its value for small δ. Solution for "vorticity layer" from the viewpoint of numerical methods is the most important part of Stone's theory, since by finite-difference methods it is very difficult to calculate flow in "vorticity layer", and also because this solution makes it possible to predict those cases when in solution by the method of establishment occurs a decrease in the accuracy.

For these reasons further is set forth only the theory of Stone and solution in "vorticity layer" of the first order. (Examples of calculations according to Stone's theory of the second order can be found, for example, in [73]).

999tab 8.2. Theory of Stone of first order. The theory of Stone of

first order follows as special case from the method of linearized characteristics, presented in point/item 7.2. If we assume $\vartheta_{0b} = \varepsilon$, then for applying the method of linearized characteristics it is necessary to only find expansion ϑ_b in Fourier series (7.10) for the surface of the cone, inclined to angle δ (See Fig. 46, 47).

page 166.

It is easy to show [116] that communication/connection between coordinates ϑ, φ and θ, Φ is given by the formulas

$$\left.\begin{aligned} \vartheta &= \theta + \delta\cos\Phi + \frac{1}{2}\delta^2 \operatorname{ctg}\theta \sin^2\Phi + o(\delta^2), \\ \varphi &= \Phi - \delta\operatorname{ctg}\theta\sin\Phi + \\ &\quad + \frac{1}{2}\delta^2 \sin\Phi\cos\Phi(2\operatorname{ctg}^2\theta + 1) + o(\delta^2). \end{aligned}\right\} \quad (8.3)$$

Since on the surface of cone $\theta = \varepsilon$, from (8.3) we obtain:

$\vartheta_b = \varepsilon + \delta\cos\varphi + o(\delta^2)$, i.e., $\vartheta_{0b} = \varepsilon$, $\vartheta_{1b} = \delta$, $\vartheta_{mb} = 0$, $m > 1$, and in expansions (7.4) remains on one member, who contains to cos φ or sin φ.

In Stone the unknown values are u, v, w, p, ρ, which are represented by expansions (8.2). Let us note that the coefficients of these expansions satisfy one and the same equations both in the system of coordinates R, ϑ, φ and in system R, θ, Φ. These equations will be extracted with the necessity for the adopted here designations.

8.3. Solution in "vorticity layer" of first order. If we instead of entropy S introduce the unknown function s = $S[\gamma(\gamma-1)c_v]^{-1}$ and to examine the case of ideal gas, then equations (1.15), (1.2), (1.5), (1.16) accept the following form:

$$\left.\begin{aligned}
L_1 &= (a^2 - V^2)\sin\theta V_\theta - (W^2 - a^2)W_\Phi - \\
&\quad - VW(\sin\theta W_\theta + V_\Phi) - \\
&\quad - (V^2 + W^2 - 2a^2)u\sin\theta + a^2 V\sin\theta = 0, \\
L_2 &= \sin\theta V u_\theta + W u_\Phi - \sin\theta(V^2 + W^2) = 0, \\
L_3 &= \sin\theta V s_\theta + W s_\Phi = 0, \\
L_4 &= \sin\theta V W_\theta - a^2 s_\Phi - u u_\Phi - V V_\Phi + \\
&\quad + W(\sin\theta u + \cos\theta V) = 0, \\
a^2 &= \frac{\gamma-1}{2}(V_{\text{пр}}^2 - u^2 - V^2 - W^2).
\end{aligned}\right\} \quad (8.4)$$

Outside "vorticity layer" the parameters of gas search for in the form of expansions (8.2), moreover

$$s = s_0 + \delta s_1\cos\Phi + \delta^2(s_2 + s_3\cos 2\Phi), \qquad (8.5)$$

where

$$\begin{aligned}
s_1 &= \frac{1}{\gamma(\gamma-1)}(P_1 - \gamma R_1), \\
s_2 &= \frac{1}{\gamma(\gamma-1)}\left(\frac{\gamma}{4}R_1^2 - \frac{1}{4}P_1^2 + P_2 - \gamma R_2\right), \\
s_3 &= \frac{1}{\gamma(\gamma-1)}\left(\frac{\gamma}{4}R_1^2 - \frac{1}{4}P_1^2 + P_3 - \gamma R_3\right).
\end{aligned}$$

Page 167.

Substituting (8.2), (8.5) in $L_3 = 0$ system (8.4), we will obtain

$$s_{0\theta} = 0, \quad s_{1\theta} = 0, \quad s_{2\theta} = -s_{3\theta} = \frac{W_1 s_1}{2V_0\sin\theta}. \qquad (8.6)$$

Let us subsequently designate the value of function when $\theta = \varepsilon$ by a superscript cross: $f(\varepsilon) = f^{\times}$.

In the vicinity of point $\theta = \varepsilon$ we have

$$V_\theta = -2U_0^\times(\theta-\varepsilon)+O[(\theta-\varepsilon)^2],$$

therefore from (8.6) we will obtain in this vicinity

$$s_2=-s_3=-\frac{\vec{W}_1^\times s_1}{4U_0^\times \sin\varepsilon}\ln(\theta-\varepsilon)+O(1). \qquad (8.7)$$

Let us find now the solution of system of equations (8.4) in vicinity $\theta=\varepsilon$ and let us require so that outside this vicinity it would be converted into the solution of Stone. During the determination of solution, let us make the assumptions which will be checked subsequently:

1. The components of velocity vector in the case of the inclined cone differ from the appropriate components with $\delta = 0$ by values $O(\delta)$.

2. Theory string is correctly determined W with accuracy $O(\delta)$ everywhere.

3. Is realized diagram Ferri's flow about.

From these assumptions and expansion (8.2) it follows that in vicinity $\theta=\varepsilon$ the component of velocity W can be presented in the form

$$W=\delta W_1^\times \sin\Phi + O[\delta(\theta-\varepsilon)^{1/2}]+o(\delta), \qquad (8.8)$$

while from equation L_1 = of 0 system (8.4) we will obtain

$$V = -2U_0^{\times}(\theta-\varepsilon)+O[(\theta-\varepsilon)^2]+O[\delta(\theta-\varepsilon)]. \quad (8.9)$$

end section.

142D

Page 168.

The first of two members in (8.9) are V_0; last/latter term appears because of the slope/inclination of cone. Substituting (8.8) and (8.9) into equation $L_3 = 0$ of the system (8.4), we will obtain this equation:

$$\sin\varepsilon\{-2U_0^\times(\theta-\varepsilon)+O[(\theta-\varepsilon)^2]+O[\delta(\theta-\varepsilon)]\}s_\theta + \\ +\{\delta W_1^\times \sin\Phi + O[\delta(\theta-\varepsilon)^{1/2}]+o(\delta)\}\, s_\Phi = 0, \tag{8.10}$$

which after the deletion of low values from coefficients it is possible to write in the form

$$(\theta-\varepsilon)s_\theta+\delta h\cdot\sin\Phi s_\Phi=0,\quad h=-\frac{W_1^\times}{2U_0^\times\sin\varepsilon}>0. \tag{8.11}$$

The evaluation of the effect of the reject/thrown terms will be made later.

The general solution of equation (8.11) takes the form

$$s-s_0=f(\zeta),\ \zeta=\frac{1+\cos\Phi}{1-\cos\Phi}\cdot(\theta-\varepsilon)^{2h\delta}. \tag{8.12}$$

When $\theta - \varepsilon = 0(\delta)$ ζ can be decomposed in power series in δ:

$$\zeta = \frac{1+\cos\Phi}{1-\cos\Phi} e^{2h\delta\cdot\ln(\theta-\varepsilon)} = \frac{1+\cos\Phi}{1-\cos\Phi}[1 + 2h\delta\ln(\theta-\varepsilon) + \ldots]. \quad (8.13)$$

If Stone's theory is still accurate when $\theta - \varepsilon = O(\delta)$, the expression (8.12) should be transformed with $\theta - \varepsilon = O(\delta)$ to a Stone solution.

Expanding expression $s - s_0$ from (8.12) into a series according to degrees δ with $\theta - \varepsilon = O(\delta)$ taking into account (8.13) we will obtain

$$s - s_0 = f\left(\frac{1+\cos\Phi}{1-\cos\Phi}\right) + \cdots;$$

by equalizing the first term of this expansion to the first term of expansion for $s - s_0$ in the solution of Stone (8.5), we have

$$f\left(\frac{1+\cos\Phi}{1-\cos\Phi}\right) = \delta s_1 \cos\Phi$$

or

$$f(\zeta) = \delta s_1 \frac{\zeta-1}{\zeta+1}. \quad (8.14)$$

Page 169.

Thus,

$$s - s_0 = \delta s_1 \frac{\zeta-1}{\zeta+1} + q. \quad (8.15)$$

If we decompose $s - s_0$ (see (8.15)) in a series according to degrees δ when $\theta - \varepsilon = 0(\delta)$, we will obtain

$$s - s_0 = \delta s_1 \cos\Phi + \delta^2\left[\frac{s_1 h}{2}\ln(\theta-\varepsilon) - \frac{s_1 h}{2}\ln(\theta-\varepsilon)\cos 2\Phi\right] + q + o(\delta^2). \quad (8.16)$$

If we consider expression (8.11) for h and formulas (8.5), (8.7),

then of (8.16) it follows that q = $O(\delta^2)$ when $\theta - \varepsilon = O(\delta)$.

Substituting (8.15) in (8.10), we will obtain equation for q in the form

$$\{\theta - \varepsilon + O[(\theta - \varepsilon)^2] + O[\delta(\theta - \varepsilon)]\}\, q_\theta + \\ + \{h\sin\Phi + O[\delta(\theta - \varepsilon)^{1/2}] + o(\delta)\} q_\Phi = O[\delta^2(\theta - \varepsilon)^{2h\delta}],$$

along characteristics of which are fulfilled the equalities

$$\frac{d(\theta - \varepsilon)}{\{\theta - \varepsilon + O[(\theta - \varepsilon)^2] + O[\delta(\theta - \varepsilon)]\}} = \\ = \frac{d\Phi}{\{h\sin\Phi + O[\delta(\theta - \varepsilon)^{1/2}] + o(\delta)\}} = \frac{dq}{O[\delta^2(\theta - \varepsilon)^{2h\delta}]}.$$

Hence q it is possible to express in the form

$$q = \int_{\theta^*(\theta,\Phi)}^{\theta} O[\delta^2(\theta - \varepsilon)^{2h\delta - 1}]\, d\theta + q^*(\theta, \Phi).$$

Here the integral is taken in terms of the line of constant entropy, passing through the point (Φ, θ^*), where $\theta^* - \varepsilon = O(\delta)$ and $q^* = O(\delta^2)$.

Page 170.

Since the lines of constant entropy on single sphere take the form, depicted on Fig. 48, $\theta - \theta^* = O(\delta)$, то $q = O(\delta^2)$ with $0 \leqslant \theta - \varepsilon \leqslant O(\delta)$, and formula (8.15) is written in this form:

$$\left.\begin{aligned} & s - s_0 = \delta s_1 \frac{\zeta - 1}{\zeta + 1} + O(\delta^2), \\ & 0 \leqslant \theta - \varepsilon \leqslant O(\delta), \\ & \zeta = \frac{1 + \cos\Phi}{1 - \cos\Phi}(\theta - \varepsilon)^{2h\delta}. \end{aligned}\right| \qquad (8.17)$$

Let us find now the components of velocity u in "vorticity layer". Let us present u here in the form $u = U_0 + U'$. From equation $L_4 = 0$, (8.4), and the made assumptions it follows that

$$W = \frac{a_0^{\times 2}}{U_0^{\times} \sin \varepsilon} s_\Phi + \frac{1}{\sin \varepsilon} U'_\Phi + O[\delta(\theta - \varepsilon)^{1/2}] + o(\delta). \quad (8.18)$$

Substituting (8.18) in $L_2 = 0$, $L_3 = 0$ and reject/throwing low values, we will obtain

$$-\sin \varepsilon\, 2U_0^{\times}(\theta - \varepsilon) U'_\theta + \left(\frac{a_0^{\times 2}}{U_0^{\times} \sin \varepsilon} s_\Phi + \frac{1}{\sin \varepsilon} U'_\Phi\right) U'_\Phi =$$
$$= \sin \varepsilon \left(\frac{a_0^{\times 2}}{U_0^{\times} \sin \varepsilon} s_\Phi + \frac{1}{\sin \varepsilon} U'_\Phi\right)^2, \quad (8.19)$$
$$-\sin \varepsilon\, 2U_0^{\times}(\theta - \varepsilon) s_\theta + \left(\frac{a_0^{\times 2}}{U_0^{\times} \sin \varepsilon} s_\Phi + \frac{1}{\sin \varepsilon} U'_\Phi\right) s_\Phi \cong 0. \quad (8.20)$$

Multiplying (8.20) by $\left(a_0^{\times 2} / U_0^{\times} \sin \varepsilon\right)$, a (8.19) - on $1 / \sin \varepsilon$ and by store/adding up, we will obtain the equation

$$-\sin \varepsilon\, 2U_0^{\times}(\theta - \beta)\left(\frac{a_0^{\times 2}}{U_0^{\times} \sin \varepsilon} s_\theta + \frac{1}{\sin \varepsilon} U'_\theta\right) = 0.$$

Page 171.

Hence, after the equating of zero bracketed expression, it follows

$$U' = -\frac{a_0^{\times 2}}{U_0^{\times}}(s - s_0) + H(\Phi),$$

where $H(\Phi)$ - arbitrary function Φ. It is more determinable of the condition that u when $\theta - \varepsilon = 0(\delta)$ is converted into the expression, given by the theory of Town. Let us assume

$$H(\Phi) = \delta\left(\frac{a_0^{\times 2}}{U_0^{\times}} s_1 + U_1^{\times}\right)\cos\Phi,$$

then

$$U' = u - U_0 = -\frac{a_0^{\times 2}}{U_0^{\times}}(s - s_0 - \delta s_1 \cos\Phi) + \\ + \delta U_1^{\times}\cos\Phi + O(\delta^2), \qquad (8.21)$$

where $s - s_0$ is given by formula (8.17). Estimation $O(\delta^2)$ in (8.21) can be obtained analogous with estimation for $s - s_0$. If we present in V in "vorticity layer" in the form $V = V_0 + V'$, then of the equation $L_1 = 0$ follows the formula:

$$V' = \int_{\varepsilon}^{\theta}\left(\frac{W_{\Phi}}{\sin\varepsilon} - 2U'\right)d\theta + o(\delta^2), \qquad (8.22)$$

with the aid of which it is possible to determine V in "vorticity layer". Pressure p, as it was already noted, is correctly determined by the solution of Stone in "vorticity layer". If we ρ in "vorticity layer" present in the form $\rho = R_0 + R'$, then from the equation of Bernoulli

$$\frac{u^2 + V^2 + W^2}{2} + \frac{\gamma}{\gamma - 1}\cdot\frac{p}{\rho} = \frac{V_{\text{np}}^2}{2} \qquad (8.23)$$

we will obtain

$$R' = \delta P_1 R_0 + \frac{\gamma - 1}{\gamma} U_0 R_0^2 P_0^{-1} U' + O(\delta^2) = \\ = \delta P_1^{\times} R_0^{\times} + \frac{\gamma - 1}{\gamma} U_0^{\times} R_0^{\times 2}\frac{1}{P_0^{\times}} U' + O(\delta^2). \quad (8.24)$$

It is checked now the assumptions, made during the determination of solution in "vorticity layer". From formulas (8.8), (8.17), (8.21), (8.22) and (8.24), that determine the solution in "vorticity layer", follows that flow parameters differ from the parameters feast $\delta = 0$

by values $O(\delta)$.

PAGE 172.

Let us show that W is determined by first-order of Stone theory correctly. From formulas (8.18), (8.21) it follows that W in "vorticity layer" is represented by the expression

$$W = -\sin\Phi \frac{\delta}{\sin\varepsilon}\left(\frac{a_0^{\times 2}}{U_0^{\times}} s_1 + U_1^{\times}\right) + O[\delta(\theta-\varepsilon)^{1/2}] + o(\delta). \tag{8.25}$$

Let us show that formulas (8.25) and (8.8) are identical. Let us find $U_1^{\times}$. Bernoulli's notation of equation (8.23) for a flow about the inclined cone, deducting from it the equation of Bernoulli for $\delta=0$, by substituting in the result of expansion (8.2) and by equalizing to zero coefficient when δ, we will obtain

$$U_0U_1 + V_0V_1 + \frac{\gamma}{\gamma-1}\frac{p_0}{\rho_0}(P_1 - R_1) = 0,$$

whence when $\theta = \varepsilon$ it follows

$$U_1^{\times} = \frac{a_0^{\times 2}}{\gamma-1}\frac{R_1^{\times} - P_1^{\times}}{U_0^{\times}}.$$

By substituting $U_1^{\times}$, s_1, (8.5), in (8.25), we will obtain

$$W = \delta\frac{a_0^{\times 2}P_1^{\times}}{U_0^{\times}\gamma\sin\varepsilon}\sin\Phi + O[\delta(\theta-\varepsilon)^{1/2}] + o(\delta). \tag{8.26}$$

We convert W, given by Stone's theory. Equation (1.7) from work [116] in the adopted here designations takes the form

$$\frac{dU_0}{d\theta}\frac{dW_1}{d\theta} + \left(U_0 + \frac{dU_0}{d\theta}\operatorname{ctg}\theta\right)W_1 - \frac{a_1^2P_1}{\gamma\sin\theta} = 0.$$

Since

$$\frac{dU_0}{d\theta} = V_0 = -2U_0^{\times}(\theta-\varepsilon) + O[(\theta-\varepsilon)^2]$$

in vicinity $\theta = \varepsilon$, of this equation it follows that

$$W_1 = \frac{a_0^{\times 2}P_1^{\times}}{\gamma U_0^{\times}\sin\varepsilon} + O[(\theta-\varepsilon)^{1/2}] \tag{8.27}$$

in vicinity $\theta = \varepsilon$.

Page 173.

Consequently, (8.8) it is possible to present in the form

$$W = \delta \frac{a_0^{\times 2} P_1^{\times}}{\gamma U_0^{\times} \sin \varepsilon} \sin \Phi + O[\delta (\theta - \varepsilon)^{1/2}] + o(\delta). \quad (8.28)$$

Comparing (8.26) and (8.28), we establish/install their identity. We investigate the behavior of line of constant entropy near the surface of cone in the obtained solution. From formulas (8.17) we will obtain

$$s - s_0 = \delta s_1 \frac{(1 + \cos \Phi)(\theta - \varepsilon)^{2h\delta} + \cos \Phi - 1}{(1 + \cos \Phi)(\theta - \varepsilon)^{2h\delta} - \cos \Phi + 1} + O(\delta^2), \quad h > 0. \quad (8.29)$$

When $\Phi = \pi$ and on the surface of the cone where $\theta = \varepsilon$, from formula (8-29) we obtain $s - s_0 = -\alpha s_1 + 0(\delta^2)$; with $\Phi = 0$ we have $s - s_0 = \alpha s_1 + 0(\delta^2)$ (neq) 0 lines s = const converge into point $\Phi = 0$, $\phi = \varepsilon$, since according to (8.29) the equations of lines s = const in the vicinity of point $\phi = \varepsilon$, $\Phi = 0$ take the form

$$\theta - \varepsilon = \left[\frac{1 + k}{2(1 - k)}\right]^{\frac{1}{2h\delta}} \Phi^{\frac{1}{h\delta}} + \ldots, \qquad k = \frac{s - s_0}{\delta s_1} = \text{const}.$$

Thus, in the obtained solution is realized the diagram Ferry's flow about. The substantiation of the validity of the theory of Stone of outside "vorticity layer" is carried out in work [118].

8.4. Final observations. The order of the thickness "vorticity layer" can be determined more strictly as follows. If values s - s_0, given by the solution of Stone, are designated $(s - s_0)_C$, and s are s_0 for "vorticity layer" - $(s - s_0)_B$, then let us agree to consider that point (θ, ϕ) it belongs (it does not belong) "vorticity layer", if

$$\lim_{\delta \to} \frac{(s-s_0)_C}{(s-s_0)_B} \neq 1 \quad (=1).$$

Page 174.

As can be seen from formulas (8.17), this requirement is equivalent to the condition that for the point "vorticity layer"

$$\lim_{\delta \to 0} (\theta - \varepsilon)^{2h\delta} \neq 1,$$

i.e.

$$\theta - \varepsilon = O(e^{-\frac{c}{\delta}}), \quad c > 0.$$

Thus, the thickness "vorticity layer" is of the order $O(e^{-\frac{c}{\delta}})$. The indicated approach to the determination of the order of the thickness "vorticity layer", although it is strict, is to a considerable extent formal, since always it is necessary to deal not with infinitesimal, but with final δ. On the boundary "vorticity layer" $(\theta - \varepsilon)^{2h\delta}$ must differ little from unity; therefore it is possible to assign the number, close to unity, for example, 0.95, and the thickness "vorticity layer" to determine from the condition

$$(\theta - \varepsilon)^{2h\delta} = 0,95$$

In each specific case.

For values $2h\delta < 1\zeta_\theta = \infty$ when $\theta = \varepsilon$ and the derivatives u_θ, ρ_θ, s_θ go to infinity on the surface of cone, since u_θ, ρ_θ, $s_\theta = 0\,[(\theta - \varepsilon)^{2h\delta - 1}]$ when $\theta \longrightarrow \varepsilon$, $\{W_0 = 0\,[(\theta - \varepsilon)^{-1/2}]$, $\theta \longrightarrow \varepsilon\}$. This fact impedes the application/use of finite-difference methods and can lead to the loss of accuracy in the results of calculations with small $2h\delta$, since the low accuracy of the approximation of the derivatives u_θ, ρ_θ, s_θ in "vorticity layer" by finite differences manifests itself the solution outside "vortex/eddy layer. After the nonlicked off transformations $2h\delta$, it is possible to present in the form

$$2h\delta = \frac{1}{M_0^{\times 2}} \frac{-\delta P_1^\times}{\gamma \sin^2 \varepsilon} \qquad \left(M_0^\times = \frac{U_0^\times}{a_0^\times}\right). \qquad (8.30)$$

First-order of the solution of Stone comparison with the results of experiments and calculations according to the method of establishment shows that expression $p/p_0 = 1 + \delta P_1 \cos \Phi$ well determines pressure on the surface of cone at $\Phi = 135°-150°$; therefore value $-\delta P_1^\times$ can be replaced with the expression

$$\left[\left(1 - \frac{p^\times}{p_0^\times}\right) \frac{1}{\cos \Phi}\right]_{\Phi = 135°} = \sqrt{2}\left(\frac{p^\times_{\Phi = 135°}}{p_0^\times} - 1\right).$$

Page 175.

Then finally for $2h\delta$ we obtain the formula

$$2h\delta \approx \frac{\sqrt{2}}{\gamma M_0^{\times 2} \sin^2 \varepsilon} \left(\frac{p^{\times}_{\Phi=135^\circ}}{p_0^{\times}} - 1 \right) \qquad (8.31)$$

(where $p^{\times}_{\Phi=135^\circ}$ is pressure on the surface of cone at $\Phi = 135^\circ$; $p_0^{\times}$ is pressure on the surface of cone with δ - = 0; M_0 - Mach number for δ = 0, $\theta = \varepsilon$). If we use the tables of the flow about the cone [29], to designate by indices 1, 2, 3 values of flow parameter on the surface of cone and those who follow the nearest to the surface of cone node/units riding-crops during by finite-difference method, then it is possible with the aid of formula (8.31) to compose for $\Phi = 90^\circ$ following table:.

Tables 3.

$M_1 = 2$	3	3	3	5	5	7	7
$\varepsilon = 10^\circ$	35°	30°	10°	15°	25°	30°	30°
$\delta = 2^\circ 30'$	5°	15°	5°	10°	5°	5°	15°
$2h\delta \approx 6{,}6$	0,14	0,48	0,77	0,66	0,14	0,09	0,48
$\frac{\rho_2 - \rho_1}{\rho_3 - \rho_2} \approx 0{,}40$	17	2,5	2	2,5	14	17	3

From this table it is evident, that in the majority of cases $2h\delta < 1$ and that in certain cases, for example, with $M_1 = 3$, $\varepsilon = 35^\circ$, $\delta = 5^\circ$, the density ρ sharply changes near the surface of cone.

In such cases the accuracy of solution will be, undoubtedly it is lower than, for example, in the case $M_1 = 2$, $\varepsilon = 10^\circ$, $\delta = 2^\circ 30'$.

From table it is also evident that value 2h6 determines the character of change ρ in "vorticity layer": than less 2h6, the sharper changes here ρ and the lesser the accuracy should expect from solution by the method of establishment with the fixed/recorded distance between mesh points.

Page 176.

A good coincidence of pressure p, calculated according to method of operation [29] on the surface of cone, with results of experiment does not serve as the guarantee of the accuracy of solution, since p on the surface of cone is sufficiently accurately determined even by the easiest methods; see [154, 155].

Let us note that "vorticity layer" exists also on the conical bodies, which differ little from round cone.

Name "vorticity layer" on the whole correctly reflects the flow pattern near the surface of cone, since in the majority of cases $u_{*} = \infty$, $\theta = \xi$, which indicates the powerful eddying of flow near the surface of cone.

§9. Flow about the conical bodies with flow breakaway.

9.1. Preliminary observations. The experimental studies of the flow pattern of conical bodies of the uniform supersonic flows of gas showed that under specific conditions the flow blows away from the surface of the streamlined bodies, forming complex vortex/eddy systems. Since the large role in the formation of detachel flows plays the viscosity of gas, in the general case the conical character of flow is disrupted. However, in a series of important special cases, separated flow retains conical symmetry and just as in two-dimensional problem of hydrodynamics, appears the possibility of its study at the basis of the equations of nonviscous gas. Is most well investigated, as it is experimental, so also theoretically, the case of the delta wing. For this reason subsequently, we will be restricted to the examination of the flow about the delta wing, after demonstrating on this problem those methods which are utilized to account for flow breakaway.

In work [113] are systematized the experimental data on the possible types of the flow about the delta wing at the moderate angles of attack.

Page 177.

Before transfer/converting to the analysis/selection of these types of flow, let us note that for wings with blunted leading edges the flow can break itself both along an entire leading edge (at high angles of attack) and along the part of the leading edge (at the moderate angles of attack); in this case is disrupted the conical character of flow (are possible and the more complex types of flows). For wings with sharp leading edges, the flow blows away along an entire edge, forming conical flow. Since us it interests the possibility of the reliable use of methods of conical flow theory in the problem in question, it is necessary to stop at wings with sharp leading edges.

The flow about the sharp leading edge of the delta wing in many respects is similar to the flow about the sharp leading edge in two-dimensional problem of gas dynamics; therefore it is expedient to examine also flow in the plane, perpendicular to leading wing edge. Let us designate for this flow angle of attack by δ_N and the Mach number of the undisturbed flow by M_{1N}. For a triangular plate δ_N and M_{1N}, they are determined in the absence of slip along the formulas

$$\left.\begin{aligned} \delta_N &= \operatorname{arctg}\left[\frac{\operatorname{tg}\delta}{\cos\Lambda}\right], \\ M_{1N} &= M_1\cos\Lambda\sqrt{1+\sin^2\delta\operatorname{tg}^2\Lambda}, \end{aligned}\right\} \quad (9.1)$$

where δ is an angle of attack of wing, Λ - the angle of its sweepback [$(\pi - 2\Lambda)$ - apex angle of plate], M_1 - a Mach number of undisturbed flow.

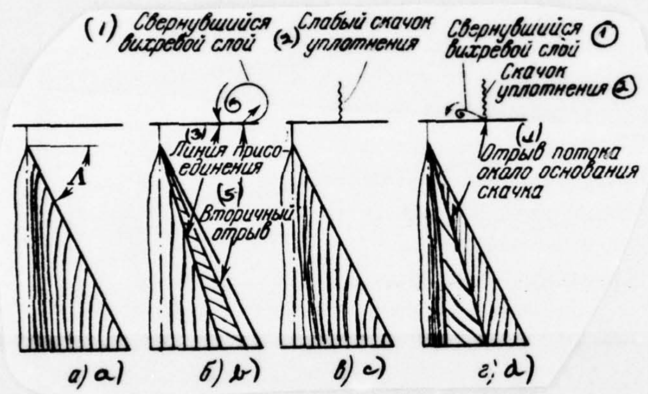


Fig. 49.

Key: (1). Rolled up vorticity layer. (2). Weak shock wave. (3). Line of connection. (4). Breakaway of flow about the basis/base of jump. (5). Secondary breakaway.

Page 178.

Let us note that for narrow wings δ_N at the moderate angles of attack is considerably more than δ, for example, for $\Lambda = 80°$ and $\delta = 8.2°$ $\delta_N = 40°$; for $\Lambda = 75°$ and $\delta = 15°$ $\delta_N = 46°$.

Figures 49a-d depicts the basic types of flows about fine/thin delta wing with sharp leading edges at the moderate angles of attack [113]. At the low angles of the attack δ of noticeable flow breakaway, it does not occur (see Fig. 49a). With large δ the flow blows away from the leading wing edges and forms vorticity layer which are coagulated spirally above suction side of wing. Because of a pressure drop about leading edge, appears the secondary breakaway of flow (see Fig. 49b).

In more detail this case is depicted on Fig. 43. At smaller sweep angles, the vorticity layer is connected to surface, forming the "locked" region like by that that is in two-dimensional problem. At large values of M_1, when $M_{1N} > 1$, flow is connected to leading edge and above suction side of wing is formed the weak shock wave (Fig. 49c), which an cause flow breakaway about its basis/base (Fig. 49d).

Besides these basic types of flow about fine/thin wings, are possible mongrels, for example, when shock wave is formed above the rolled up vorticity layer (Fig. 49b, etc.).

The most complex flows appear in cases when $M_{1N} < 1$. Here

appears local conical-supersonic zone near wing edge, with weak shock waves, and small separating zone. Let us designate flows of this type by symbol b-d. For "thick" wings (or at high angles of attack) in the case of $M_{1N} > 1$, when shock wave canno be that which was connected to pressure side of wing, near edge also appears local conical-transonic region with the detached powerful shock wave and are possible flows of the type" a" or b (Fig. 49), sometimes with shock wave above the rolled up vorticity layer. Later we will return to the analysis/selection of the types of flows in cases when leading wing edges supersonic ($M_{1N} > 1$), and here let us be interested in flows of the type b and b, c,.

Page 179.

Work [113] gives the experimental data from which it follows that the boundary between the flows of types b and b-c barely depends on Reynolds number of undisturbed flow; moreover, if $\delta > 2°$, $M_{1N} < 0.6 + 0.2\,\delta_N$, then is realized flow of the type b (Fig. 49); if $M_{1N} > 0.8 + 0.2\,\delta_N$, then occurs flow of the type b-c; between these boundaries lie/rests the transition region where are possible mongrels of flows. (Experiments [113] are carried out at values δ_N and M_{1N}, which they satisfy the conditions: $2° < \delta_N < 40°$, $0.2 < M_{1N} < 1.5$).

In view of large complexity of flows for types b and b-c theoretical results are obtained only in those cases when the rolled up vorticity layers play the main role in shaping of flow about wing and when their effect can be disregarded. In the latter case is utilized at the moderate angles of attack the linear theory (for example, see [3]), in which the lift coefficient of wing C_L is proportional to angle of attack δ. For the narrow wings of values C_L, obtained from experiment, they are obtained greater than gives linear theory (theory of the elongation of dzhonsona - Ward's bodies) and C_L already depends on δ nonlinearly. In work [156] is communicated about experiments with M - 1.9 and values π/2 - Λ = 5°-31°.75 on the models which had very sharp leading edges and for which the parameter

$$k' = \operatorname{ctg}\Lambda \operatorname{ctg}\alpha_1, \ \overset{\text{where}}{\text{где}}\ \operatorname{tg}\alpha_1 = \frac{1}{m_1}\,[m_1 = (M_1^2 - 1)^{1/2}],$$

varied from 0.141 to 1.00. The disagreement between results of experiments and by values C_L, given by linear theory, became noticeable (more than 5o/o), when $k' < 0.3$ and $\delta \operatorname{tg}\Lambda > 0.6$.

These numerals can serve as reference point for the determination of those values of the parameters of wing and undisturbed flow in which is realized the flow of the type b (Fig. 49).

The first attempt at the determination theoretically of

dependence C_L on δ for a triangular plate in the case of b (Fig. 49) was conducted by R. Legendre [111].

Page 180.

Since the wing was assumed to be narrow and angles of attack small, was applied the known theory of dzhonsa - Ward's extended bodies (for example, see [121]), in which the modified velocity potential of disturbance/perturbation ϕ_0 [see point/item 9.1] it satisfies the equation of Laplace in each plane, the direction of standard to which coincides with direction of undisturbed flow or is close to it; in figure 50a, is selected the coordinate system, connected with the surface of the triangular plate, establish/installed at low angle of attack δ; therefore with taken accuracy $\frac{\partial^2\varphi_0}{\partial x^2}+\frac{\partial^2\varphi_0}{\partial y^2}=0$. In view of the conical flow pattern, is sufficient to examine one of the planes z = const, for example, z = 1 on Fig. 50b.

The coagulated vorticity layers Legendre schematized by two isolated/insulated vortex/eddy threads (intensity of which was proportional to apex distance of wing).

AD-A066 205 FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO F/G 20/4

NONLINEAR CONICAL FLOWS OF GAS,(U)

MAR 78 B M BULAKH

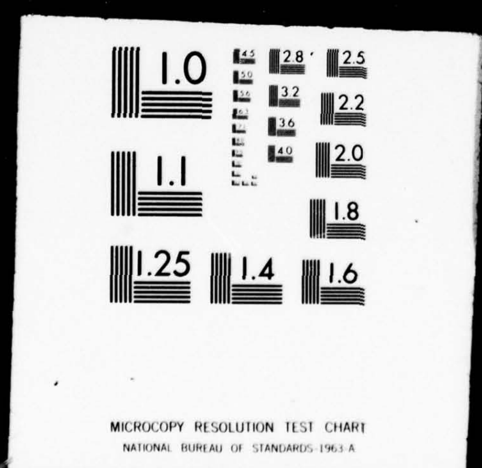
UNCLASSIFIED FTD-ID(RS)T-0266-78 NL

4 OF 7

AD A066205

4 OF 7

AD
A066205



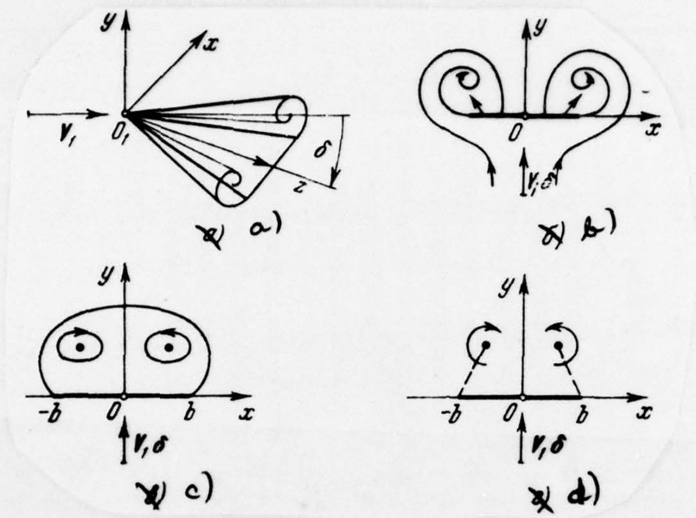


Fig. 50.

Page 181.

Then the problem of determination of φ_0 in plane z = 1 in accuracy coincided with the problem of determining the velocity potential during the flow of incompressible flow about the plate whose speed at infinity was equal to $V_1\delta$ and directed perpendicular to the surface of plate, in the case when after plate are formed two isolate/insulated eddy/vortices. (If we utilize a continuous operation of coordinates, as is done in monograph [121], then is obtained the problem of the motion of plate in the quiescent liquid

with a speed of $V_1 \delta$). The solution of this problem is conducted by the methods of the theory of complex variable functions, developed for two-dimensional problem of hydrodynamics, and it is not connected with great difficulties. Central place in this theory are those assumptions from which they are determined the circulation of eddy/vortex g and of its coordinate x_C, y_C (word goes on the strength of symmetry about flow with x > 0). One Condition for determining g, x_C, y_C is obtained from the requirement for the finiteness of speed on the edge of plate (flow smoothly it converges from edge), an additional two conditions are obtained from the assumption that the vortex filament moves together with flow, i.e., velocity vector for the points of filament is directed along filament. This condition (after linearization) determines the composite speed in point $t_C = x_C + iy_C$ for flow on plane $t = x + iy$ ($z = 1$). This speed is equal to $V_1 t_C$.

The formulated conditions are sufficient for determination g, x_C, y_C and problem becomes determined. However, Mak K. Adams noted [157] that Legendre too simplified the picture of flow, in consequence of which in expression for pressure appeared log term from $t - t_C$, and it became multiple-valued function. If we formally introduce on plane t the cut/section, which connects eddy/vortex with the edge of plate, then pressure on it will suffer discontinuity/interruption after circuit/bypass around eddy/vortex.

Physically this indicates, that even so eddy/vortices themselves will not bear power load, entire vortex/eddy system is not free from it.

Page 182.

For these reasons and the expression for C_L is obtained ambiguous; it takes the form

$$C_L = \frac{\pi}{2}\lambda\delta + \pi\lambda^{1/3}h\delta^{5/3}, \qquad (9.2)$$

where λ is wing aspect ratio [λ = 4 ctg Λ]; h = 1 + 1/2 $\sqrt{2}$, if we C_L calculate according to the distribution of pressure (Adams corrected the available at Legendre miscalculation C_L), or h = 4^1/3, if we C_L calculate according to the method of Munk's momentum/impulse/pulses.

A further improvement of the model of the flow of Legendre made R. Edwards [158], and also K. Braun and U. Maykya [159]. They introduced into examination planes vorticity layer, the connecting vortex filament with leading edges (see Fig. 50d), and they required instead of satisfaction of the condition of the absence of the forces, acting on vortex filaments, the equality of zero resultant force acting from liquid on entire vortex/eddy system. This condition gave for composite speed in point t_C another value (equal to $V_1(2t_C - b)$), and expression for C_L took the form

$$C_L = \frac{\pi}{2}\lambda\delta + \pi\lambda^{1/3}\delta^{5/3}. \qquad (9.3)$$

Comparison with experiment showed that formula (9.3) qualitatively correctly determines change C_L with an increase δ, but increase C_L is obtained excessive. The theory of the extended bodies where $C_L = \pi/2\ \lambda\delta$, gives decreased values for C_L.

The quantitative coincidence of values C_L, determined in formula (9.3), with experimental values is observed only for very narrow wings ($\Lambda \approx 85°$). This fact is explained by the fact that for wings with large spread/scopes the rolled up vorticity layers occupy the considerable region above the wing surface and schematization by their vortex lines becomes too rough. A following space in the development of theory made K. Mangler and D. Smith [119]; they utilized the model of flow, in which is considered the form of the vorticity layers, considered as contact discontinuity/interruptions (Fig. 50b). (The problem of determining ϕ_0 in plane z = 1 cannot already be considered as problem of the steady flow of plate with the spiral vorticity layers, which begin on its edges, due to a difference in the conditions on vorticity layers).

Page 183.

For Mangler and Smith it was not impossible to fully solve

stated problem, since the form of vorticity layers it was necessary to postulate, but the entering the solution parameters to determine by the satisfaction of conditions on vorticity layers at isolated points. However, this theory is most ideal from those who exist (of resting on the theory extended bodies), and its basic condition/positions will be presented further.

The results of the theory of Mangler and Smith do not depend on the Mach number M_1 of the incident to wing flow (and other authors's results, which were resting on the theory of the extended bodies). However, from experiment it is known that the vortex/eddy systems above the wing with an increase M_1 (or by an increase in the wingspan with that which was fix/recorded M_1) "are pressed" against the wing surface and the type of flow gradually changes, approaching type b-c of figure 49. For this reason the theory of Mangler and Smith is suitable only for narrow wings. The effect of number M_1 on stream conditions about triangular plate was registration/accounting by L. Skvaer [160].

If Mangler and Smith attempted to most completely include/connect in the theory of the part of flow about wing, then Skvayer following D. Kyukheman's idea, was examined certain averaged picture of flow. The rolled up vorticity layers above the wing are led to a pressure drop in the vicinity of leading wing edges. The

same phenomenon will cause an increase of the local angle of attack in some vicinities of leading edges.

D. Kyukheman examines certain auxiliary airfoil at whose local angle of attack changes abruptly: on one part of the wing, it is equal to δ_1 = const, on another δ_2 = const. Values of δ_1 and δ_2 are located from the conditions of the finiteness of speed on leading edges and the condition that the mean incidence of the auxiliary airfoil is equal to the angle of attack of the assigned wing, i.e.,

$$\delta = \delta_1 \cdot x_0 \operatorname{tg} \Lambda + \delta_2 (1 - x_0 \operatorname{tg} \Lambda),$$

where x_0 there is a coordinate of point on plane $z = 1$, where occurs the discontinuity/interruption of local angle of attack.

Page 184.

For determination x_0, is made the assumption (actually, arbitrary), that

$$x_0 \operatorname{tg} \Lambda = 1 - 2\frac{\delta}{\pi},$$

i.e. the line of discontinuity the local angle of attack of the auxiliary airfoil is moved from leading edge to the central line of the wing when δ grow/rises from 0 to $\pi/2$.

Kyukheman acted within the framework of the theory of the extended bodies; Skvayer utilized formulas of linear theory and

obtained in the final analysis a good coincidence of theoretical and experimental results (see Section 9.3).

The information about quite last/latter works in this region can be found in report [161].

9.2. Results of theory of extended bodies for triangular plastic. Following K. Mangler and D. Smith [119], let us examine the narrow triangular plate with sweep angle Λ, placed into the uniform flow of the gas, which has speed V_1, at low angle of attack δ without slip (Fig. 51).

The axes of the Cartesian system of coordinates O_1xyz are selected so that the wing lie/rests at plane y = 0, and axis O_1z is directed along its central line. The flow of gas of outside vorticity layers is considered irrotational, and its velocity potential ϕ is conveniently presented in the form

$$\varphi = V_1 \cos\delta z + \varphi_0, \qquad (9.4)$$

where ϕ_0 is the modified potential of velocity of disturbance/perturbation, which differs from the usual velocity potential of disturbance/perturbation, equal to $\phi - V_1 \cos \delta z - V_1 \sin \delta y$, by component $V_1 \sin \delta y$. According to the theory of the extended bodies, the velocity potential of disturbance/perturbation and, consequently, also ϕ_0, satisfies the two-dimensional equation of

Laplace

$$\frac{\partial^2 \varphi_0}{\partial x^2} + \frac{\partial^2 \varphi_0}{\partial y^2} = 0. \qquad (9.5)$$

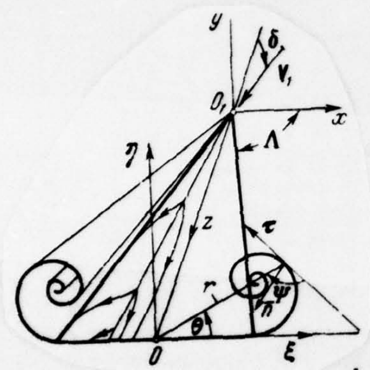


Fig. 51.

Page 185.

The components of vector of speed through the appropriate axes of coordinates u, v, w are located through the formulas

$$u = \frac{\partial \varphi}{\partial x} = \frac{\partial \varphi_0}{\partial x}, \quad v = \frac{\partial \varphi}{\partial y} = \frac{\partial \varphi_0}{\partial y}, \quad w = \frac{\partial \varphi}{\partial z} = V_1 \cos \delta + \frac{\partial \varphi_0}{\partial z}. \qquad (9.6)$$

In view of the conical flow pattern about wing, it is convenient immediately to pass to plane $\xi = x/z$, $\eta = y/z$, and to introduce into examination the conical potential $F(\xi, \eta) = z^{-1}\phi(x, y, z)$. Then formulas (9.4)-(9.6) take the form

$$
\left.\begin{aligned}
&F = V_1\cos\delta + F_0, \text{ где } F_0(\xi, \eta) = z^{-1}\varphi_0,\\
&\frac{\partial^2 F_0}{\partial\xi^2} + \frac{\partial^2 F_0}{\partial\eta^2} = 0,\\
&u = \frac{\partial F_0}{\partial\xi}, \quad v = \frac{\partial F_0}{\partial\eta}, \quad w = V_1\cos\delta + F_0 - \xi\frac{\partial F_0}{\partial\xi} - \eta\frac{\partial F_0}{\partial\eta}.
\end{aligned}\right\} \tag{9.7}
$$

Key: (1). where.

If we utilize polar coordinates r, θ on plane ξ η (see Fig. 51), then boundary condition for F_0 far from wing it is possible to write in the form

$$\frac{\partial F_0}{\partial\xi} \to 0, \frac{\partial F_0}{\partial\eta} \to V_1\sin\delta \approx V_1\delta \text{ при } r \to \infty. \tag{9.8}$$

Key: (1). with.

The surfaces of wing and coagulated vorticity layers which are considered as contact discontinuity/interruptions, are conical stream surfaces; therefore along them must be fulfilled the condition

$$\frac{d\xi}{u - \xi w} = \frac{d\eta}{v - \eta w},$$

which, taking into account equalities $d\eta/d\xi = tg(\theta + \Psi)$ (see Fig. 51), $\xi = r\cos\theta$, $\eta = r\sin\theta$, it can be written in the form

$$v\cos(\theta + \Psi) - u\sin(\theta + \Psi) + wr\sin\Psi = 0. \tag{9.9}$$

Page 186.

If we designate by n internal standard to the line, which corresponds to stream surface on plane $\xi\eta$, to consider formulas (9.7), to replace under condition (9.9) w for V_1 (t.e. to conduct the linearization of condition), then we will finally obtain from it the formula

$$\frac{\partial F_0}{\partial n} = -V_1 r \sin\Psi. \qquad (9.10)$$

Specifically, to wing

$$\frac{\partial F_0}{\partial n} = 0, \qquad (9.11)$$

since there $\Psi = 0$ or π.

The expansion pressure coefficient $C_p = \frac{p - p_1}{\rho_1 \frac{V_1^2}{2}}$ according to degrees the component of perturbation rate u', v', w' takes the form

$$C_p = -2\frac{w'}{V_1} + (M_1^2 - 1)\left(\frac{w'}{V_1}\right)^2 - \left(\frac{u'}{V_1}\right)^2 - \left(\frac{v'}{V_1}\right)^2 + \dots$$

For our case within the framework of the theory of the extended bodies, it is record/written in the form

$$C_p = -\frac{2}{V_1}\left(F_0 - \xi\frac{\partial F_0}{\partial \xi} - \eta\frac{\partial F_0}{\partial \eta}\right) - \frac{1}{V_1^2}\left[\left(\frac{\partial F_0}{\partial \xi}\right)^2 + \left(\frac{\partial F_0}{\partial \eta}\right)^2\right] + \delta^2 + \dots \qquad (9.12)$$

Let us designate the jump of certain value f during transition through the contact surface by symbol Δf; then on vorticity layer $\Delta C_p = 0$ and from formula (9.12) it is obtained

$$\Delta F_0 - \Delta\left(\xi\frac{\partial F_0}{\partial \xi} + \eta\frac{\partial F_0}{\partial \eta}\right) + \frac{1}{2V_1}\Delta\left[\left(\frac{\partial F_0}{\partial \xi}\right)^2 + \left(\frac{\partial F_0}{\partial \eta}\right)^2\right] = 0. \qquad (9.13)$$

If we introduce on plane $\xi\eta$ along with the normal direction n, also tangent τ (Fig. 51), then

$$\Delta\left(\xi\frac{\partial F_0}{\partial \xi}+\eta\frac{\partial F_0}{\partial \eta}\right)=\left(\xi\frac{\partial \xi}{\partial \tau}+\eta\frac{\partial \eta}{\partial \tau}\right)\Delta\left(\frac{\partial F_0}{\partial \tau}\right)=r\cos\psi\Delta\left(\frac{\partial F_0}{\partial \tau}\right),$$

then since derived $\frac{\partial F_0}{\partial n}$ according to formula (9.10) it is continuous on vorticity layer.

Page 187.

Further, $$\Delta\left[\left(\frac{\partial F_0}{\partial \xi}\right)^2+\left(\frac{\partial F_0}{\partial \eta}\right)^2\right]=\Delta\left[\left(\frac{\partial F_0}{\partial n}\right)^2+\left(\frac{\partial F_0}{\partial \tau}\right)^2\right]=2\left(\frac{\partial F_0}{\partial \tau}\right)_m\Delta\left(\frac{\partial F_0}{\partial \tau}\right),$$

where the index of "m" indicates the half-sum of the values, undertaken from both sides of vorticity layer.

From formula (9.13) we obtain then the continuity condition of pressure on vorticity layer in the form

$$\Delta F_0=\Delta\left(\frac{\partial F_0}{\partial \tau}\right)\left[r\cos\Psi-\frac{1}{V_1}\left(\frac{\partial F_0}{\partial \tau}\right)_m\right]. \qquad (9.14)$$

Thus, initial three-dimensional problem came to the two-dimensional problem of the determination of the harmonic function F_0 in the region, depicted on Fig. 52, which is plane with cut/section along the axis ξ from -b to +b, that are continued in the form of unknown previously spiral-shaped curves (secondary flow breakaway from wing is not considered). Boundary conditions for determining F_0 at infinity are assigned by formula (9.8), on cut/section $\eta = 0$, $b < \xi < b$ - by formula (9.11), in spiral-shaped curves - by formulas (9.10)

and (9.14).

The formulated problem of determination F_0 still remains complex; therefore Mangler and Smith produce its further simplification. It is assumed that basic part of the vorticity layer, which begins on the edge of plate, has simple form, and its folding about the center of curve occurs within the circle, which has small diameter 2R. Within this circle it is accepted that the spiral-shaped curve has equation $r_1 = c\mu/\theta_1$, where r_1, θ_1 are the polar coordinates with beginning in the center of curve, c - distance from this center to the origin of coordinates, μ - certain constant.

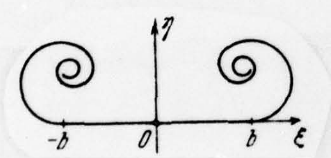


Fig. 52.

Page 188.

The investigation, carried out by Mangler and Smith, showed that the effect of the rolled up vorticity layer, arrange/located within the mentioned circle, on the flow of outside circle in the first approximation, will be the same as effect of the point eddy/vortex,

placed to the center of curve. The intensity of eddy/vortex is determined from formula $g = 2\pi\mu R V_1 \operatorname{tg} \nu$, where R is a radius of the circle in question, ν - the angle, composed by half-line, that corresponds in space xyz to the center of spiral-shaped curve, with the central line of wing. Peripheral speed with $r = R$ is equal then $G/2\pi R = \mu V_1 \operatorname{tg} \nu$. Furthermore, on leaving of spiral-shaped curve from circle (with $r_1 = R$) must be fulfilled also the conditions

$$\left.\begin{aligned} \Delta F_0 &= 2\pi\mu R V_1 \operatorname{tg} \nu, \\ \Delta\left(\frac{\partial F_0}{\partial \tau}\right) &= -2\pi V_1 \operatorname{tg} \nu \frac{R}{c}. \end{aligned}\right\} \tag{9.15}$$

The initial diagram, depicted on Fig. 52, is replaced more idle time, depicted on Fig. 53a.

Instead of spiral-shaped curved, beginning on leading edge of plate, is examined the curve BED (see Fig. 53a) and the point eddy/vortex, placed at point C.

At point D, the curve BED concerns the circle of radius R, with center at point C. (On the strength of symmetry are examined the points of region from $\xi \geqslant 0$).

Let us consider plane $\xi\eta$ the plane of complex variable

$$t = \xi + i\eta$$

and let us pass from it to plane $t^* = \xi^* + i\eta^*$ with the aid of the conformal transformation

$$t^{*2} = t^2 - b^2.$$

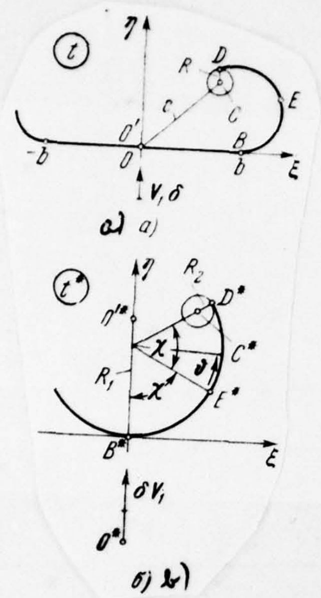


Fig. 53.

Page 189.

On plane t *, cut OB (Fig. 53a), which corresponds to wing, is depicted as the cut of the imaginary axis between points t * = $\pm$ ib. If we are distracted from conditions in the vortex/eddy word BDE (see Fig. 53a), then F_0 it is convenient to consider as streaming potential of the incompressible fluid about plate and to introduce composite potential and the composite speed of this flow. On plane t *, the axis ξ * = 0 must then to be the flow line of flow, and

conditions (9.100, (9.14) take the form [119]

$$\left.\begin{aligned}\frac{\partial F_0}{\partial n^*} &= -V_1 r^* \sin\Psi,\\ \Delta F_0 = \Delta F_0^* &= \Delta\left(\frac{\partial F_0}{\partial \tau^*}\right)\left(\frac{r}{r^*}\right)^2\left[r^*\cos\Psi - \frac{1}{V_1}\left(\frac{\partial F_0}{\partial \tau^*}\right)_m\right],\end{aligned}\right\} \quad (9.16)$$

where τ * is an arc length and n * - internal standard to the curve B*E*D * on plane t *.

Further simplification in the problem consists in the fact that instead of searching for the of harmonic function F_0 (or the corresponding it analytic function of t *), which satisfies conditions (9.16) in the unknown previously curve B*E*D * in figure 53b, Mangler and Smith counted the curve B*E*D * of another circumference, which has a center on the imaginary axis, radius R_1, central angle 2χ. The effect of vorticity layer B*E*D * in field of flow in plane t * is accepted approximately such which it will be from vorticity layer in two-dimensional problem of hydrodynamics. This means that on plane t * in the final analysis is examined the incompressible flow, obtained by the imposition of uniform flow, flow from two isolated/insulated eddy/vortices and two vorticity layers (for that in order to axis ξ * - 0 it would be flow line, it is necessary to symmetrically add the isolated/insulated eddy/vortex and vorticity layer).

The intensity of vorticity layer B*E*D * is accepted in the form

$$\Delta\left(\frac{\partial F_0}{\partial \tau}\right) = V_1 \operatorname{ctg} \Lambda \, (\gamma_0 + \gamma_1 \cos\vartheta + \gamma_2 \sin\vartheta),$$

where γ_0, γ_1, γ_2 are unknown constants, angle ϑ is shown to Fig. 53b.

end section.

Page 190.

Function F_0 corresponding to the indicated flow, depends on seven parameters: R_1, χ, R_2, γ_0, γ_1, γ_2, μ, which are located by the satisfaction of conditions (9.16) in the isolated points of the curve B*E*D . Sending away the reader after details in [119], let us note that the authors made calculations for a series of the values of parameter $\delta \operatorname{tg} \Lambda$, lying within limits from 0.1 to 2.65, and were obtained the results, which are somewhat better than the results Maykl and Braun. For example, the values of the coefficient of normal force for a plate with that which was fix/recorded $\delta \operatorname{tg} \Lambda$ were obtained somewhat smaller than in Maykl and Braun. However, it must be noted that the essential expansion of the field of the applicability of theory did not occur, what is, apparently, the consequence of the hypothesis that the region of intense convolution of vorticity layer is small in comparison with the distance of its center of leading edge.

9.3. Results of linear theory for triangular plate. Following L. Skvaer [160], let us examine the triangular plate with sweep angle

Λ, placed into the uniform flow of the gas, which has speed V_1, the Mach number M_1 and of so forth, under angle of attack δ without slip.

It is assumed that with flow about plate is realized the mode/conditions of 6 Figure 49. As it was already noted, due to the effect of the coagulated vorticity layers in some vicinities of leading edges, occurs a pressure drop, which is led to the nonlinear dependence of the lift coefficient C_L of wing on angle of attack δ.

For obtaining this dependence Skvayer proposes the semi-empirical approach: is utilized the setting of problem and equations of usual linear conical flow theory (see, for example [3], [4]), but instead of the assigned wing is examined auxiliary on which local angle of attack piecewise-is constant. If we pass to the plane

$$\xi' = \frac{x}{z} \cdot \mathrm{tg}\,\Lambda, \quad \eta' = \frac{y}{z} \cdot \mathrm{tg}\,\Lambda,$$

that wing it is depicted here as axis intercept $O'\xi'$:

$$0 < |\xi'| < 1.$$

For the assigned wing local angle of attack is equal to δ, while for auxiliary it is obtained by the imposition of two distributions:

$$\delta_d = \begin{cases} 0, & 0 < |\xi'| \leqslant \xi_0', \\ \delta_1, & \xi_0' < |\xi'| \leqslant 1, \end{cases} \quad (9.17)$$

$$\delta_c = \delta_2, \quad 0 < |\xi'| < 1, \quad (9.18)$$

thus, when $\xi' = \xi'_0$ proceeds a change in the local angle of attack from value δ_2 to value $\delta_1 + \delta_2$.

Page 191.

As a result of the linearity of fundamental equations, the parameters of flow about the auxiliary airfoil are obtained by the addition of the same in the problems where the local angle of attack is assigned by distributions (9.17) and (9.18).

Pressure coefficients on wing for distributions (9.17), (9.18) according to linear theory are given by the formulas

$$C_{pd} = \frac{2\,\mathrm{ctg}\,\Lambda (k')^2 c \xi_0' \delta_1}{\pi D} \left\{ \frac{2\Pi(-D^2, k)}{E(k)} \frac{1}{\sqrt{1-(\xi')^2}} + \right.$$

$$\left. + \frac{1}{(k')^2 c} \ln \frac{c - \sqrt{1-(\xi')^2}}{c + \sqrt{1-(\xi')^2}} \right\}, \quad (9.19)$$

$$C_{pc} = \frac{2\,\mathrm{ctg}\,\Lambda \delta_2}{E(k) \sqrt{1-(\xi')^2}}, \quad (9.20)$$

where

$$k' = m_1 \mathrm{ctg} \Lambda, \quad m_1 = (M_1^2 - 1)^{1/2}, \quad k = (1 - m_1^2 \,\mathrm{ctg}^2\, \Lambda)^{1/2},$$
$$c = [1 - (\xi_0')^2]^{1/2}, \; D = [1 - m_1^2 \,\mathrm{ctg}^2\, \Lambda\, (\xi_0')^2]^{1/2}, \quad E(k), \quad \Pi(-D^2\, k) -$$

complete elliptic integrals of the second and third kind of module/modulus k. The determination of values δ_1 and δ_2 is conducted from following conditions. The mean incidence of the auxiliary airfoil to the angle of attack of the assigned wing, i.e.

$$\delta = \delta_2 + (1 - \xi_0')\delta_1. \tag{9.21}$$

Vorticity layer smoothly leaves edge, i.e., pressure coefficient on edge it is equal to zero:

$$C_p = C_{pd} + C_{pc} = 0, \qquad \xi' = 1,\ \eta' = 0.$$

Since on the leading edge of plate the log term in (9.19) is equal to zero, the terms, which contain the factor

$$[1 - (\xi')^2]^{-1/2},$$

must average out. This condition gives one additional dependence between δ_1 and δ_2:

$$\delta_2 = -\frac{2c\xi_0'(k')^2}{\pi \cdot D}\Pi(-D^2k)\cdot\delta_1. \tag{9.22}$$

Page 192.

The lift coefficient of wing, c_L, obtained by integration c_p for its surface, after exception/elimination δ_1 and δ_2 with the aid of (9.21), (9.22), will be located from the formula

$$C_L = \frac{\pi\lambda\delta}{2}\left\{\frac{\xi_0'[1-(\xi_0')^2]^{1/2}}{m_1^2 \operatorname{ctg}^2\Lambda\cdot\xi_0'[1-(\xi_0')^2]^{1/2}\Pi(-D^2, k) - \frac{\pi}{2}(1-\xi_0')D}\right\}. \quad (9.23)$$

where λ is wing aspect ratio $(\lambda = 4 \operatorname{ctg}\Lambda)$.

When $m_1 \operatorname{ctg}\Lambda \to 0$, i.e., we pass to the theory of the extended bodies, term $m_1^2 \operatorname{ctg}^2 \Lambda\Pi$ (-D², k) approaches arccos ξ'₀/ξ'₀ $[1-(\xi_0')^2]^{1/2}$ and D ⟶ 1, so that (9.23) he converts to the formula

$$C_L = \frac{\pi\lambda\delta}{2}\left\{\frac{\xi_0'[1-(\xi_0')^2]^{1/2}}{\arccos\xi_0' - \frac{\pi}{2}(1-\xi_0')}\right\}. \quad (9.24)$$

which was equal obtained by D. Kyukheman. If ξ'₀ ⟶ 1, then expression in the curly braces in formula (9.23) approaches 1/E(k). Assuming that the derivative $\frac{\partial\xi_0'}{\partial\delta}$ is final when ξ'₀ = 1 (with δ = 0), we will obtain

$$\left(\frac{\partial C_L}{\partial\delta}\right)_{\delta=0} = \frac{\pi\lambda}{2E(k)}. \quad (9.25)$$

that there is a result of usual linear theory.

In formula (9.23) enters the only unknown parameter of ξ'_0 which, however, plays the dominant role, since with its aid Skvayer proposes to consider nonlinearity in dependence C_L from δ. On that, will be how selected function $\xi' = \xi'(\delta)$, depends the quality of proposed theory.

Examining numerical results of Mangler and and Smith [119], that relate to narrow plates (i.e. with m_1 $\mathrm{ctg}\,\Lambda \to 0$), Skvayer comes to the conclusion that the values C_L are approximated well by the formula

$$C_L = \frac{\pi\lambda\delta}{2} + 4\delta^2 = \frac{\pi\lambda\delta}{2}\left(1 + \frac{2}{\pi}\frac{\delta}{\mathrm{ctg}\,\Lambda}\right). \quad (9.26)$$

Comparing formulas (9.24), (9.26), we obtain

$$1 + \frac{2}{\pi}\frac{\delta}{\mathrm{ctg}\,\Lambda} = \frac{\xi'_0\,[1 - (\xi'_0)^2]^{1/2}}{\arccos \xi'_0 - \frac{\pi}{2}(1 - \xi'_0)}. \quad (9.27)$$

Further Skvayer makes the main assumption that the dependence ξ'

on δ, determined by formula (9.27), will be valid at all values $m_1 \operatorname{ctg} \Lambda$, but not only at the low values of this parameter.

Page 193.

Thus, formulas (9.23), (9.27) uniquely determine the dependence C_L on δ, which can be represented in the form

$$\frac{C_L}{\operatorname{ctg}^2 \Lambda} = \frac{2\pi}{E(k)} \frac{\delta}{\operatorname{ctg} \Lambda} \Phi\left(\frac{\delta}{\operatorname{ctg} \Lambda}, m_1 \operatorname{ctg} \Lambda\right), \qquad (9.28)$$

where

$$\Phi\left(\frac{\delta}{\operatorname{ctg} \Lambda}, m_1 \operatorname{ctg} \Lambda\right) = \frac{\xi_0' [1 - (\xi_0')^2]^{1/2} E(k)}{m_1^2 \operatorname{ctg}^2 \Lambda \xi_0' [1 - (\xi_0')^2]^{1/2} \Pi(-D_*^2 k) - \frac{\pi}{2} (1 - \xi_0') [1 - m_1^2 \operatorname{ctg} \Lambda (\xi_0')^2]^{1/2}}.$$

Figures 54 depicts the dependence of ξ'_0 on $\frac{\delta}{\operatorname{ctg} \Lambda} = \delta \operatorname{tg} \Lambda$ that corresponds to equation (9.27).

Figure 55, undertaken work [160], gives the comparison of the results of the examined theory, usual linear theory and results of U. Maykl's experiments [156], carried out with M_1 = 1.9 for models with the very sharp leading edges and parameter $m_1 \operatorname{ctg} \Lambda$, that which are changing within limits from 0.14 to 1.00.

As can be seen from Fig. 55, theoretical results will agree well with experimental. For the wings, which have "final" thickness, agreement it is worse.

The theory of Skvayer is inapplicable when on suction side of wing pressure falls below zero and flow transfer/converts from type b to type b-c (in more detail, see in [160]).

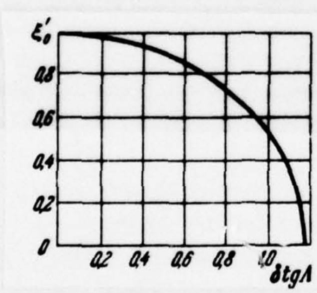


Fig 54

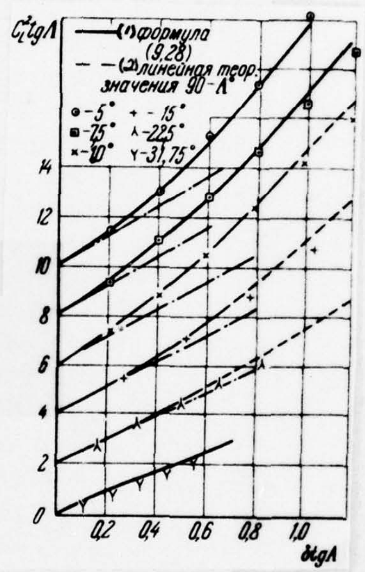


Fig 55

Fig. 54.

Fig. 55.

Key: (1). formula. (2). linear theoretically of value.

Page 194.

9.4. Final Notes The most important conclusion/derivation from materials §9 lies in the fact that a conical flow theory of nonviscous gas can be utilized for determining field of flow about conical bodies with sharp leading edges when flow blows away from the leading edges of body. Of course, just as in two-dimensional problem of hydrodynamics, a series of the parameters is required to determine from experiment or to introduce into the theory of the assumptions which are checked in the final analysis experimentally.

The existing theories rest on linear equations for the velocity potential of disturbance/perturbation and bear, therefore, hybrid character. Is necessary the further development of nonlinear theory, especially for the cases not of very narrow wings with subsonic leading edges.

§10. Numerical methods in inverse problem.

10.1. Lead-in observations. The problem of determining of flow parameter about the conical body of the assigned form he is called straight line. In the inverse problem of known, is counted the form of leading shock wave, unknown are the parameters of flow, and also the form of the body, which creates the assigned shock wave. Knowing the front of bow shock and the parameters of undisturbed flow, it is possible to find flow parameters immediately behind shock wave from known formulas, and to obtain for determining the parameters of flow in an entire region between the leading shock wave and the body surface problem with initial data (Cauchy problem) for the system of equations of conical flows [see, for example, (1.1)-(1.5)]. Since in the larger part of the flow (or in all flow) about body the flow conical-is subsonic, i.e., real are only entropy characteristics - flow line, situation is completely analogous to that that store/adds up during the solution of the reverse problem of the flow of supersonic flow about the blunt-nosed body of gas in the flat/plane case. The appearing difficulties and the methods of their overcoming are discussed in detail in monograph [5]; see also [162]; therefore here we will be restricted only to the presentation of basic results on this question.

Page 195.

As is known, the Cauchy problem for eliptic equations is incorrectly

placed, since how conveniently small changes under initial conditions can give to how conveniently to large changes in the solution of problem. In the flat/plane case this fact is exhibited in the low sensitivity of the form of leading shock wave to a change in the duct of the streamlined body. If we solve inverse problem by finite-difference methods, moving step by step from bow shock toward the surface of the streamlined body, then the incorrectness of the Cauchy problem for elliptical type of equations is led to the instability of numerical solution. The small errors, allowed at any space during the solution of problem, grow/rise exponentially in the subsequent spaces. The reason for the indicated instability are the higher harmonics in the expansion of initial data in Fourier series. The basic method of the elimination of instability consists in the fact that at each space of the process of the construction of the solution before transition with the aid of finite-difference diagram to the following space in solution is conducted "filtering" of the upper harmonics, which is equivalent to the smoothing of solution at each space. In practice the smoothing of solution is carried out with the aid of the use of multiexact diagrams for numerical differentiation along curved, carrying Cauchy's data. As a result of this solution, is obtained the body whose surface is smoothed. For these reasons finite-difference methods in inverse problem can be used for the determination of the parameters of flows only about bodies with smooth ducts without discontinuity/interruptions or

abrupt changes. Does not help here the known method of Garabedyan-Libersteyn in which the stability of numerical solution is obtained because of the introduction of the composite three-dimensional space in planes of which, that intersect the flow plane, equation they will be hyperbolic type, since for its use it is necessary initial data to analytically continue into complex domain that it is equivalent to the solution of the problem of Cauchy for the equation of Laplace.

Page 196.

Thus, finite-difference methods in inverse problem are of interest faster not as instrument for the determination (by cut-and-try method) of flow about the assigned body, but as method of explaining the qualitative special feature/peculiarities of flows about conical bodies. By reverse/inverse methods it is possible, for example, to investigate the behavior of flow parameter near body surface, to find the location of the peculiarities of the Ferry, to determine form and location of conical-sonic lines and so forth, that difficultly or generally it is not possible to make by tha existing direct methods.

Let us note that for the large numbers of Mach M_1 of the incident to body flow the form of bow shock is more sensitive to a

change in the duct of the streamlined body, than with those who were moderated M_1, so that in the case $M_1 >> 1$ reverse/inverse methods can be utilized for the determination of flow about the assigned (smooth) body.

In I. Nochevkinoy's work [163] is examined the case, when the front of bow shock is the round cone whose axis forms certain angle with direction of undisturbed flow. In work is utilized the system of spherical coordinates R, θ, Φ, whose axis ($\theta = 0$) coincides with the axis of shock wave; the unknown functions are the components of velocity u, v, w and entropy S. From system of equations in partial derivatives for u, v, w, S, they transfer/convert to the system of ordinary differential equations by means of the replacement of derivatives in terms of Φ by finite differences. Then for each fixed/recorded angle θ they are determined in the first and second approach/approximations of the value of the unknown functions by Euler's method. (With space $\Delta\Phi = 1°$ problem came to the solution to 718 differential equations.). As a result of calculations, it was explained that the entropy very sharply changes near body surface, especially with $M_1 >> 1$; the body, which excites bow shock in the form of round cone, with $M_1 = \mu$ is close to round cone. (With the final M_1, for example, with $M_1 = 4$ and the half-angles of the shock wave and slope/inclination of its axis to direction of undisturbed flow, equal to with respect 60° and 6°, the duct of the body strongly

differs from circumference.). In V. Syagaeva's work [164] is utilized, actually, the same method, as in work [163], but shock wave in the principle of "arbitrary" form. Furthermore, here is given the algorithm of the determination of approximate solution of direct problem with the aid of the solution of a series of inverse problems.

Page 197.

As examples are examined the flow about the round cone at an angle of attack and the flow about the elliptical cone at zero angle of attack. Let us note that work gives some results of the flow-field analyses of round cone at $\varepsilon = 20°$, $\delta = 15°$ and numbers M_1, which are changed from 3.53 to 20, from which it is possible to obtain the representation of emergence and development of the zone in which the flow is conical-supersonic.

In the named works, and also in analogous works [165], G. Radkhakrishnan [166] and B. Briggs [167], as independent variables are utilized the variables of physical space; because of this in resolving problem, intersects the vorticity layer near the surface of the streamlined body, which is led to large complications in calculation. In the work of P. Stokker and F. Modzher [114] is proposed the method, free from this deficiency/lack. This method, fine/thinnest of the existing methods of the solution of the reverse

problem, is set forth further.

10.2. Method of two functions of current. In work [114] initial are the equations (1.1)-(1.5). The equation of continuity (1.1) contains density derivative ρ in the form of combination $v\rho_\theta + w \operatorname{cosec} \theta \cdot \rho_\Phi$, which is derived ρ along the lines of the constant entropy S. But with S = const $d\rho = a^{-2}dp$, where a - the local velocity of sound, and equation (1.1) can be presented in the form

$$v_\theta + \operatorname{cosec} \theta w_\Phi + \frac{v}{\rho a^2} p_\theta + \frac{w}{\rho a^2} \operatorname{cosec} \theta \cdot p_\Phi = -2u - v \operatorname{ctg} \theta. \tag{10.1}$$

Let us examine the equation of energy (1.5)

$$vS_\theta + w \operatorname{cosec} \theta S_\Phi = 0.$$

If we introduce function ψ, which satisfies the equation

$$v\psi_\theta + w \operatorname{cosec} \theta \psi_\Phi = 0, \tag{10.2}$$

that $\psi =$ const on the lines of constant entropy (flow lines).

Page 198.

By analogy with two-dimensional problem, let us call/name ψ function of current ($\psi =$ const on conical stream surfaces in the physical space), but, unlike the flat/plane case, ψ it is determined not

unambiguously, since any function ψ, $\Psi(\psi)$ it also satisfies equation (10.2) it can be accepted as the function of current.

If we utilize ψ as independent variable, then is the region where searches for solution, it is automatically limited to the surface of the streamlined body (where $\psi = \text{const}$) and drop off the difficulties, connected with the isolation/liberation of the surface of body in vorticity layer. As the second independent variable it is possible to take, for example, Φ, as is done in work [163]. Certain inconvenience of such selection consists in the fact that the case of round cone with $\delta = 0$ must be obtained as maximum from solution with $\delta \neq 0$, when $\delta \longrightarrow 0$, and in the planes of the symmetry of the flow of line $\Phi = \text{const}$ and $\psi = \text{const}$ coincide.

As independent variables the authors [114] utilize two functions of current ψ, χ, which exist in three-dimensional/space flow. The physical sense of the function of current ψ was already explained, and function χ let us introduce with the aid of the equations

$$\left.\begin{aligned} \chi\psi_{\omega} &= -\rho v \sin\theta, \\ \chi\psi_{\theta} &= \rho w. \end{aligned}\right\} \qquad (10.3)$$

From relationship/ratios (10.3) it follows that ψ satisfy the equation (10.2), and χ — to the equation

$$\left(\frac{\rho v \sin\theta}{\chi}\right)_{\Phi} + \left(\frac{\rho w}{\chi}\right)_{\theta} = 0,$$

which with the aid of (1.1) can be represented in the form

$$-\frac{1}{\chi}(v\chi_{\theta} + w \operatorname{cosec}\theta \cdot \chi_{\Phi}) = 2u,$$

or

$$\frac{1}{\chi}\left(\frac{d\chi}{d\Phi}\right)_{\psi=\mathrm{const}} = -2\frac{u\sin\theta}{w}. \qquad (10.4)$$

Instead of χ it is convenient to introduce alternating/variable η by the formula

$$\eta = \ln\left\{\frac{F(\psi)}{\chi}\right\}, \qquad (10.5)$$

where $F(\psi)$ is the function which will be determined later.

Page 199.

According to (10.4) η satisfies the equation

$$\left.\begin{aligned}\left(\frac{d\eta}{d\Phi}\right)_{\psi=\mathrm{const}} &= \frac{2u\sin\theta}{w},\\ \left[\left(\frac{d\eta}{d\theta}\right)_{\psi=\mathrm{const}} \right.&= \left.2\frac{u}{v}\right].\end{aligned}\right\} \qquad (10.6)$$

or

$$v\eta_{\theta} + w \operatorname{coses} \theta \cdot \eta_{\Phi} = 2u. \quad (10.7$$

As can be seen from formula (10.6), lines $\eta = \text{const}$ do not coincide with lines $\psi = \text{const}$ in the flows where u ≠ 0, and therefore η it is possible to take as independent variable along with ψ.

Let us assume that on bow shock $\eta = 0$; i.e. $\chi = F(\psi)$ on jump. If we assign function $F(\psi)$, then function ψ it will be determined on jump from relationship/ratios (10.3) with an accuracy to additive constant; if we assign ψ on jump, then $F(\psi) = \chi$ will be determined unambiguously from (10.3) (and the equations of consistency on the shock wave). At the point where there is a special feature/peculiarity of Ferry, w = v = 0, but $\psi_{\theta} \neq 0, \psi_{\Phi} \neq 0$, consequently, as can be seen from (10.3), here $\chi = 0, \eta = \infty$. [For a round cone at zero angle of attack (w ≡ 0), $\psi_{\theta} \equiv 0) \psi$ it is possible to take in the form $\psi = \Phi$, then $\chi = -\rho v \sin\theta = \chi(\theta)$.] Equations of motion from ψ and η as independent variables take the following form:

$$\left.\begin{aligned}
&u_{\eta} = \frac{v^2 + w^2}{2u},\\
&v_{\eta} - \frac{\sin\theta}{\chi}\Phi_{\psi}p_{\eta} = \frac{w^2 \operatorname{ctg}\theta}{2u} - \frac{v}{2} - \frac{w}{2u\chi}p_{\psi},\\
&w_{\eta} + \frac{v\sin\theta}{w\chi}\Phi_{\psi}p_{\eta} + \frac{1}{\rho w}p_{\eta} = -\frac{vw\operatorname{ctg}\theta}{2u} - \frac{w}{2} + \frac{v}{2u\chi}p_{\psi},\\
&\frac{1}{\rho a^2}p_{\eta} + \frac{1}{w}\left(1 + \frac{\rho v \sin\theta}{\chi}\Phi_{\psi}\right)w_{\eta} - \frac{\rho\sin\theta}{\chi}\Phi_{\psi}v_{\eta} =\\
&\qquad = \frac{\rho v}{2u\chi}w_{\psi} - \frac{\rho w}{2u\chi}v_{\psi} - 1 - \frac{v\operatorname{ctg}\theta}{2u},\\
&\rho_{\eta} = \frac{1}{a^2}p_{\eta}, \quad \Phi_{\eta} = \frac{w}{2u\sin\theta}, \quad \theta_{\eta} = \frac{v}{2u}.
\end{aligned}\right\} \quad (10.8)$$

Page 200.

If we designate by ω the angle between normal to the surface of shock wave and plane $\Phi = \text{const}$, that passing through the point in question ($\omega > 0$, if standard is directed to the side of increase Φ), through α - the angle between standard and direction of undisturbed flow, through δ - an angle of attack, i.e., the angle between axis $\theta = 0$ and direction of undisturbed flow, then the conditions of consistency on shock wave for an ideal gas can be written as follows:

$$\left.\begin{aligned}
\frac{u}{V_1} &= -\sin\theta\cos\Phi\sin\delta + \cos\theta\cos\delta,\\
\frac{v}{V_1} &= -\frac{\rho_1}{\rho}\cos\omega\{(\cos\theta\cos\Phi\sin\delta + \sin\theta\cos\delta)\cos\omega -\\
&\quad - \sin\Phi\sin\omega\sin\delta\} - \sin\omega\{(\cos\theta\cos\Phi\sin\delta +\\
&\quad + \sin\theta\cos\delta)\sin\omega + \cos\Phi\cos\omega\sin\delta\},\\
\frac{w}{V_1} &= -\frac{\rho_1}{\rho}\sin\omega\{(\cos\theta\cos\Phi\sin\delta + \sin\theta\cos\delta)\cos\omega -\\
&\quad - \sin\Phi\sin\omega\sin\delta\} + \cos\omega\{(\cos\theta\cos\Phi\sin\delta +\\
&\quad + \sin\theta\cos\delta)\sin\omega + \cos\Phi\cos\omega\sin\delta\},\\
\frac{p}{\rho_1 V_1^2} &= \frac{2}{\gamma+1}\cos^2\alpha - \frac{\gamma-1}{\gamma(\gamma+1)}\frac{1}{M_1^2},\\
\frac{\rho}{\rho_1} &= \frac{\cos^2\alpha}{\frac{\gamma-1}{\gamma+1}\cos^2\alpha + \frac{2}{\gamma+1}\frac{1}{M_1^2}}.
\end{aligned}\right\} \quad (10.9)$$

Here index "1" designated the parameters of undisturbed flow. If the equation of the front of the leading shock wave we write in the form

$$g(\theta, \Phi) = 0,$$

then

$$\left.\begin{aligned} \cos\varkappa &= -(\cos\theta\cos\Phi\sin\delta + \sin\theta\cos\delta) \times \\ &\times g_\theta\left[g_\theta^2 + g_\Phi^2\frac{1}{\sin^2\theta}\right]^{-1/2} + \frac{\sin\Phi\sin\delta}{\sin\theta} g_\Phi\left[g_\theta^2 + \frac{g_\Phi^2}{\sin^2\theta}\right]^{-1/2}, \\ \cos\omega &= g_\theta\left[g_\theta^2 + \frac{g_\Phi^2}{\sin^2\theta}\right]^{-1/2}. \end{aligned}\right\} \tag{10.10}$$

Page 201.

The equation, which lays out of bow shock, can be assigned with the

aid of the cutting off of Fourier series

$$\sin^2\theta = b_0 + \sum_1^n b_k \cos k\Phi \qquad (10.11)$$

or in another convenient manner (see [115]). after using arbitrariness in the selection of the function of current ψ, always it is possible to count that on shock wave $\psi = \Phi$; then on jump $F(\psi) = \chi = -(\rho v \sin\theta + \rho w\, g_\Phi/g_\theta)$ and in field of flow according to (10.5)

$$\chi = F(\psi)\exp(-\eta). \qquad (10.12)$$

If we assign the values of coefficients b_0, b_k in equation (10.11) and the parameters of undisturbed flow, then through formulas (10.9)-(10.12) are located initial data (when $\eta = 0$) and function χ for a system of equations (10.8). Knowing the unknown values when $\eta = 0$, they find their derivatives in terms of ψ with the aid of the nine-point difference formula

$$\Delta\psi(f_\psi)_0 = A_1(f_1 - f_{-1}) + A_2(f_2 - f_{-2}) + A_3(f_3 - f_{-3}) + + A_4(f_4 - f_{-4}),$$

where A_1 = 0.8; A_2 = -0.2; A_3 = 0.03809524; A_4 = -0.00357143, after which value of all quantities on line $\eta = h$ are found by the formulas

$$f(h) = f(0) + (f_\eta)_{\eta=0}\, h,$$

where the derivatives $(f_{\eta})_{\eta=0}$ are determined from system (10.8).

After accepting the values of the unknown quantities when $\eta = h$ as initial, find their values $\eta = 2h$ in a described manner, etc. The region in which searches for the solution, is determined by inequalities $0 \leqslant \Phi \leqslant 2\pi,\ 0 \leqslant \eta < \infty.$ Large values η correspond to the vicinity of Ferry's special feature/peculiarity, so that on plane (ψ, η) occurs the "elongation" of this region in comparison with physical space, which is favorable for using finite-difference method.

In work [114] it is communicated about solution by the method of two functions of the current of the reverse/inverse and straight line of problems (iteratively) in the following cases.

Page 202.

Inverse problem:

1. $M_1 = \infty$; $b_0 = 0.166$; b_2 is given a series of values, large - 0.055; the others $b_k = 0.$

2. Shock wave is close to elliptical cone:

a). $M_1 = 10$; $tg\,\theta_{max} = 0{,}5$; $tg\,\theta_{min} = 0{,}4$; $\delta = 0$.

b). $M_1 = 6$; $tg\,\theta_{max} = 0{,}962$; $tg\,\theta_{min} = 0{,}577$; $\delta = 0$.

Direct problem:

3. The streamlined body is approximately elliptical cone, $M_1 = 6$; $tg\,\theta_{max} = 0{,}400$; $tg\,\theta_{min} = 0{,}226$.

4. Streamlined body is round cone:

a). $M_1 = 3.53$; $\varepsilon = 20°$; $\delta = 5°$;

b). $M_1 = 3.53$; $\varepsilon = 20°$; $\delta = 10°$.

Without being stopped on the results of calculations in each of enumerated cases, let us formulate general the conclusion/derivations which can be made, relying on these calculations.

The behavior of the lines of constant entropy - flow lines clearly indicates the existence of Ferry's special feature/peculiarities in field of flow. In the vicinity of all investigated bodies, there is a thin vorticity layer, falling into which flow line very rapidly they approach a surface of the

streamlined body, so that it is created impression, that they they form envelope, concerning body surface. Pressure little is changed across vorticity layer, while the components of velocity and density change rapidly and their airfoil/profiles here cannot be found by this method which is also inapplicable near Ferry's special feature/peculiarities. In the case of round cone with $M_1 = 3.53$; $\varepsilon = 20°$; $\delta = 15°$ were met difficulties in the determination of body surface which the authors are explained by the displacement/movement of Ferry's special feature/peculiarity from the surface of cone in field of flow; see [169].

The accuracy (stable) of solution was monitored with the aid of the picture of the formation/education "of envelope" by flow lines and by the degree of pressure change in the close points of vorticity layer, which on plane $\varphi\eta$ they are far located from each other. Furthermore, in a number of cases was conducted comparison with the results of the calculations of Modzher according to method, the close known method of Garabedyan in which there is stability of numerical solution.

Page 203.

The accuracy of solution was changed over wide limits; with the large M_1 it was above, with those who were moderated M_1 - is below.

Apparently, by this method (as by other reverse/inverse methods) it is possible to solve the problem of flow only for the bodies of quite simple form.

In the work of D. Eastman and M. Omar [115] the method of two functions of current is applied for determining flow about the windward face of round cone at high angles of attack. Being oriented toward analogy with two-dimensional problem, the authors assumed that the form of the cross section of shock wave can be represented by the equation of the conic section which contains three parameters. These parameters are selected so that would be obtained the correct form of the streamlined body. The results of the calculations, made by Eastman and Omar for cases $M_1 = 7.95$; $\varepsilon = 10°$; $0 < \delta < 24°$, they render/showed in excellent agreement with the results of experiments. Let us note that for a cone with $\varepsilon = 10°$ at flight speed $V_1 = 5500$ m/s on height/altitude 15000 m and the account the real properties of gas the calculations were made to values $\delta = 60°$, which indicates the large stability of conical flow.

§11. Numerical methods in direct problem.

11.1. Lead-in observations. For the solution to direct problem

of the flow about the conical bodies, there exist two basic methods. This is known methods of A. Dorodnitsin's integral relationship/ratios [170] and the finite-difference method, proposed to K. Babenko with colleagues [29]. Each of these methods has its merits and deficiency/lacks. The simplest version of the method of the integral relationship/ratios when the unknown values are approximated linearly in terms of their values on shock wave and the streamlined body, is developed by P. Chushkin and V. Shchennikov [107]. In this form the method of integral relationship/ratios gives good results only at the large values of the Mach numbers M_1 of the incident flow when bow shock is located near the surface of the streamlined body, and does not consider the special feature/peculiarity of the Ferry and vorticity layer.

At the moderate values M_1, the accuracy of method falls and are required higher approach/approximations.

Page 204.

The finite-difference method [29] in principle considers vorticity layer, but during the replacement of the derived unknown values by finite differences the accuracy of approximation can become low in cases when derivatives go to infinity on the surface of the streamlined body, which will pronounce on the accuracy of entire

solution. Especially strongly this fact is exhibited with $M_1 >> 1$. If we want to correctly consider vorticity layer in the method of final differences, near body surface, it is necessary to take extra-fine space, which considerably complicates calculation. (in the tables [29] in many instances the thickness of vorticity layer is less than the space of the difference grid.). On the other hand, the method of operation [29] draws by the universality of the algorithm of the solution of problem and by the possibility of the automatic correction of the errors, allowed in the process of count.

There are other methods of the solution to direct problem (see for example, [171]), but axes possess smaller generality than named.

For those cases when flow occurs with the "closing" of flow about the windward face of body, problem stops completely analogous to two-dimensional problem of the flow of supersonic flow about the blunted bODY of gas, and for its solution can be used the methods, used in two-dimensional problem of gas dynamics.

In work of A. Bazzhin and I. Chelyshevoy [109] for this purpose is used the method of operation [172].

11.2. Method of integral relationship/ratios. Since the essence of the method of integral relationship/ratios is well known at

present, let us turn immediately to its use in the problem of the flow about the conical bodies, following [107].

The equations of motion of gas (1.1)-(1.5) are converted to divergent **form and are record/written in the form**

$$(\rho uv\sin\theta)_\theta + (\rho uw)_\Phi = (V^2-3u^2)\rho\sin\theta, \tag{11.1}$$

$$[(\rho v^2 + p)\sin\theta]_\theta + (\rho vw)_\Phi = (\rho w^2 + p)\cos\theta - 3\rho uv\sin\theta, \tag{11.2}$$

$$(\rho vw\sin\theta)_\theta + (\rho w^2 + p)_\Phi = -\rho(vw\cos\theta + 3uw\sin\theta), \tag{11.3}$$

$$(\rho v\sin\theta)_\theta + (\rho w)_\Phi = -2\rho u\sin\theta, \tag{11.4}$$

$$v(p\rho^{-\gamma})_\theta + w\operatorname{cosec}\theta\,(p\rho^{-\gamma})_\Phi = 0. \tag{11.5}$$

Page 205.

[Equation (11.5) is not given to divergent form, but this, as we will see further, it is unessential].

Here

$$V^2 = u^2 + v^2 + w^2;$$

all values we consider **as dimensionless, considering** that the speed is referred to the critical speed of **sound** a_*, **density** - to the density of the incident flow ρ_1, **pressure - to value** $\rho_1 a_*^2$.

Instead of equation (11.1) subsequently is utilized Bernoulli's integral

$$p = \left(\frac{\gamma+1}{2\gamma} - \frac{\gamma-1}{2\gamma}V^2\right)\rho. \qquad (11.6)$$

System of equations (11.2)-(11.6) must be integrated in the region between the leading shock wave and the surface of the streamlined conical body. If the equation of the duct of the section of body by the sphere of a single radius is $\theta = \theta^{\times}(\Phi)$, then the condition of the nonseparated flow of body he is record/written in the form

$$\frac{v^{\times}}{w^{\times}} = \frac{1}{\sin\theta^{\times}}\frac{d\theta^{\times}}{d\Phi} = \Psi(\Phi). \qquad (11.7)$$

Cross designated the values when $\theta = \theta^{\times}(\Phi)$.

Page 206.

If the equation of the spherical section of leading shock wave is $\theta = \theta^{o}(\Phi)$, then gas-dynamic values behind its front are determined from the formulas

$$\left.\begin{aligned}
u^{\circ} &= V_1(-\sin\delta\cos\Phi\sin\theta^{\circ} + \cos\delta\cos\theta^{\circ}),\\
v^{\circ} &= V_1 n\sin\beta\cos\beta - l,\\
w^{\circ} &= V_1 n\cos^2\beta + l\,\mathrm{tg}\,\beta,\\
\rho^{\circ} &= V_1\frac{m\cos\beta}{l},\\
p^{\circ} &= \frac{2}{\gamma+1}V_1^2 m^2\cos^2\beta - \frac{\gamma-1}{\gamma+1}\left(1 - \frac{\gamma-1}{\gamma+1}V_1^2\right),
\end{aligned}\right\} \qquad (11.8)$$

where are introduced the designations

$$\begin{aligned}
\mathrm{tg}\,\beta &= \frac{1}{\sin\theta^{\circ}}\frac{d\theta^{\circ}}{d\Phi},\\
m &= \cos\delta\sin\theta^{\circ} + \sin\delta(\cos\Phi\cos\theta^{\circ} + \sin\Phi\,\mathrm{tg}\,\beta),\\
n &= \sin\delta\sin\Phi - (\sin\delta\cos\Phi\cos\theta^{\circ} + \cos\delta\sin\theta^{\circ})\,\mathrm{tg}\,\beta,\\
l &= \frac{1 - \frac{\gamma-1}{\gamma+1}V_1^2(1 - m^2\cos^2\beta)}{V_1 m};
\end{aligned}$$

δ is an angle between axis $\theta = 0$ and direction of undisturbed flow; by zero are designated the values at $\theta = \theta^0(\Phi)$.

The gas-dynamic functions for which subsequently will be required the values of derivatives on shock wave, let us designate through χ_i, a their derived let us represent in the form

$$\frac{d\chi_k}{d\Phi} = A_k + B_k \frac{d^2\theta^0}{d\Phi^2}, \qquad (11.8a)$$

where k = 1, 2, 3, 4 are related respectively to functions w^0, $\rho^0 w^0$, v^0, p^0. Values A_k, B_k are determined with the aid of relationship/ratios (10.8).

Equations (11.2)-(11.4) let us write in the uniform form

$$F_\theta + f_\Phi = \omega. \qquad (11.9)$$

for equation (11.2), for example, $F = (\rho v^2 + p))\sin\theta$, $f = \rho v w$, $\omega = (\rho w^2 + p)\cos\Phi - 3\rho u v \sin\theta$ so forth].

Integrating equations (11.9) for θ from the known duct of the streamlined body $\theta = \theta^\times(\Phi)$ to shock wave $\theta = \theta^0(\Phi)$, let us compose three integral relationship/ratios of the form

$$\frac{d}{d\Phi}\int_{\theta^\times}^{\theta^0} f\,d\theta + f^\times \frac{d\theta^\times}{d\Phi} - f^0 \frac{d\theta^0}{d\Phi} + F^0 - F^\times = \int_{\theta^\times}^{\theta^0} \omega\,d\theta. \qquad (11.10)$$

For the solution of problem in the first approximation,, all the integrands f and ω are approximated on θ linearly in terms of their values on shock wave and over surface of the streamlined body.

Page 207.

As a result from equations (10.10) we will obtain the system of ordinary differential equations for the values of functions on shock wave and the body surfaces of the form

$$\frac{df^{\circ}}{d\Phi}+\frac{df^{\times}}{d\Phi}=\Omega, \qquad (11.11)$$

where

$$\Omega=\omega^{\circ}+\omega^{\times}+ +\frac{2}{\theta^{\circ}-\theta^{\times}}\left[-F^{\circ}+F^{\times}+\frac{1}{2}(f^{\circ}-f^{\times})\left(\frac{d\theta^{\circ}}{d\Phi}+\frac{d\theta^{\times}}{d\Phi}\right)\right]. \qquad (11.12)$$

The concrete/specific/actual form of the function Ω is determined from equations (11.2)-(11.4). Let us substitute in three equations (11.11) the appropriate values of functions $f^{\circ}, f^{\times}$, let us compute derivatives on shock wave with the aid of conditions (11.8), (11.8a),

the component of speed $v^{\times}$ it is expressed by formula (11.7), and density and pressure - by formulas (11.6) and

$$\vartheta^{-\gamma} = p\rho^{-\gamma},$$

where ϑ - a function of entropy; as a result we will obtain the system of three ordinary differential equations for determining of θ^{o}, $u^{\times}, w^{\times},$ which can be represented in the form

$$\left.\begin{aligned}
\frac{d\theta^{\circ}}{d\Phi} &= \frac{b_1\sigma_1 + v^{\times}[\sigma_2(b_2 + \Psi b_1) + A_2 - \Omega_1]}{a_1\sigma_1 + v^{\times}[\sigma_2(a_2 + \Psi a_1) - B_2]},\\
\frac{du^{\times}}{d\Phi} &= \frac{1}{\rho^{\times}u^{\times}}\left[(a_2 + \Psi a_1)\frac{d\theta^{\circ}}{d\Phi} - (b_2 + \Psi b_1) - \frac{d\Psi}{d\Phi}\rho^{\times}w^{\times}v^{\times}\right],\\
\frac{dw^{\times}}{d\Phi} &= \frac{1}{\rho^{\times}\sigma_1}\left[-B_2\frac{d\theta^{\circ}}{d\Phi^2} + \rho^{\times}\sigma_2\left(u^{\times}\frac{du^{\times}}{d\Phi} + \frac{d\Psi}{d\Phi}w^{\times}v^{\times}\right) + \right.\\
&\qquad\left. + \Omega_1 - A_2\right].
\end{aligned}\right\} \quad (11.13)$$

where

$$\begin{aligned}
a_1 &= B_2(v^{\circ} - v^{\times}) + B_3\rho^{\circ}w^{\circ},\\
a_2 &= B_2(w^{\circ} - w^{\times}) + B_1\rho^{\circ}w^{\circ} + B_4,\\
b_1 &= -A_2(v^{\circ} - v^{\times}) - v^{\times}\Omega_1 + \Omega_3 - A_3\rho^{\circ}w^{\circ} - \frac{d\Psi}{d\Phi}\rho^{\times}w^{\times 2},\\
b_2 &= -A_2(w^{\circ} - w^{\times}) - w^{\times}\Omega_1 + \Omega_2 - A_1\rho^{\circ}w^{\circ} - A_4,\\
\sigma_1 &= 1 - \frac{\rho^{\times}}{\gamma p^{\times}}(w^{\times 2} + v^{\times 2}), \quad \sigma_2 = \frac{\rho^{\times}}{\gamma p^{\times}}w^{\times}.
\end{aligned}$$

Page 208.

If the streamlined body has a plane of symmetry, in which let us assume that $\Phi = 0$, π, then the necessary solution of the system of equations (11.13) satisfies the conditions

$$\frac{d\theta^{\circ}}{d\Phi} = u^{\times} = 0 \text{ with } \Phi = 0, \pi, \qquad (11.14)$$

and the integration of equations (11.13) must be conducted on segment $0 \leqslant \Phi \leqslant \pi$.

Four conditions (11.14) are sufficient for determining the solution of system (11.13) in an only manner. The actual determination of solution is realized by means of the reducing of problem with boundary conditions (11.14) to problem with initial data. When $\Phi = \pi$ they are assigned by values of θ°, $u^{\times}$ they integrate equations (11.13) to value $\Phi = 0$. If when $\Phi = 0$ $\frac{d\theta^{\circ}}{d\Phi}$ or $w^{\times}$ are not equal to zero, then values θ°, $u^{\times}$ when $\Phi = \pi$ change until are satisfied conditions (11.14). (Process of requisition θ°, $u^{\times}$ when $\Phi = \pi$ is realized automatically by a computer; see [107]).

In equations (11.13) enters value $\vartheta = p^{1/\varkappa}\rho^{-1}$ when $\theta = \theta^{\times}(\Phi)$, designated $\vartheta^{\times}$. Since during nonseparated flow the body surface is conical stream surface, here $\vartheta^{\times} = \text{const}$. For the simplest bodies of the type of round or elliptical cone $\vartheta^{\times}$ is taken one and the same value on an entire body surface. For example, for a round cone (see Fig. 48) $\vartheta^{\times} = \vartheta^{\circ}|_{\Phi=\pi}$ is known function $\theta^{\circ}|_{\Phi=\pi}$. For the bodies of more complex form $\vartheta^{\times}$ it is piecewise constant on the surface of body and these constants must be found in resolving equations (11.13).

Work [107] gives the results of the calculations of pressure coefficient on body surface for a round cone with M_1 = 3.53; $\varepsilon = 20°$, δ = 5°, and the cone, cross section of which is ellipse with the relation of semi-axes, equal to 1.788 with M_1 = 6, δ = to 0, and also is conducted their comparison with the available experimental data.

For a round cone the agreement of theoretical and experimental data satisfactory (it is better - on windward, worse - on the lee sides of cone), but for an elliptical cone (where M_1 = 6, but not 3.53) agreement is distinct.

Page 209.

In the third equation of system (11.13) in the denominator of right side is expression $\beta_1 = 1 - \frac{\rho}{\gamma p}(u'^2 + v'^2)$, which will become zero, when a conical-sonic line touches at certain point of body surface. The authors [107] consider that at high angles of attack or the large values M_1 this method it is not possible to use. This, apparently, not thus. Since the flow about the body will occur with the "closing" of flow about its windward face, problem will become completely to the analogous problem of the flow of supersonic flow about the cylinder of gas in which successfully is applied the method of integral relationship/ratios. [The part of conditions (11.14) is replaced by the conditions of the regularity of solution at singular points.]

The problem of the determination of the second approach/approximation is considerably more complex than the problem of first approximation. It should also be noted that the method of integral relationship/ratios in that form in which it is given in work [107], gives only averaged picture of flow.

11.3. Finite-difference method (method of establishment). The method of the calculation of conical flows, proposed to K. Babenko with colleagues in book [29], is based on the results, which relate to the more Common/general/total problem of determining the three-dimensional/space supersonic flows of nonviscous gas about smooth bodies. This problem is formulated in the following manner. To the smooth body AOB (Fig. 56) attacks the uniform supersonic flow of nonviscous gas, which has speed V_1; the pressure of gas p_1, density ρ_1 so forth.

About body is formed leading shock wave CDEF, after which the flow remains supersonic, with the exception perhaps certain region ODE near the nose of body. Let us break now the region where the flow is agitated, by certain surface p on region I and II (see Fig. 56) in such a way that in all its points of rate of flow they would be supersonic and surface Π would have three-dimensional/space type

(tangential plane to surface П at any point of it it lie/rests outside the Mach cone, constructed for this point according to the value of local Mach number).

Page 210.

Let the problem be solved for domain I by any method and are obtained values of V, p, ρ so forth on surface Π. Is required to find the parameters of flow, and also the form of leading shock wave, in vicinity of surface Π, i.e., in certain layer, limited by surface and by the surface Π_1, satisfying the same requirements, as Π. After the determination of solution in this layer, it is possible to pass to following so forth. Entering so, let us construct the solution in domain II up to the emergence of special feature/peculiarities in solution (for example, inversion into infinity, acceleration on certain surface). Virtually stated problem is solved as follows. The equations of the inviscid adiabatic steady flow of ideal gas are record/written in cylindrical coordinates (z, r, ϕ), where axis Oz is selected so that as the surfaces $\Pi, \Pi_1, \Pi_2, \ldots$ it would be possible to take planes z = const (that always it can be made for the bodies of the simple form); see Fig. 56. Then they transfer/convert to the new coordinate system, selected in such a way that the domain in which searches for solution, it would have the fixed/recorded boundaries. If the equation of body surface in cylindrical coordinates is $r = G(z, \phi)$, and the unknown previously leading shock

wave $r = F(z, \phi)$, then as the simplest variables such type it is possible to take x, ξ, ϑ, where

$$x = z,\ \xi = \frac{r - G(z, \varphi)}{F(z, \varphi) - G(z, \varphi)},\qquad \vartheta = \varphi. \qquad (11.15)$$

After the replacement of variables the domain in which searches for the solution, is determined by the inequalities

$$x > x_0;\ 0 \leqslant \vartheta \leqslant 2\pi;\ 0 \leqslant \xi \leqslant 1 \qquad (11.16)$$

($x = x_0$) is an equation of surface П), but into the equations of motion of gas, will enter the unknown previously function $F(z, \phi) = F(x, \vartheta)$.

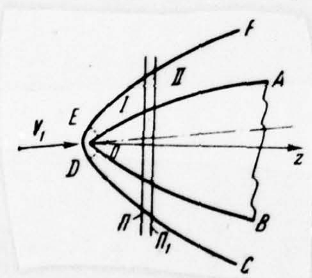


Fig. 56.

Further in the domain, given by inequalities (11.16), is introduced rectangular grid with spaces Δx, $\Delta\xi = 1/m$, $\Delta\vartheta = 2\pi/n$, where m, n are the integers, and the equation of motion of gas and boundary conditions they are record/written in finite-difference form. As a

result for determination $5mn + 6n$ unknowns, which determine the parameters of gas and the position of shock wave at $x = x_0 + (k + 1)\Delta x$ in the known values at $x = x_0 + k\Delta x$ (k, integer,) is obtained the complex system $5mn + 6n$ of nonlinear equations. The development of the procedure of the solution of this system composes the essence of problem. Sending away the reader to primary source [29], let us pause only at the most essential torque/moments of this procedure. Mentioned above system of nonlinear equations is solved with the aid of the iterative process in which the system decomposes to n of independent subsystems of $5m + 6$ equations each (the so-called "equations on ray/beam"). Last/latter equations are solved by dispersion method. The authors [29] conducted extensive study and distances in an explicit form of the conditions, assuring the correctness of the system of the used difference equations and the stability of screw die (under some natural simplifying assumptions).

As a result is obtained the well substantiated finite-difference method of the solution to the spatial problem of supersonic gas dynamics.

The use of this method for the calculation of conical flows is based on following considerations. It is well known from experiments and calculations for the simplest cases, that during the flow about the body, which differs from cone only in certain vicinity of

apex/vertex, the flow at large distances from it will be close to conical everywhere, with the exception of certain domain, adjacent to the surface of the body ("vortex/eddy" or "entropy" layer for bodies with the small blunting); the thickness of this domain rapidly it decreases with an increase in the distance from the apex/vertex of body; the form of body near its apex/vertex it affects (even then is small) only to the speed of the establishment of conical flow. These facts give grounds for the assumption that if we with $x = x_0$ assign on the whole the "arbitrary" values of the gas parameters, and with these initial conditions to find the solution of the problem of the flow about the cone (satisfying the boundary conditions on body and leading shock wave with $x \geqslant x_0$), then it with $x \to \infty$ will approach the solution of the problem of the flow about the infinite cone, although it is uneven.

Page 212.

It is hence clear as to use the numerical method of the calculation of three-dimensional/space flows on the calculation of conical flows. Let us assign at $x = x_0$ the more or less adequate/approaching initial values of the parameters of gas, and let us with $x \geqslant x_0$ solve the problem of the flow about the conical body, transfer/converting with the aid of the described numerical algorithm, from $x_k = x_0 + k\Delta x$ to $x_{k+1} = x_0 + (k+1)\Delta x$ (k - integer). Calculations are

conducted to this value k_0, for which the differences among the unknown values in the k_0- layer and the subsequent layers at those who were fix/recorded ξ and ϑ they will not become (on module/modulus) the less assigned magnitudes. [Recall that the parameters of conical flow depend only on the angular variables, which they are here

$$\xi = \frac{r - G(z, \varphi)}{F(z, \varphi) - G(z, \varphi)} \quad \text{and} \quad \vartheta = \varphi.$$

For the conical bodies $G(z, \phi) = zg(\phi)$, $F(z, \phi) = zf(\phi)$ and

$$\xi = \frac{\frac{r}{z} - g(\varphi)}{f(\varphi) - g(\varphi)}.]$$

By the final criterion of correctness of the obtained thus values of the unknown quantities is satisfaction them the differential equations of conical flows and boundary conditions with the error, allowed with approximation. A large quantity of calculations, made by the authors [29], confirms the convergence of the values of the unknown quantities to the same for conical flows.

Convergence indicated earlier nonuniform near body surface is exhibited in the fact that the establishment of the values of functions on body (with increase in x) occurs more slowly than in remaining zone of flow. In work [29] it is noted that "on body is establish/installed the correct value of entropy, whatever its distribution at first. This is connected with the fact that in the

differential equations of flow line, exiting/waste from wave, asymptotically approach a body surface, while in the difference equations of the disturbance/perturbation of entropy they are transferred from shock wave to body for the finite number of spaces on coordinate x".

Page 213.

This fact, however, also indicates the inadequacy of differential and difference equations near body surface, since in precise solutions of the differential equations of motion of gas for the blunted and sharpened cones of the value of entropy on their surfaces they will be different with any x.

As already mentioned in §8 (point/item 8.4). Here it is possible to still add that the position deteriorates for the large values of the Mach numbers M_1 of the incident flow, since "vorticity layer" on cones at hypersonic speeds is very fine, but the value of entropy across it strongly changes from the constant value, available on the surface of cone (the discussion concerns the round cone), to different values on the outer edge "vorticity layer". For this reason when $M_1 \gg 1$ would be more more interestingly obtain the algorithm of the solution of problem, which would give the variable values of entropy "on the surface" of cone, disregarding thereby the "vortex

layer", as it takes place in work [168]. For the moderate values of M_1 and angles of attack, the described finite-difference method, called another method of establishment, is at present, apparently, best. In book [29] by the method of establishment are designed the tables of the flow about the round cones at angles of attack for the values of numbers M_1, equal to 2, 3, 5, 7. Half-angles of cone are changed in the tables from 10° to 40° with space 5°. and angles of attack δ within limits of from 5° to 20° with a spacing of 5°. In appendix 1 of book [29] are placed the tables for Mach numbers, equal to 4 and 6, obtained by quadratic interpolation. Gas is assumed to be ideal, $\gamma = c_p/c_v =$ 1.4. For convenience in the use of these tables, are given also the fields of flow about cones with $\delta = 0$. Furthermore, in book [29] are given the results of calculations for the body, comprised of the head cone, connected with base cone of the smaller semiangle, streamlined both with air with $\gamma = 1.4$ and for flow taking into account equilibrium chemical reactions.

These calculations represent large value, since they make it possible to reveal/detect/expose the dependence of field of flow on the different parameters.

Page 214.

The detailed analysis of such dependences (for example, gas-dynamic functions of the value of coordinate ξ and others) is given in the

book. It should also be noted that the tables make it possible to calculate the parameters of gas at any point of flow and are convenient for use.

11.4. Method of straight lines. For the solution of the problem of the supersonic flow about the blunted body of gas G. Talenin with colleagues they developed the method which conditionally can be call/named the method of straight lines; see [172]. Essentially this method is similar to the method of integral relationship/ratios, but does not require the reductions of equations to divergent form. A. Bazzhin and I. Chelysheva in work [109] used the method of straight lines on the calculation of the flow about the cone with the smooth duct of cross section when flow occurs with flow choking about the windward face of cone. As examples are given the results of the flow-field analyses of round and elliptical cones at high angles of attack. It is presented the first briefly basic condition/positions of the method of straight lines, and then let us discuss the results of calculation and possibility of method.

The equations of motion of gas they are record/written in Cartesian coordinates $\xi=\frac{x}{z}$, $\eta=\frac{y}{z}$. As the unknown values are selected the components of velocity vector u, v, w, pressure p and temperature T. Then instead of ξ and η are introduced alternating/variable n and s, where s - the arc length, calculated off certain point along

the counterflowed body to the standard, passing through point $(\xi;\eta)$, n - the distance along the normal of this point, in reference to distance of bow shock N = N(s) (Fig. 57).

(Alternating/variable s, n usually are utilized in boundary-layer theory, only here n is calibrated so that the domain between the body surface and the unknown previously surface of shock wave would correspond to values of n, determined by inequalities $0 \leqslant n \leqslant 1$).

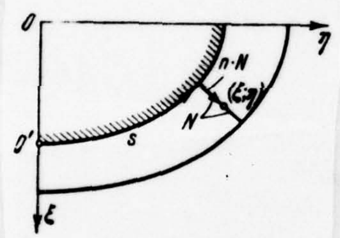


Fig. 57.

Page 215.

Further equations of motion are permitted relative to derivatives in terms of n of the unknown values, i.e., they are reduced to the form

$$\frac{\partial f_k}{\partial n} = F_k\left(s, n, f, \frac{\partial f}{\partial s}, \frac{dN}{ds}, N\right) \quad (k = 1, 2, \ldots, 5), (11.17)$$

where by f_k they are understood by p, u, v, w, T.

In the range of perturbed flow, determined by the inequalities

$$0 \leqslant s \leqslant s_*, \quad 0 \leqslant n \leqslant 1, \qquad (11.18)$$

are carried out several coordinate ray/beams

$$s = s_i = \text{const} \quad (i = 0,\ 1,\ 2, \ldots,\ l-1),$$

and as the unknown values are taken values f_k on ray/beams $s = s_i$, i.e. $(f_k)_{s=s_i} = f_{ki}(n)$, but functions f_k are approximated by polynomials on s, the node/units of interpolation polynomials are defined by values $s = s_i$. Derivatives $\frac{\partial f_k}{\partial s}$ are determined by differentiation with respect to s of these polynomials. As a result f_k and $\frac{\partial f_k}{\partial s}$ at any point of flow, are expressed through $f_{ki}(n)$. For determining functions $f_{ki}(n)$ system (11.17) is satisfied on ray/beams $s = s_i$ (i = 0.1, ..., l-1), in consequence of which for $f_{ki}(n)$ is obtained the system 5l of ordinary differential equations. The order of the solution of this system is following. Given the position of bow shock, i.e., function N = N(s); of usual relationship/ratios on shock wave are located initial conditions for a system (11.17); it is conducted the integration of system (11.17) from value of n = 1 to n = 0; with n = 0, is checked the condition of the flow about the body on each ray/beam $s = s_i$; in the case of its nonfulfillment it changes the form of bow shock and entire calculation is repeated, while the boundary

condition on the body will not be satisfied. Constant $s_{\circ}$ in (11.18) is selected in such a way as most to correctly consider a conical-transonic zone of flow.

Page 215.

As the illustration of the possibilities of method work [109] gives separate of the results of calculations according to the nine-beam diagram of the flow about the round and elliptical cones with the Mach number of undisturbed flow M_1 = 7 and 7.95.

The results of the calculations, obtained by the method of straight lines, are close to the results given by other methods (see [115, 29]) and to the results of experiment. Most interesting are the figures in which is shown the form of shock waves and conical-sonic lines in flows about round cone with half-angle $\varepsilon = 20°$ at the angles of attack δ = 30, 40, 50° and M_1 = 7, and of elliptical cone (with the relation of the semi-axes of cross section, equal to two). The form of conical-sonic lines was obtained the same type as during the flow about the cylinder.

Although the method of straight lines in work [109] is utilized for the solution of the problem of the flow about the bodies at high angles of attack, when is accomplished flow choking about the

windward face of the cone, not is evident reasons on which it was not possible to utilize this method for the solution of problem at the moderate angles of attack. For this purpose it is necessary only functions ξ_k to approximate not by polynomials s, but by the trigonometric polynomials s, which provide the periodicity of solution by s. This modification of method is desirable even because the method of the straight lines is convenient to account for vorticity layer near the surface of the streamlined body. Let us note that the method of straight lines is suitable for the case of the inadequate gas.

B. Flow about the conical bodies, arrange/located outside the Mach cone of undisturbed flow.

§12. Classification and the methods of the solution of the problems of the flow about the conical bodies.

12.1. Lead-in observations. In section B, are examined those cases of the flow of uniform supersonic flows about conical bodies of gas with the moderate values of the Mach numbers M_1, at which the streamlined bodies lie/rest completely outside the Mach cones of

undisturbed flow, constructed for apex/vertex bodies downstream (Fig. 58).

Page 217.

The problem of the flow about the body in variables ξ, η here is very similar to two-dimensional problem of the supersonic uniform flow about the airfoil/profile of gas, and about it it is possible to say literally the same as the plane problem. Specifically, one should distinguish the cases in which the bow shock is connected to the leading edge of body (Fig. 59a) and the cases in which the flow occurs with the detached shock wave (Fig. 59b).

In the case of a Fig. 59 flow of body by a conical-supersonic (if body surface convex), and it possible are calculated by perturbation method or (it is more accurately) method of characteristics.

In the perturbation method (small parameter method) of first-order disturbance/perturbation of the components of the rate of pressure, and so forth they satisfy within the Mach cone of undisturbed flow the two-dimensional equation of Laplace, but outside him - to one-dimensional wave equation.

The determination of higher approach/approximations of outside Mach cone (i.e. in the which interests us domain) is reduced to the solution of heterogeneous one-dimensional wave equations, and A. Avdonin [173] found here the third approach/approximation in the locked form (see also [174]).

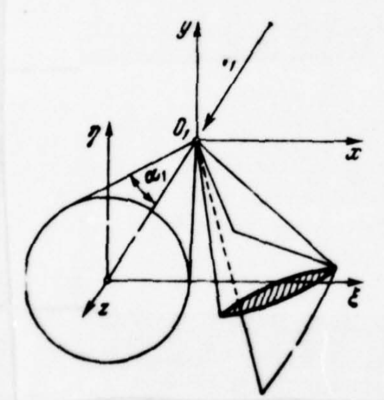


Fig. 58.

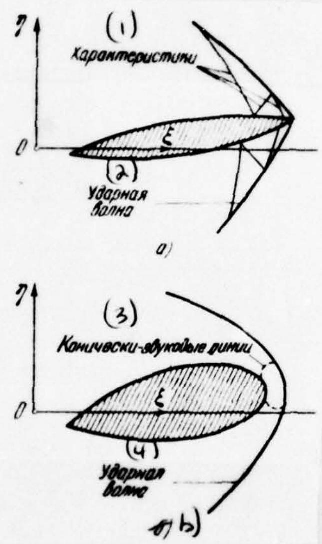


Fig. 59

Fig. 58.

Fig. 59.

Key: (1). Characteristics. (2). shock wave. (3). conical-sonic lines. (4). Shock wave.

Page 218.

Method of characteristics for conical flows was proposed by S. Vallander in work [175]. If body is arrange/located relative to flow so that above one of its sides the flow is expanded, then for this side of wing flow will be simple conical wave (see [98], [100]).

For the case b of figure 58, position is considerably more complex, since near the leading edge of body is formed a conical-subsonic zone, and for determining flow near edge it is necessary to solve the conical-transonic problem.

For a rounded edge this problem can be solved, for example, by the method of straight lines (see [109]). For the pointed leading edge, but streamlined at high angle of attack, so that shock wave cannot be that which was connected, apparently, better to utilize a method of integral relationship/ratios (see [176, 177]). After the determination of flow in the vicinity of leading edge for the calculation of the remaining conical-supersonic part of the flow, can be used the method of characteristics.

12.2. Method of characteristics. As initial let us take the

equations (1.17)-(1.20), which are most close to the equations of the flat/plane eddy of gas. The unknown values are here the components of velocity u, v, w along the axes of the Cartesian system of coordinates O_1xyz (see Fig. 2) and specific enthropy S.

As is known, system performances of differential equations they are called also the curves for which certain linear combination of initial equations contains the derivatives of the unknown functions only along these curves.

In the form of equations (1.18), (1.20) immediately it is possible to say that the flow lines, defined by equation $\frac{d\xi}{u-\xi w}=\frac{d\eta}{v-\eta w}$, are dual system performances of equations (1.17)-(1.20). Along these lines are fulfilled the obvious equalities

$$\xi\,du + \eta\,dv + dw = 0, \qquad (12.1)$$

$$dS = 0. \qquad (12.2)$$

Page 219.

The other two characteristics are defined by equation (1.43)

$$A(\eta')^2 - 2B\eta' + C = 0$$

(see point/item 1.8), where $\eta' = \frac{d\eta}{d\xi}$ — an angular coefficient of characteristic.

After elementary, but bulky and long calculations, it is

possible to write the relationship/ratio between the differentials of the unknown values along the characteristics, determined by equation (1.43), in the form

$$(\xi\,du + \eta\,dv + dw)[w(C - B\eta') + a^2\eta'(v\xi - u\eta)] + \\ + (C - B\eta')\,TdS + \\ + [v - \eta w - \eta'(u - \xi w)]\,(C\,dv + A\eta'\,du) = 0, \quad (12.3)$$

where A, B, C are given by formulas (1.30). Relationship/ratios (12.1)-(12.3), (1.43) are valid for eddy and the inadequate gas. If gas ideal, then T in equation (12.3) should replace by $\frac{a^2}{\gamma R}$ (see point/item 1.3).

If the flow of gas is irrotational, then dS = 0, ξ du + η dv + dw = 0, and relationship/ratio (12.3) is converted into Cdv + Aη'du = 0, which can be written in the form

$$\frac{dv}{du} = -\frac{A}{C}\eta'.$$

If one considers that $\eta'_+ \cdot \eta'_- = C/A$, then finally we will obtain

$$\left(\frac{dv}{du}\right)_+ = -\frac{1}{\eta'_-}; \quad \left(\frac{dv}{du}\right)_- = -\frac{1}{\eta'_+}, \quad (12.4)$$

where η'_+, η'_- they are located from equation (1.43). Furthermore,

$$dw \equiv -(\xi\,du + \eta\,dv). \quad (12.5)$$

For irrotational conical flow the calculations according to method of characteristics differ from analogous calculations for two-dimensional gas-dynamic problem only in the facts that after

determination u, v, for example in thirds to point, from assigned values in two others, w it is determined at the third point with the aid of formula (12.5). For the vortex/eddy conical flows in comparison with of flat/plane vortex/eddy problems of gas dynamics, is obtained certain complication, since dw enters in relationship/ratios on characteristics along with du and dv.

Page 220.

Furthermore, conditions on shock wave and the surface of the streamlined body are here more complex.

For this reason let us examine the basic operations of method of characteristics (first approximation) for conical flows. Let are known to u, v, w, S at two points: $(\xi_1;\ \eta_1)$, $(\xi_2;\ \eta_2)$; it is required to find u, v, w, S in close point $(\xi_3;\ \eta_3)$. Indices 1, 2, 3 let us designate function values at these points. From equation (1.43) we find $(\eta'_+)_1$, $(\eta'_-)_2$ and, replacing the arcs of characteristics by segments of tangent, let us find point $(\xi_3;\ \eta_3)$ (Fig. 60a).

Further through formula $\eta' = \frac{v - \eta w}{u - \xi w}$ we find angular coefficients η'_1 and η'_2 the flow lines, passing through points 1 and 2 (Fig. 60a), also, with the aid of linear interpolation on the cut straight

line, connecting point 1 and 2, we find point 4, for which the tangent to flow line is passed through point 3. At point 4 with the aid of linear interpolation we define u, v, w, S. Then $S_3 \approx S_4$. Replacing, now differentials by finite differences, from relationship/ratios (12.1), (12.3) we obtain the system of three linear equations for u_3, v_3, w_3:

$$\xi_4(u_3 - u_4) + \eta_4(v_3 - v_4) + w_3 - w_4 = 0, \quad (12.6)$$

$$[\xi_1(u_3 - u_1) + \eta_1(v_3 - v_1) + w_3 - w_1]\{w_1[C_1 - B_1(\eta'_+)_1] + a_1^2(\eta'_+)_1(v_1\xi_1 - u_1\eta_1)\} + [C_1 - B_1(\eta'_+)_1]T_1(S_3 - S_1) + [v_1 - \eta_1 w_1 - (\eta'_+)_1(u_1 - \xi_1 w_1)][C_1(v_3 - v_1) + A_1(\eta'_+)_1(u_3 - u_1)] = 0, \quad (12.7)$$

$$[\xi_2(u_3 - u_2) + \eta_2(v_3 - v_2) + w_3 - w_2]\{w_2[C_2 - B_2(\eta'_-)_2] + a_2^2(\eta'_-)_2(v_2\xi_2 - u_2\eta_2)\} + [C_2 - B_2(\eta'_-)_2]T_2 \times (S_3 - S_2) + [v_2 - \eta_2 w_2 - (\eta'_-)_2(u_2 - \xi_2 w_2)][C_2(v_3 - v_2) + A_2(\eta'_-)_2(u_3 - u_2)] = 0. \quad (12.8)$$

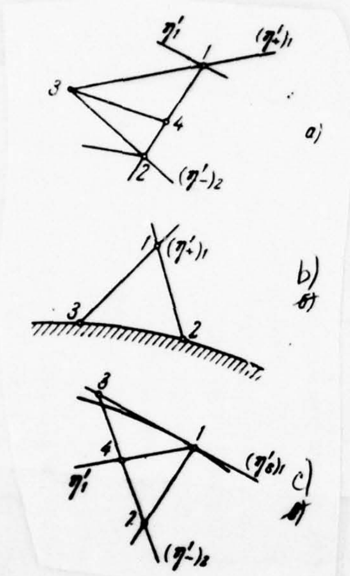


Fig. 60.

Page 221.

Method of characteristics is well known, so that here and throughout we are not stopped on the questions of the refinement of solution, being interested only in fundamental side of matter.

Let the point of 3, figure 60b is locate on the surface of the streamlined body. Since points 2 and 3 lie/rest on one flow line,

$$S_3 = S_2, \quad \xi_2(u_3 - u_2) + \eta_2(v_3 - v_2) + w_3 - w_2 = 0.$$

Furthermore, $v_3 - \eta_3 w_3 = (\eta_b')_3(u_3 - \xi_3 w_3)$, where $(\eta_b')_3$ the known angular coefficient of the surface of the streamlined body at point 3. One Additional Condition is obtained from relationship/ratio on characteristic 1-3, this is an equation (12.7).

Let the point of 3 figure 60c, lie/rest on the surface of leading shock wave. Let us examine the case of ideal gas, then conditions on shock wave are assigned by the formulas (1.47)-(1.49), which for our case are conveniently written in this form:

$$\left.\begin{aligned}
&u_3 = u_\infty - (\eta_s')_3 P_3, \quad v_3 = v_\infty + P_3,\\
&w_3 = w_\infty + [\xi(\eta_s') - \eta]_3 P_3, \quad \left(\eta_s' = \frac{d\eta_s}{d\xi}\right),\\
&P = \frac{2}{\gamma+1}\left[\frac{a_\infty^2}{v_\infty - \eta_s' u_\infty + w_\infty(\xi\eta_s' - \eta_s)} - \frac{v_\infty - \eta_s' u_\infty + w_\infty(\xi\eta_s' - \eta_s)}{1 + \eta_s'^2 + (\xi\eta_s' - \eta_s)^2}\right],\\
&S_3 - S_\infty = c_v\left\{\ln\left[\frac{2\gamma q_n^2 - \gamma + 1}{\gamma + 1}\right] - \gamma \ln\left[\frac{2 + (\gamma - 1) q_n^2}{(\gamma + 1) q_n^2}\right]\right\}\Big|_3,\\
&q_n^2 a_\infty^2 = [v_\infty - u_\infty\eta_s' + w_\infty(\xi\eta_s' - \eta_s)^2][1 + \eta_s'^2 + (\xi\eta_s' - \eta_s)^2]^{-1}.
\end{aligned}\right\} \tag{12.9}$$

In formulas (12.9) the parameters of undisturbed flow are designated in index "∞", and their values for point 3 have an index "3".

Page 222.

The position of point 3 on shock wave will be determined by the intersection of tangent to characteristic at the point of 2, figure 60c, and tangent to shock wave at point 1. For determination S_3, u_3, v_3, w_3 $(\eta_i')_3$ are five relationship/ratios (12.8), (12.9). The solution of this system five of nonlinear equations is conducted iteratively. After conducting through the point of 1 figure 60c, tangent to flow line, we find point 4 on the cut of characteristic 2-3. Through values $S_4 \approx S_1$ and S_2 by linear extrapolation is found zero approximation for $S_3^{(0)}$. Substituting $S_3^{(0)}$ and u_3, v_3, w_3 by formulas (12.9) into equation (12.8), and we again find more precise value $(\eta_i')_3$, so forth, after which compute u_3, v_3, w_3 by formulas (12.9).

For irrotational approach/approximation the calculation strongly is simplified, since equation (12.8) here assumes the form

$$(v_2 - v_3)(\eta_i')_3 + (u_3 - u_2) = 0. \qquad (12.8a)$$

12.3. Fine/thin wing with sharp leading edge. As an example let us examine order of calculation of flow near the fine/thin wing with sharp leading edge, depicted on Fig. 61. Let the wing be placed into flow in such a way that about its upper surface is the flow of evacuation/rarefaction, and about lower - a flow with attached shock wave.

Page 223.

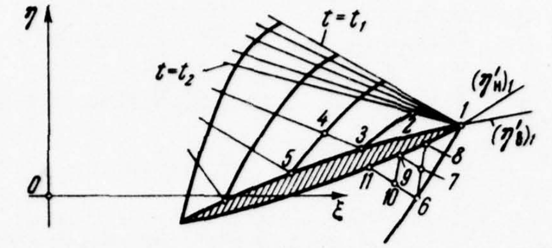


Fig. 61.

The equation of the upper part of the surface let us take in the form $\eta = \eta_в(\xi)$, lower $\eta = \eta_н(\xi)$. Since leading wing edge supersonic and the bow shock is connected to it, flows about the upper and lower parts of the surface do not interact and can be designed separately. Let us examine first flow about the upper part of the surface. On leading edge (point 1 in Fig. 61) appears centered simple conical wave (see Section 2.17), in which the conical potential F is represented in the form

$$\left.\begin{aligned} &F = (\xi - \xi_1)\Phi(t) + F_1, \\ &t = \frac{\eta - \eta_1}{\xi - \xi_1}, \qquad F_1 = \text{const}, \end{aligned}\right\} \quad (12.10)$$

a the components of velocity u, v, w are located through formulas (2.78)

$$u = \Phi - t\Phi', \quad v = \Phi', \quad w = F_1 - \xi_1 u - \eta_1 v, \Phi' = \frac{d\Phi}{dt}.$$

If we orient axis O_1z in direction of undisturbed flow, then $F_1 = V_1$. Substituting u, v, w by formulas (2.78) into the equation (2.79), we will obtain the relationship/ratio

$$a^2[1 + t^2 + (\xi_1 - t\eta_1)^2] - [w(\xi_1 t - \eta_1) - ut + v]^2 = 0, \quad (12.11)$$

which is differential first-order equation for determining $\Phi = \Phi(t)$ and

easily can be solved relatively Φ'.

Let us designate by t_1 value of t for the characteristic, which separate/liberates undisturbed flow from centered wave, then with $t = t_1$

$$w = V_1,\ u = 0,\ v = 0,\ a = a_1. \quad (12.12)$$

From formulas (12.12), (2.78) it follows that with $t = t_1$ $\Phi = \Phi' = 0$, and value t_1 will be located from the equation

$$a_1^2[1 + t_1^2 + (\xi_1 - t_1\eta_1)^2] - [V_1(\xi_1 t_1 - \eta_1)]^2 = 0, \quad (12.13)$$

which is obtained by means of substitution (12.12) in (12.11).

page 224.

Further is conducted the integration of equation (12.11) with the initial condition $t = t_1$, $\Phi = 0$ to such a $t = t_2$ by which is satisfied the equality

$$(\eta_n')_1 = \frac{v - \eta_1 w}{u - \xi_1 w},$$

meaning that the velocity vector with $t = t_2$ is parallel to tangential plane to the body surface, constructed for points leading edge. It is possible, of course, to use by those by the fact that for an ideal gas oblique flow of Prandtl-Mayer is described by known formulas. Characteristic $t = t_2$, it adjoins further the region of the

simple conical wave whose calculation can be produced by the short-cut method of characteristics.

On characteristic $t = t_2$, is taken point 2 (see Fig. 61) near body surface; for it it is located $(\eta'_-)_2$, is carried out the cut of characteristic 2-3 to point 3, which lies on body surface, and here are determined flow parameters (see Section 12.2). Then is calculated $(\eta'_+)_3$ and through point 3 is carried out rectilinear characteristic 4-3, on which flow parameters the same as at point 3.

Further on characteristic 3-4, is taken point 4, is located value $(\eta')_4$ and they are determined the position (and flow parameters) of close point 5, which lies on the body surface and, etc.

In the problem of the flow about the lower part of the surface, let us examine only order of calculation near the leading edge, since the subsequent operations are examined in p. 12.2. Just as in two-dimensional problem of gas dynamics, the small section of the airfoil/profile of the body of 1-8 figure 61 is replaced by tangent, carried out at point 1. Then the section of head impact oxen 1-6 is rectilinear, and flow in domain 1-6-7-8, where 6-7-8 is a rectilinear characteristic, it is uniform. The angular coefficient of shock wave $(\eta'_c)_1$ in point 1 will be located from the condition that after shock wave the velocity vector is parallel to tangential plane to the body

surface, constructed for point 1. This condition takes the form

$$v_6 - \eta_1 w_6 = (\eta_H')_1 (u_6 - \xi_1 w_6).$$

Page 225.

Substituting here values u_6, v_6, w_6 from formulas (1.47), where it is necessary index "2" to replace by index "6" and to place $u_1 = v_1 = 0$, $w_1 = V_1$, we will obtain the algebraic equation of cube for $(\eta_H')_1$, one of roots of which it corresponds to the unknown weak shock wave. Further order of calculation does not differ from such for two-dimensional problem gas dynamics.

It is necessary, however, to keep in mind that, unlike two-dimensional problem where the flow about the airfoil/profile, depicted on Fig. as 61, will be supersonic at the points of airfoil/profile, in conical flows can be met the cases, when in certain point of the airfoil/profile of the section of body by plane z = 1 appears a conical-sonic point and further calculation according to method of characteristics it becomes impossible. This situation appears, for example, if jump 1-6 (see Fig. 61) sufficiently intense, and airfoil/profile - fine/thin and little bent).

These cases are examined in section C.

C. The flow about the conical bodies, which partially protrude from the Mach cones of undisturbed flow.

§13. Classification and observations about the diagrams of the flow about the conical bodies.

13.1. Classification. In section C will be examined the cases when the streamlined bodies are partially arrange/located outside the Mach cones of undisturbed flow, constructed for apex/vertex bodies downstream. Here are not examined the cases, when bodies fill entire interior of Mach cones and partially they lie/rest outside them, since conical flows under these conditions can arise only at hypersonic speeds ($M_1 \gg 1$). Thus, are examined conical bodies of the type "wing", "wing with fuselage", "fuselage with tail assembly", that have supersonic leading edges, in the uniform flows of gas with moderate values of Mach numbers $M_1 > 1$. The leading edges of bodies we assume in the majority of cases acute/sharp, since the blunting of edge is led to the local effect on flow (near edge) which is examined into §12.

Page 226.

Angles of attack δ will be assumed to be those who were moderated, with those so that the bow shocks would be connected to the leading sharp edges of bodies.

Will be also examined separately the flow about the triangular plate at high angles of attack.

13.2. Observations about diagrams of flow about conical bodies. The chief characteristic of the flow about the bodies in the cases, component section C, is a conical-transonic pattern of the appearing flows. (About the part of the surface of the streamlined body the flow of a conical-supersonic, and about another part - conical-subsonic, moreover to flow parameters in a conical-subsonic domain affect the flow parameters of the part of a conical-supersonic zone of flow).

These facts were already noted by A. Ferri (see [178]). Are well known the difficulties, connected with setting and solution of the problems of the transonic flow about the airfoil/profiles. For conical flows these difficulties are aggravated, since although the unknown values depend here on two variables (ξ, η or θ, Φ), flows bear all the same three-dimensional nature. The construction of the

diagrams of flow and the setting of the corresponding boundary-value problems for the equations of conical flows for the cases of section C is by no means trivial matter. At moderate values of M_1 and δ, conical-transonic flows occupy the significant part of the domain where the flow is agitated by the streamlined body; when $M_1 \gg 1$ these domains also can play noticeable role, besides flows in the simplest cases, which, for example, is the case of triangular plate.

Each concrete/specific/actual body, for example, delta wing, the crossed wing, requires special approach, but there are common/general/total features, which are inherent in the conical flows of section C. For this reason into §14, are examined in detail the diagram of flow, the setting of the appearing boundary-value problems and other questions for the triangular plate which served as the object of a large quantity of investigations. In the subsequent paragraphs are examined the diagrams of the flow about other conical bodies (taking into account the results of §14) and the methods of the solution of the corresponding problems of flow.

Page 227.

§14. Diagrams of the flow about the triangular plate (deltoid flat/plane wing).

14.1. Lead-in observations. Let us examine a triangular plate with sweep angle Λ, placed without slip into the supersonic flow of gas, determined by speed V_1, by Mach number M_1 (by speed of sound a_1).

Axis O_1z of the Cartesian system of coordinates O_1xyz it is directed along the line of the symmetry of wing, the angle of attack of wing let us designate by δ (Fig. 62). It is assumed that the wing edges supersonic, i.e., the component V_1 in the plane, perpendicular to leading wing edge, are more a_1.

The Mach number of undisturbed flow and angle of attack for flow in the plane, perpendicular to leading edge, are expressed by the formulas

$$\left.\begin{aligned} M_{1N} &= M_1 \cos\Lambda \sqrt{1+\sin^2\delta\,\mathrm{tg}^2\Lambda}, \\ \delta_N &= \mathrm{arctg}\left(\frac{\mathrm{tg}\,\delta}{\cos\Lambda}\right). \end{aligned}\right\} \qquad (14.1)$$

Consequently, is examined the totality of such values of M_1, Λ, δ, which satisfy the inequality

$$M_1 \cos\Lambda \sqrt{1+\sin^2\delta\,\mathrm{tg}^2\Lambda} > 1. \qquad (14.2)$$

Angles of attack δ (with those who were fix/recorded M_1, Λ) let us

divide into two classes: for the first class is fulfilled inequality $0<\delta<\delta_d$, for the second $\delta>\delta_d$, where δ_d is the angle of attack at which occurs the departure/withdrawal of bow shock from the leading edge of plate. Affiliation/accessory δ with a defined class is establish/installed as follows. For plane flow is well known the dependence of the maximum angle of rotation of flow in oblique shock wave, $\delta_{\text{кр}}$, then the Mach number of undisturbed flow.

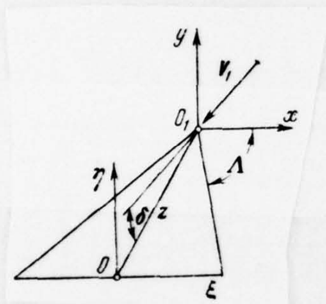


Fig. 62.

Page 228.

Substituting the value of M_1, Λ, δ into formulas (14.1), we determine M_{1N}, δ_N; if $\delta_N<\delta_{\text{кр}}$, where $\delta_{\text{кр}}$ it is found by the formulas of oblique shock wave for M_{1N}, then δ belongs to the first class otherwise - to the second class.

If $\delta < \delta_d$, then the shock wave, which appears during the flow about pressure side of wing, will be connected to leading edge; conical flows about upper and pressure sides of wing do not interact and they must be calculated separately. (Suction side of wing we call here that part of the surface, about which the flow is expanded).

Subsequently there will first be examined the case $\delta < \delta_d$, and then - case $\delta > \delta_d$.

Case $\delta < \delta_d$ is set forth in essence from B. Bulakh's works [9, 104, 106, 179-183], case $\delta > \delta_d$ - on the basis of A. Bazzhin's works [176, 177] and G. Chernogo [184]; references to other authors's results are given in the text of the corresponding point/items.

14.2. Flow about upper surface of delta flat/plane wing. Let us examine the flow pattern of the upper surface of triangular plate in the case $\delta < \delta_d$ on plane $\xi\eta$; on the strength of the symmetry of flow this picture we depict for $\xi > 0$. On plane $\xi\eta$ the wing is depicted as the cut of 0-3 axes 0ξ (Fig. 63).

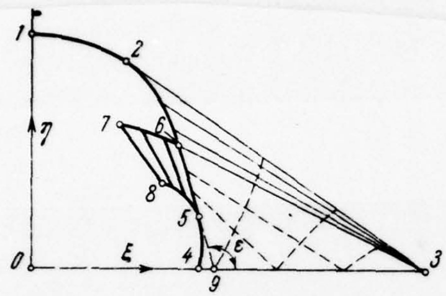


Fig. 63.

Page 229.

The boundary of the region, in which the flow is agitated by wing, is determined by the construction of the envelope of the Mach cones of undisturbed flow with apex/vertexes at the points of leading edge. It consists of the arc of Mach cone 1-2 (with apex/vertex at point O_1; Fig. 62) and the plane of Mach 2-3, passing through the leading edge. Here and in further by Mach cone, the plane of Mach and so forth are understood the images of these surfaces on plane $\xi\eta$. i.e., traces from the intersection of these surfaces with plane z = 1.

During the flow about the sharp leading edge of plate, is formed

oblique flow of Prandtl - Mayer (centered simple conical wave), which continues until velocity vector becomes parallel to the surface of plate. This flow has a beam of rectilinear characteristics of equation (1.29), passing through the point of 3 figure 63 (see point/items 2.17, 12.3). Characteristic 3-6 is the boundary of the flow of Prandtl - Mayer after whom follows the uniform flow, which adjoins the wing surface. above the wing center section, there is conical flow of the general view, in which flow parameters are complex functions ξ and η. This flow is separate/liberated from undisturbed flow by the arc of Mach cone 1-2, which is conical-sonic line or, otherwise, the parabolic line of equation (1.29) and its characteristic. Further the flow of general view is separate/liberated from the flow of Prandtl - Mayer by the curved characteristics of this flow, which emerges from point 2, and by the cut of rectilinear characteristic 6-5 for a uniform flow about the wing surface. Point 5 is parabolic. It appears at small angles of attack 6. If angle of attack is sufficiently great, then angle ε in figure 63 becomes acute/sharp and rectilinear characteristic 6-9 must be continued to the wing surface. But if parabolic point is, then further boundary with uniform flow is the arc of its Mach cone 4-5.

For the inclusion into the boundary of arc 4-5 are basis/bases both physical and mathematical nature. If arc 4-5 is is not included in boundary, then when 6 $\rightarrow$ 0 conical field of general view would

fill entire region 0-1-2-3-0.

Page 230.

Furthermore, for determining the conical potential F in the vicinity of point 9 we would obtain the problem in which on characteristic 5-9 was assigned the function and its normal derivative, but on segment 4-9, - normal derivative $F_\eta=0$. This problem has the unique solution, which corresponds to uniform flow, since the characteristics of the second family, constructed for the points of cut 5-9 near point 9, intersect cut 4-9. (Characteristics of uniform flow in region 3-6-5-4-3 are depicted on Fig. as 63 broken lines). Thus, if the flow about the upper surface of triangular plate occurs without shock waves, then for determining the conical potential F in region 0-1-2-6-5-4-0 is obtained the following boundary-value problem: required: to find the solution to equation (1.29), $L[F]=AF_{\xi\xi}+2BF_{\xi\eta}+CF_{\eta\eta}=0$, in the class of the functions, which possess piece-wise continuous second derivatives in terms of ξ and η; in terms of the boundary conditions: on cuts 0-1, 0-4 respectively $F_\xi=0$, $F_\eta=0$; on arcs 1-2, 2-6, 6-5, 5-4 they are assigned F, F_ξ, F_η, satisfying conditions strips $(dF=F_\xi d\xi+F_\eta d\eta)$ and to the conditions, which are satisfied on the characteristics of equation L [F] = 0 (see point/item 12.2). Previously it is known that equation L [F] = 0 changes its type from elliptical in the part of the region, which

contains point O, by hyperbolic in the vicinity of boundary 2-6. Furthermore, $AC - B^2 = 0$ on arcs 1-2, 4-5. At present the uniqueness theorems and existence for similar problems do not exist.

Will be further given considerations in favor of the fact that the formulated above problem is redetermine/redefined and, therefore, the unstressed flow about suction side of wing is impossible already with small δ. These considerations, although they are not completely strict from the viewpoint of mathematics, sufficiently, in our opinion, are convincing.

Happy fact is that into boundary of the region 0-1-2-6-5-4-0 in Fig. 63 enters the cut of rectilinear characteristic 6-5. According to theorem 2 of p. 2.17 to characteristic 5-6, must adjoin simple conical wave (region 5-6-7-8 in Fig. 63). This simple wave must somehow be clamped with the remaining part of the flow.

Page 231.

Here there are these possibilities: all the curved characteristics of simple wave, 6-7, 5-8 and others, converge into one point; characteristic 7-8, it adjoins the range of the uniform flow, detached from common type flow by Mach cone. Let us examine the first possibility. The point into which converge the curved

characteristics, must be simultaneously parabolic point and the point of the envelope of the rectilinear characteristics of simple wave, since otherwise curved characteristics cannot emerge one point. All attempts to construct so complex singular a point were finished unsuccessfully and, most probable, that this point does not exist.

So/such unlikely an opportunity arises when characteristic 7-8 adjoins uniform flow. hence it is possible to draw the conclusion that the simple wave cannot close with the flow of general view and, therefore, continuous flow in region 0-1-2-6-5-4-0 does not exist. unlikely existence and the generalized (in any sense) solution previously formulated boundary-value problem, since in the point/item 2.17 was demonstrated theorem 1, according to consequence of which, during motion along the characteristics 6-7, 5-8 and other curved characteristics of simple wave inside region 0-1-2-6-5-4-0 encountered parabolic points cannot form continuous parabolic line.

(There is another a series of the considerations, which indicate the impossibility of continuous flow above suction side of wing, see, for example [12, 183]). To the existence of swallowed shock with $\delta \to 0$, indicate also asymptotic theories [10], [182]. It is presented briefly the results [182]. When $\delta \to 0$ arc 2-6-5 in Fig. 63 disappears and all the flow is determined by arcs 1-2, 4-5.

Within region 0-1-2-6-5-4-0 parameters of flow can be expanded in series according to degrees δ. The first term of this expansion is given by usual linear theory. Expansion in terms of δ near the Mach cones and weak shock waves is examined in point/item 5.2. Let us allow now as the possible boundary of the flow of general view along with Mach cones 1-2, 5-4 weak shock waves, located near these curves.

Page 232.

Let us take the first terms of expansions in terms of δ, that occur within region 0-1-2-6-5-4-0, and let us attempt to connect them with the first terms of the expansions in terms of δ, which are the parameters of flow near Mach cones 1-2, 4-5 or near the weak shock waves, which pass about these cones. Thus establish/installed that various expansions can be connected only in such a case, when the boundary of the conical flow of general view is the Mach cone 1-2 and the weak shock wave, which passes near Mach cone 4-5. (See analogous reasonings in point/item 5.3). With large δ the impossibility of shock-free flow follows from the fact that characteristic 2-6 in Fig. 63 is bent so/such strongly, that cut 6-9 the secant of the symmetry of flow 0-1. This will occur for example, at $M_1 = 6$, $\delta = 20°$, $\Lambda = 60°$ (see [185]). From preceding/previous it follows that during the flow about the upper part of the surface of plate is formed the swallowed shock, but the question of its relatively position and form requires separate examination.

Here, of specific help can be the analogy with the problem of the flow about the symmetrical airfoil/profile at zero angle of attack by the subsonic flow of gas with the formation/education of local supersonic zones. It is known that continuous flow of such type about the airfoil/profile of "arbitrary" form does not exist in local supersonic zones, as a rule, appear shock waves. It is also known that for the assigned Mach number of the incident flow it is possible to fit the airfoil/profiles flow about which is continuous. If we compare the problems of determining the velocity potential ϕ in flow of approximately one half of airfoil/profile (after supplying boundary condition $\frac{\partial\varphi}{\partial n}=0$ on the axis of the symmetry of flow) and the problem of determination F in region 0-1-2-6-5-4-0, then the role of local supersonic zone here plays the part of the region, which adjoins arc 2-6-5. In both problems boundary conditions are assigned on the "locked" ducts, and, if it is additional to boundary conditions to assign one additional function, connected with solution, then solution in the vicinity of boundary completely will be determined.

Page 233.

Since boundary of 1-2 is determined by undisturbed flow, and boundary

2-6-5-4 Fig. 63 is determined by flow in region 2-3-4-5-6-2, it plays the problem of determination F in region 0-1-2-6-5-4-0 role of the airfoil/profile of the assigned form in two-dimensional problem. Relying on this analogy, it is possible to expect that are possible two versions of the location of swallowed shock in the problem of delta wing.

In the first version the shock wave begins at point 2 and continues to the wing surface at the point of 10 figures 64a.

In this case the problem of defining F in region 0-1-2-10-0 corresponds to the problem of determination ϕ for the airfoil/profile whose surface is previously unknown, and it is establish/installed so as to assure continuous flow with local supersonic zone. In the second version the shock wave 11-12 (Fig. 64b) is partially arrange/located in region 0-1-2-6-5-4-0, moreover the passage of jump inside this region is completed at certain point on characteristic 6-5. This case corresponds to the flow about the airfoil/profile with shock wave in local supersonic zone.

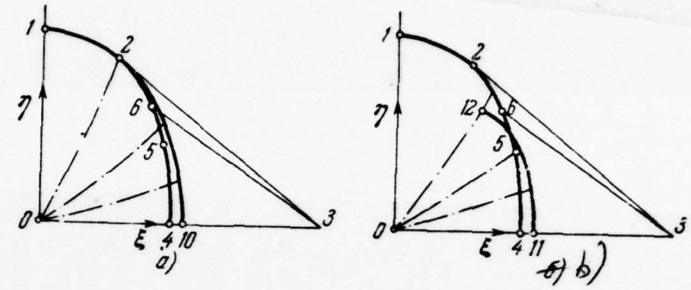


Fig. 64.

Page 234.

The diagrams of flow, depicted on Fig. 64, were proposed by B. Bulakh in works [9, 179, 181]. Is curious the history of question. For the first time the problem of delta wing examined S. Maslen [186], but he did not understand that the unstressed flow about suction side of wing is impossible. Then the author in [9] proposed diagram a of Fig. 64. In this diagram additionally was introduced "possible shock wave" about Mach cone 1-2 (see also [179]). This jump was included in diagram because at that time were known only axisymmetric flows, adjoining the Mach cone 1-2, which could not close with other parts of flow. Furthermore, if this jump is absent, then during the solution of problem it automatically degenerates into

the arc of Mach cone 1-2. It was later explained that there is no this jump and it was replaced by Mach cone 1-2 (see [106] and [182]). L. Fauell [187] instead of characteristic 2-6-5 erroneously accepted as the boundary of conical flow of general view the parabolic stream line of Prandtl-Meyer and examined continuous flow. Shock wave in it appeared, when angle of attack became more than certain critical angle. Diagram b in Fig. 64 was proposed by the author in work [181]. This same diagram was later proposed in [188]. Here it is not mentioned the small errors and the changes, which concern diagram a Fig. 64. The existence of swallowed shock during about the upper surface of triangular plate was for the first time confirmed L. Fauell's experiments [187], who by the measurement of pressure field found that the jump appears already at low angles of attack, which completely corresponds to the diagrams, depicted on Fig. 64. Subsequently in the experiments of a number of the authors [188, 189, 113], this shock wave was also found by pressure measurement in field of flow. In addition, D. Pierce and D Treadgold [190] with the aid of the optical shadow method, developed specially for the visualization of conical flows, knew how to photograph jump. Specifically, it turned out that about the wing surface the jump slightly is split, having lambda-shaped form.

If relative to the existence swallowed shock of there are no doubts, then relative to its form and the positions of information

still little; therefore which diagram one should give preference, a or b Fig. 64, the determined judgments cannot be made.

Page 235.

The existing experimental data ([187], [188]) they show that at high angles of attack, apparently, is realized diagram b, Fig. 64. To Fig. 65, undertaken work [188], shows form and and the shock position, found by the measurements of pressure field for a wing when $\Lambda = 45°$; δ = 14.2°; M_1 = 2.96.

By marks ⊡, △, ⊙, ▽ in Fig. 65 is noted the shock position at different distances from the apex/vertex of wing; by marks x - the position of characteristic 3-6; by marks + - the position of leading shock wave. The fact is that when δ = 14.2°, $M_{1N} = 2{,}15$, δ_N = 19°40', and the apex angle of the plane-wedge profile of model in the section, perpendicular to leading edge, is equal to 8°, that in sum is somewhat more than $\delta_{кр} \approx 25-26°$ for $M_{1N} = 2{,}15$. Because of this the shock wave, available about pressure side of wing, will move away from leading edge, which, as can be seen from Fig. 65, does not disrupt the common picture of flow. Let us note also that, according to [188], the force of swallowed shock first grow/rises with removal/distance from wing, and then falls. During the flow about the sharp leading wing edge, appears the small separating zone, which Fig. 65 shows.

AD-A066 205 FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO F/G 20/4
NONLINEAR CONICAL FLOWS OF GAS,(U)
MAR 78 B M BULAKH
UNCLASSIFIED FTD-ID(RS)T-0266-78 NL

5 OF 7
AD
A066205

OF 7

6205

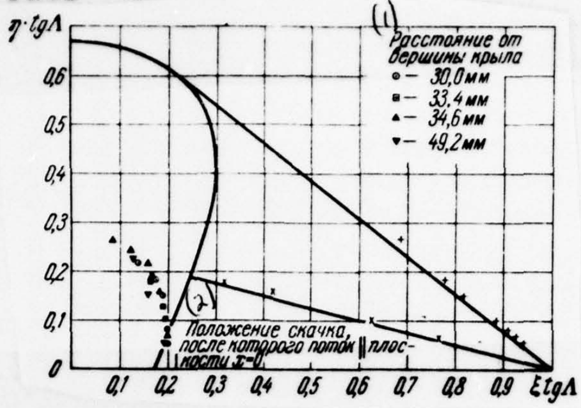


Fig. 65.

Key: (1). Distance from the apex/vertex of wing. (2). Position of the jump after which the flow of plane x = 0.

Page 236.

Let us make in conclusion several observations of the relatively boundary-value problems, which correspond to the diagrams of the flow about Fig. 64. For circuit b, the transonic character of problem is evident; for circuit a at small values Λ this is also not difficult to show. For large values Λ, possibly, and will be met the cases, where the flow in region 0-1-2-10-0 conical-subsonic. At point 2 in Fig. 64a, b are connected the flows of different nature, therefore, these points must be special. The possible construction of solution in the vicinity of these points is examined in point/item 3.4. After shock waves 2-10 (Fig. 64a) and 12-11 (Fig. 64b) flows vortex/eddy; they are connected with irrotational flows along flow lines with respect to 0-2 and 0-12. In the vortex flows the flow lines converge into points 0, where there are Ferry's special feature/peculiarities. The discontinuity/interruption of accelerations will propagate inside the conical fields of general view along the characteristics, which

emerge from points 6 in Fig. 64a, b.

14.3. Flow about lower surface of delta flat/plane wing. Let us examine now the flow pattern of the lower surface of a delta plate in cases when $\delta < \delta_d$.

Unlike the circuit of the flow about suction side of wing, the circuit of the flow about the lower surface of plate on the whole is simple. On leading wing edge, is formed the step shock of packing/seal 3-14 (Fig. 66) after which follows the uniform flow, which adjoins the wing surface. The range of common type conical flow is separate/liberated from uniform flow by its Mach cone 14-15. Beginning from point 14, shock wave 14-16 is bent, and at point 16 tangential plane to the jump is perpendicular to the plane of the symmetry of flow 0-16. Along flow line 0-14, occurs combination of irrotational flow after Mach cone 14-15 and vortex/eddy, that is formed behind shock wave 14-16. At point 0, is Ferry's special feature/peculiarity.

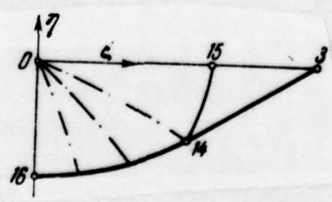


Fig. 66.

Page 237.

With decrease of M_1, region 0-15-14-16-0 (see Fig. 66) is expanded and within limit can fill entire region 0-3-14-16-0.

The circuit, depicted on Fig. 66, was, apparently, used for the first time by S. Maslen [186] during the numerical solution of the problem of the flow about the plate. To the important role of flow line 0-14 (Fig. 65), was directed attention in work [180]. This flow line is characteristic; therefore during the passage through it, suffer discontinuity/interruption not only derived S, but also derivatives the component of velocity. Neglect of this fact in works [186], [178] led to appearance in the numerical solutions of this and analogous problems of conical-supersonic zones near Mach cone 14-15, what, however, be cannot; see point/item 2.18 and [180].

The further refinements, which concern diagram in Fig. 66, are made in work [106] in which is examined the construction of flow in the vicinity of point 14. In this work it is shown, that point 14 must be person, since if one assumes that acceleration at point 14 in different directions from the range of irrotaticnal flow 0-14-15-0,

then the joining of irrotational and vortex flows in the vicinity of point 14 is impossible. Are not known at present the singular points, which ensure the joining of flows in the vicinity of point 14. The possible way out is a change in the circuit of flow in the vicinity of point 14. In work [106] is proposed one such circuit of flow, which contains additionally the weak shock wave, which passes near Mach cone 14-15, and local flow of the type of the flow of Prandtl - Mayer; flow line 0-14 is in this circuit the line of contact rupture.

It is possible to expect that in range 0-15-14-16-0 (Fig. 66) the flow will be conical-subsonic, with the exception of the vicinity of point 14, where the position not is clear.

14.4. Flow about a delta plate at high angles of attack. If we fix value of M_1 and to increase angle of attack δ, then the range of a conical-subsonic flow 0-16-14-15-0 in Fig. 66 will increase, also, when δ is smoewhat smaller δ_d it will fill entire range 0-3-14-16-0, where the flow is agitated by wing.

Page 238.

When $\delta > \delta_d$ the bow wave will move away from the leading edge of plate, which, however, strongly does not change the flow pattern of the upper part of the wing, if δ considerably does not exceed δ_d (Fig. 67).

About pressure side of wing, appears a conical-transonic zone of flow, limited by curvilinear shock wave 19-20-16, by the plane of the symmetry of flow 16-0, by the surface of wing 0-3 and by characteristic 3-19. (conical-sonic line 3-19 and by characteristic 3-19. (Conical-sonic line 3-19 is depicted as broken line). Flow in range 0-3-19-16-0 does not depend on the flow of gas in other ranges and must be calculated separately. The characteristic feature of flow in this range is the presence of point 18 where the flow branches: the flow lines, which lie to leading edge are nearer than separating line 18-20, they intersect sonic line 3-19; other flow lines converge into point 0, where there is Ferry's special feature/peculiarity (see Fig. 67a, b).

For this reason the value of specific enthropy on pressure side of wing previously not known is determined by the jump of entropy on shock wave at point 20 Fig. 67b, see [176], [184]. After characteristic 3-19, flow is expanded during the flow about the sharp edge and occurs flow breakaway at point 3; then flow again is connected near leading edge with the formation/education of weak shock waves.

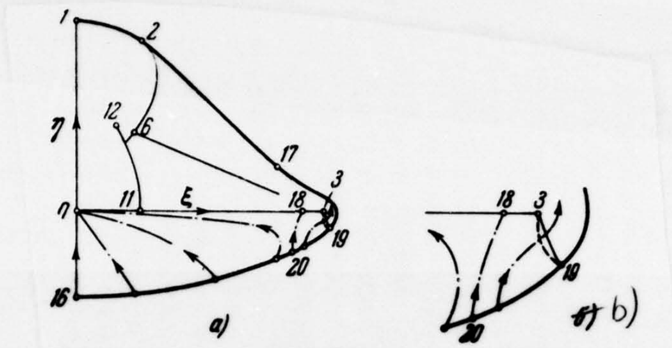


Fig. 67.

Page 239.

All these phenomena change the flow pattern of the upper surface of plate as compared with case $\delta < \delta_a$ only near the leading edge and Fig. 67 shows. Let us note that a bow shock wave 19-17 appears near characteristic 2-3 above the upper surface of plate; see also Fig. 65.

With an increase in the angle of attack δ, the flow line 18-20 is moved to the side of the line of the symmetry of wing 0-16 (Fig. 68a) and can with it merge (Fig. 68b); then at high angles of attack, flow occurs according to circuit 68b.

The circuits of flow, depicted on Fig. 68, were establish/installed by A. Bazzhin [176] and by G. Chern [184], moreover in work [184] it is noted, that if $\Lambda < \Lambda^* (M_1)$, then there is realized only circuit of flow a Fig. 68, otherwise - circuit a and b.

With an increase δ, the role of separating phenomena on the leading edge of plate increases and above the upper part of the plate is formed complex vortex flow. One should, however, note that the high angles of attack are encountered within the framework of conical

flow theory only at the hypersonic speeds when with good accuracy it is possible to place pressure on the upper part of the plate equal to zero (see [184]).

In the experiments of M. Bertram and A. Henderson (low-order) [191], carried out with M_1 = 9.6, $\Lambda = 75°$, where was conducted the visualization of flow lines on the lower surface of the delta plate, was observed the exchange of all the described mode/conditions of the flow about the plate with an increase δ from 0° to 60°. At smaller values of M_1, the experiments also confirm the given circuits of flow around of the wing.

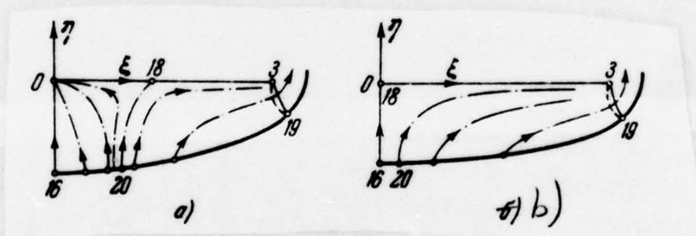


Fig. 68.

end section.

Page 240.

§15. Circuits of the flow about other conical bodies.

15.1. Circuit of flow about delta in plan/layout wing. Since compiling the circuits of the flow about the conical bodies of section β is significant matter, will be further given the circuits of the flow about some conical bodies. Let us first of all disassemble those changes in the circuit of the flow about the triangular plate which appear because of the finiteness of the thickness of body. In Fig. 69 represented delta in plan/layout wing with sharp leading edges which flows itself at such angles of attack 6, when on the one hand of wing flow is expanded, and on the other hand - it is compressed. The circuit of flow is depicted on Fig. 70. The circuit of the flow about the upper part of the wing as compared with the circuit of the flow about the triangular plate changes as follows. Instead of the range of the uniform flow, which adjoins the wing surface, appears the range of simple wave 3-5-4-3 Fig. 70 (it is

examined body with convex surface), boundary of which is its curved characteristics 5-4. For a cambered wing, the acceleration in simple conical wave non-vanishing; therefore on characteristic 2-4-5, according to the theorem of 1 point/item 2.17, parabolic point be met cannot.

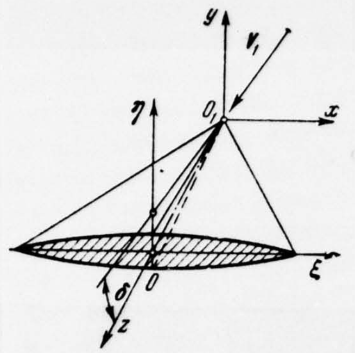


Fig. 69.

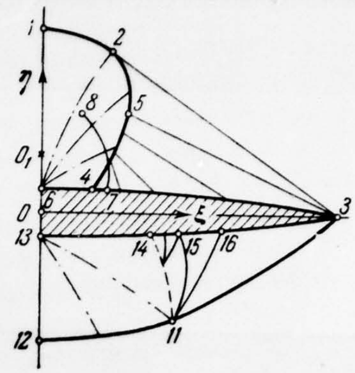


Fig. 70.

Page 241.

Just as in the case of triangular plate, one should expect the appearance of a swallowed shock 7-8 (see Fig. 70). The circuit of the flow about pressure side of wing will change more significantly. Leading shock wave 3-11-12 is bent on section 3-11, where the point of 11 figures 70 is parabolic or, it is better to say that a conical-sonic point of flow. Conical field above the wing center section 11-12-13-15-11 is separate/liberated by characteristic 11-15 (curved 11-14 is a conical-sonic line, curved 11-16 - the second characteristic), and flow bears here transonic character, which complicates the solution of the problem of flow [1].

FOOTNOTE [1]. This difficulty disappears, if we use the method of establishment (see [29]), but in this case the picture of flow in the vicinities of lines 11-15, 11-13 it will be obtained "greased". ENDFOOTNOTE.

15.2. Circuit of flow about edge of rectangular plate. Let us examine now the circuit of the flow about the edge of rectangular plate at the moderate angle of attack δ (Figs. 71, 72) according to [10, 179].

Outside the range of the wing-tip effect plate flows itself also as infinite-span wing. During the flow about the upper part of the plate, is formed the flow of Prandtl - Mayer whose rectilinear characteristics 6-12, 7-11 and others are depicted on Fig. 72.

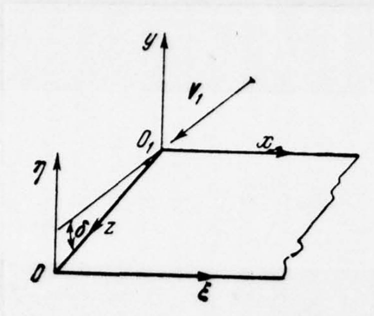


Fig. 71.

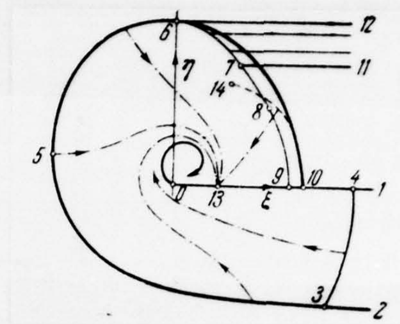


Fig. 72.

Page 242.

The flow of βrandtlya - Mayer, follows the range of uniform flow 11-7-8-9-1, which adjoins the surface of plate. During the flow about the lower part of the plate, is formed the step shock of packing/seal 2-3 after which the flow is uniform (range 1-4-3-2 in figure 72).

The range of fringe effect of plate is separated from undisturbed flow by shock wave 3-5 and by the arc of Mach cone 5-6; from the flow of Prandtl - Mayyer - by characteristic 6-7; from uniform flow - by rectilinear characteristic 7-8 and by the arc of Mach cone 8-9; from uniform flow in range 1-2-3-4 - by the arc of Mach cone for this flow 3-4. Just as in the case of triangular plate,

in the vicinity of boundary 6-7-8-9 one should expect the appearance of a swallowed shock 6-10 (or 10-14). During the flow about the flank edge of plate, the flow blows away, forming complex vortex flow. Flow lines are depicted as dot-dash lines.

15.3. Circuit of flow about crossed wing. Let us examine now the circuit of the flow about the crossed wing, formed by two identical flat/plane delta wings, which intersect at right angles (Fig. 73).

We assume that the flow about wing symmetrical (velocity vector V_1 lie/rests at plane yO_1z), angle of attack δ moderated, with those so that the appearing on the leading edges bodies of shock wave would be connected.

On the strength of last/latter condition flows about the parts of the wing, limited it is reduced to the construction of the circuits of the flow about three V-shaped wings. Let us call/name conditionally these wings upper, lateral and lower.

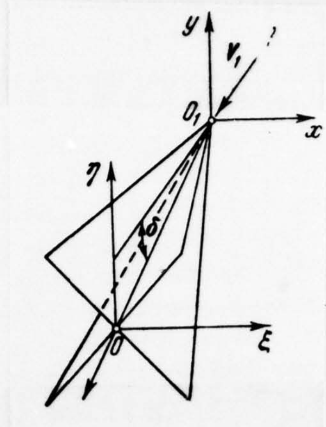


Fig. 73.

Page 243.

If the Mach number of undisturbed flow M_1 and angle of attack δ are such, that the Mach cone with apex/vertex at point O_1, constructed for an undisturbed flow, intersects the plane, passing through the leading edges of upper (lower) V-shaped wing, then the circuit of its flow in principle will be the same as circuit of the flow about the corresponding side of triangular plate.

With an increase M_1 with that which was fix/recorded δ the solution/opening of the mentioned Mach cone decreases and, beginning with certain value M_1, Mach cone lie/rests below (above) the plane,

passing through the leading edges of upper (lower) V-shaped wing. The circuit of the flow about the crossed wing for this case is depicted on Fig. 74.

On the strength of the symmetry of flow, this circuit is given only for $\xi > 0$.

Figures 74 wing depicts the line segments 0-1, 0-13. Let us examine first the circuit of the flow about the upper V-shaped wing. On leading sharp wing edge, is formed the flow of Prandtl - Mayer, range 1-2-3 in Fig. 74; it follows the range of uniform flow 1-3-4-5-1 where arcs 3-4 and 4-5 are a respectively rectilinear characteristic and the Mach cone of this flow. Range 2-3-8-2 - interaction region of the flows of Prandtl-Mayer, which are formed on the leading edges of V-shaped wing; curved 2-3 and 3-8 - characteristic of this flow. Range 8-3-4-6-8 is occupied with simple wave; curve 4-6 there is the curved characteristics of simple wave, which emerges from parabolic point 4. In range 6-7-8-6, flow is uniform. This circuit is obtained during continuous flow, but near curve 5-4-6-7 passes shock wave 18-19, which "cuts off" the part of zones of flow mentioned above.

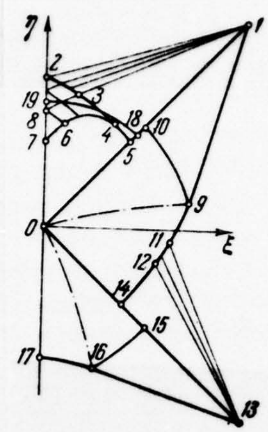


Fig. 74.

Page 244.

During the flow about the lower V-shaped wing on its leading edge, is formed the step shock of packing/seal 13-16 after which follows the range of uniform flow 13-15-16-13. Line 16-17 is a curvilinear part of leading shock wave; curved 15-16 - a part of the Mach cone for a uniform flow in range 13-15-16-13. Are possible also the mode/conditions, in which occurs the regular or Mach reflection of rectilinear jump 13-16 from the plane of symmetry 0-17. During the flow about the lateral V-shaped wing on one of its edges, is formed shock wave 1-9, on another - the flow of Prandtl - Mayer 11-12-13-11.

Line 9-11-12-14 depicts the curvilinear part of bow shock; line 9-10 is an arc of Mach cone for a uniform flow in range 1-9-10-1. Flow lines are designated in Fig. 74 in dash line.

15.4. Circuit of flow about triangular plate with fuselage in the form of half of cone. As a last/latter example let us examine the circuit of the flow about the triangular plate to which is attached the fuselage in the form of the half of round cone, at the moderate angles of attack (Fig. 75).

Thus far bow shock is connected to the leading edge of body, the upper part of the wing flows itself just as in the case of triangular plate. The circuits of the flow about the lower part of the body are depicted on Fig. 76 (see [8]).

Wing on plane $\xi\eta$ is depicted as cut 0-1-2. On its leading edge appears the step shock of packing/seal 2-3 after which there is a range of uniform flow 1-2-3-1, which adjoins the wing surface. Fuselage excites shock wave 4-3-1. At point 3, jumps intersect and is formed the line of contact rupture 3-6, at point 6, is Ferry's special feature/peculiarity, see Fig. 76a.

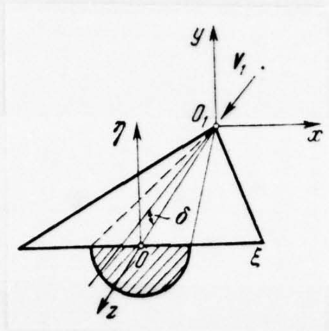


Fig. 75.

Page 245.

With an increase in the angle of attack, the shock wave 1-3 is attenuate/weakened and is shift/sheared to the leading edge of plate (Fig. 76b).

In order to better understand the picture of flow in the vicinity of point 3 in Fig. 76, let us turn to analogy with two-dimensional problem of gas dynamics and let us examine the flow about the circular cylinder to which is attached the fox tail (Fig. 77).

Here line 5-6-1-2 depicts cylinder and tapered plate. On its leading edge appears the shock wave 2-3. Line 1-3-4 depicts the basic

head shock wave, excited by cylinder. Point 3 on Fig. 77 lie/rests at the subsonic part of the flow about cylinder and is the analog of point 3 in Fig. 76 (sonic line it is designated in broken line).

§16. Methods of the solution of the problems of the flow about the conical bodies.

16.1. Lead-in observations. For the solution of the problems of the flow about the section B they are applied both numerical and analytical methods. Basic analytical method, small parameter method, is utilized for the determination of the second (irrotational) approach/approximation. Its basic features are described in §§ 5, 6 sections A; the special feature/peculiarities of its application/use for the cases of section B will be examined further.

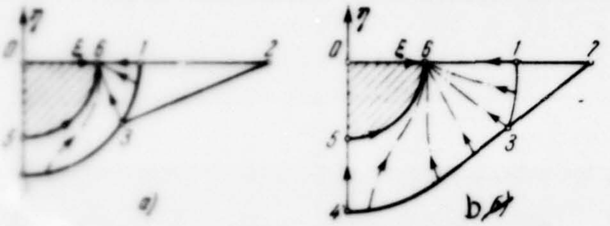


Fig. 76.

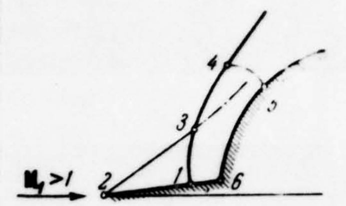


Fig. 77.

Page 246.

If for the problems of section A small parameter method (perturbation method) could compete with numerical by methods, and in certain cases it was the only possible at present method of the solution of problems, then for the problems of section B position another. If we look to the circuits of the flow about the conical bodies, depicted on Fig. 63-76, then attention is drawn to such features of the flow about the bodies which were not in the problems of section A. This is a transonic character of flows, the severe strain of the boundaries of the region of conical flow already at low angles of attack, the appearance of ranges, which contain simple conical waves, and the ranges of their interaction. Furthermore, the eddying of flow plays noticeable role already at low angles of attack, for example, for a

triangular plate - at $M_1 > 3$, $\delta > 5°$ (see [192]). Small parameter method, in which linear equations for the velocity potentials of disturbance/perturbation are eliptic equations within cone of the Mach of undisturbed flow (or in the allied range in the method of PLG), in principle cannot consider either the transonic character of flows, or the appearance of simple waves. (The discussion concerns plane $\xi\eta$.)

For these reasons the small parameter method has in the problems of section B having and main role belongs to numerical methods. Of course, can be obtained so that extrapolation of the results of small parameter method for sizable angles of attack gives good results, for example, for the distribution of pressure on wing, but this already matter of the case. Numerical methods are bulky and require the application/use of electronic digital computers, but they give reliable results. In the opinion of the author, it will be more right occupied by interpolation, having numerical results for the discrete set of the parameters of undisturbed flow and streamlined body, than by extrapolation into the range where this method of solution is unsuitable.

16.2. Survey/coverage of results, obtained by small parameter method and by other analytical methods. In view of that which was presented in point/item 16.1, in point/item 16.2 is given only

survey/coverage of the results, which relate to the determination of the second approach/approximation for the problems of section B (first approximation it is given by the usual linear theory).

Page 247.

Let us examine first of all those difficulties which appear during the use of a small parameter method for the solution of the problems of section B. Let us assume that flow irrotational let us present its conical potential F in the form $F = V_1\,[1 + f(r, \theta)]$, $r = \tilde{r}m_1$, $m_1 = (M^2{}_1 - 1)^{1/2}$, $\tilde{r}$, θ - polar plane coordinates $\xi\eta$. If axis O_1z is directed along the speed of undisturbed flow, then of the interior of the Mach cone of this flow corresponds circle $r \leqslant 1$. If gas ideal, then equation for the conical velocity potential of disturbance/perturbation $f(r, \theta)$ can be written in the form

$$\begin{aligned}
&\Big\{M_1^{-2} - (\gamma - 1)\Big[f - rf_r + \frac{1}{2}(f - rf_r)^2 + \frac{1}{2}m_1^2 f_r^2 + \\
&\quad + \frac{1}{2}m_1^2 r^{-2} f_\theta^2\Big]\Big\}(m_1^2 f_{rr} + m_1^2 r^{-1} f_r + m_1^2 r^{-2} f_{\theta\theta} + r^2 f_{rr}) = \\
&\quad = \frac{1}{2}\Big[m_1^2 f_r \frac{\partial}{\partial r} + m_1^2 r^{-2} f_\theta \frac{\partial}{\partial \theta} - (1 + f - rf_r) r \frac{\partial}{\partial r}\Big] \times \\
&\qquad \times [m_1^2 f_r^2 + m_1^2 r^{-2} f_\theta^2 + (1 + f - rf_r)^2]
\end{aligned}$$

or in the form

$$\begin{aligned}
&\Big\{1 - r^2 - (\gamma + 1) M_1^2 m_1^{-2} r^2 \Big[f - rf_r + \frac{1}{2}(f - rf_r)^2\Big] - \\
&\quad - (\gamma - 1) M_1^2 \Big[f - rf_r + \frac{1}{2}(f - rf_r)^2\Big] + \\
&\quad + \frac{1}{2}[(r^2 + m_1^2)(f_r^2 + r^{-2} f_\theta^2)] + 2M_1^2 r f_r (1 + f - rf_r) - \\
&\quad - M_1^2 m_1^2 f_r^2\Big\} f_{rr} + [1 - (\gamma - 1) M_1^2 (f - rf_r)](r^{-1} f_r + r^{-2} f_{\theta\theta}) - \\
&\quad - 2M_1^2 (r^{-2} f_\theta - r^{-1} f_{r\theta}) f_\theta + \cdots = 0, \qquad (16.1)
\end{aligned}$$

where the dots designated cubic terms (for example, $f_\theta^2 f_{r\theta}$), not containing f_{rr}. The velocity components along the axes of the Cartesian system are determined from the formulas

$$\left.\begin{aligned} w &= V_1(1+f-rf_r),\\ v &= m_1V_1(f_r\sin\theta + r^{-1}f_\theta\cos\theta),\\ u &= m_1V_1(f_r\cos\theta - r^{-1}f_\theta\sin\theta). \end{aligned}\right\} \qquad (16.2)$$

Page 248.

Let us now search for f in the form

$$f = f_1 + f_2 + f_3 + \cdots, \qquad (16.3)$$

where f_k is disturbance/perturbation of order k, which appears because of the effect of the streamlined body. Substitutiig (16.3) into equation (16.1) and equalizing the terms of identical orders, we obtain the sequence of equations for f_k:

$$(1-r^2)f_{1rr} + \frac{1}{r}f_{1r} + \frac{1}{r^2}f_{1\theta\theta} = 0, \qquad (16.4)$$

$(1-r^2)$ $f_{2rr} + \frac{1}{r}f_{2r} + \frac{1}{r^2}f_{2\theta\theta}$ = [quadratic terms in (16.1), in which f it is replaced by f_1] so forth (16.5).

Equation (16.4) is a usual equation for the conical velocity potential of disturbance/perturbation in linear theory.

When r $\longrightarrow$ 1, linear theory gives for f_1 and f_{1r}:

$$\left.\begin{aligned} f &= D(\theta) + C(\theta)(r-1) + O\left[|r-1|^{3/2}\right] \quad (r \gtrless 1), \\ f_r &= C(\theta) \pm B(\theta)(r-1)^{1/2} + O(r-1) \quad (r>1), \\ f_r &= C(\theta) + A(\theta)(1-r)^{1/2} + O(1-r) \quad (r<1), \end{aligned}\right\} \quad (16.6)$$

where $A(\theta)$, $B(\theta)$, $C(\theta)$, $D(\theta)$ is some functions θ (see [10]).

In the right side of equation (16.5) is the member

$$f_{1rr}\left[-(\gamma+1)M_1^2 m_1^2 r^{-2}(f_1 - rf_{1r}) - (\gamma-1)M_1^2(f_1 - rf_{1r}) + \right. \\ \left. + 2M_1^2 r f_{1r}\right]. \quad (16.7)$$

Since the streamlined body partially lie/rests outside Mach cone for an undisturbed flow, with r > 1 are final perturbation rates, i.e., $C(\theta) \not\equiv 0$, $D(\theta) \not\equiv 0$, and expression (16.7) with r $\longrightarrow$ 1 goes to infinity as $(1-r)^{-1/2}$. Because of this f_{2r} is a value of the order of the square of perturbation rates only if r not close to unity, but with r $\longrightarrow$ 1 $f_{2r} = O\left[(1-r)^{-1/2}\right]$, and second term in expansion (16.3) exceeds the here first. For subsequent members of expansion (16.3) the position is aggravated, for example, $f_{3r} = O\left[(1-r)^{-3/2}\right]$

with r $\rightarrow$ 1 so forth.

Page 249.

This means that expansion (16.3) is unsuitable in vicinity r = 1.

For the correction of position, M. Lighthill in work [10] proposed his, now widely known method, called the method of PLG (Puankare - Laytkhilla - Go) or the method of the strain of independent variables.

The essence of this method consists in the fact that along with f in a series is decompose/expanded independent alternating/variable r, and solution searches for in the form

$$\left.\begin{aligned} r &= R + r(\theta) = R + r_1(\theta) + r_2(\theta) + r_3(\theta) + \ldots, \\ f &= f_1(R,\theta) + f_2(R,\theta) + f_3(R,\theta) + \ldots, \end{aligned}\right\} \qquad (16.8)$$

where $r_k(\theta)$, just as $f_k(R, \theta)$, is value of order k. After the substitution of expansions (16.8) into equation (16.1) is obtained new equation with independent alternating/variable R and θ.

The coefficient when f_{RR} in it (or, it is more precise, the coefficient when $f_{1RR} + f_{2RR} + \ldots$) can be made equal to zero with R = 1 to any degree of accuracy with the aid of the following process:

1. Is determined f_1. [This function takes the same form, as in alternating/variable r, θ, since terms $r_1(\theta)$, $r_2(\theta)$ so forth they appear in equations for f_k only with k > 1].

2. It is determined by $r_1(\theta)$ from condition that first-order terms in coefficient when f_{RR} turn in zero with R = 1.

3. It is determined by f_2.

4. It is determined $r_2(\theta)$ from condition that terms of second order in coefficient when f_{RR} turn in zero with R = 1 so forth.

Equations for f_k take the form of equation (16.5), where r is replaced by R, but the right sides of these equations will be already regular functions $(1-R)^{1/2}$ and θ, since terms f_{jRR} with j < k are multiplied by the coefficients which turn in zero with R = 1. Therefore f_{kR} they will also be regular functions $(1-R)^{1/2}$ and θ, i.e., increases in the order of special feature/peculiarity with R = 1 no longer occurs, and expansion (16.8) it will represent f also in vicinity R = 1.

Page 250.

M. Lighthill [10] used his method for the determination of the force (and of position) of shock waves in conical flows. He showed that range R > 1 and R < 1, constructed according to the solutions to equation (16.4), they overlap, if we pass to alternating/variable r and θ, and in the range of overlap was found shock wave (specifically, it it can be zero force) by the satisfaction of normal conditions on shock wave. Sending away the reader after details in [10], let us note just M. Lighthill showed that the appearance of shock waves can be predicted on the basis of the behavior of usual linear solution with R $\longrightarrow$ 1.

By the combination of the method of PLG with the method of canonical systems, described in point/item 1.7, it is possible to give the problem of the determination of the second approach/approximation to the problem of the solution to the two-dimensional equations of Poisson in the range, which is the part of unit circle, in the same way as this was done in p. 6.2. One should, however, note that the process of obtaining the second approach/approximation is very laborious, and main difficulty composes obtaining particular integral for the appropriate equation of Poisson.

For some problems it is possible to also utilize the method of the connection of asymptotic expansions (see [182]) which just as is laborious as method PLG.

The further development of second-order theory pass within the framework of the theory of "three-dimensional" flows. For "three-dimensional" flows velocity potential he is record/written in the form

$$\Phi = V_1 [z + \varphi (x, y, z)],$$

where the velocity potential of disturbance/perturbation ϕ searches for as expansion

$$\left.\begin{aligned} \varphi &= \varphi_1 (x, y, Z) + \varphi_2 (x, y, Z) + \ldots, \\ z &= Z + z_1 (x, y, Z) + z_2 (x, y, Z) + \ldots \end{aligned}\right\} \quad (16.9)$$

here $\varphi_k (x, y, Z)$ and $z_k (x, y, Z)$ they are of the order of smallness k. Functions $z_k (x, y, Z)$, just as $r_k (\theta)$ in expansion (16.8), they are selected so as not to allow an increase in the orders of special feature/peculiarities in higher approach/approximations near special lines and the points, available in linear solution. Function ϕ_1 satisfies the two-dimensional wave equation:

$$-m_1^2 \varphi_{1zz} + \varphi_{1yy} + \varphi_{1xx} = 0;$$

function φ_k with k > 1 - to the nonhomogeneous wave equations

whose right sides depend on $\varphi_1, \ldots, \varphi_{k-1}, z_1, \ldots, z_{k-1}$ and their derivatives in terms of x, y, Z.

Page 251.

Since the determination of the particular integrals of equations for φ_k is more complex problem, than in the case of conical flows, were found only approximate particular integrals in the form of the combinations, containing ϕ_1 and derivatives of this function. M. Sugo in work [129] proposed this integral for ϕ_2 and examined as an example the problem of triangular plate. Another approach to the solution of the problem of the second approach/approximation proposed D. Clark and D. Wallace [193], which found in analytical form the distribution of pressure on triangular plate with supersonic edges in the second approach/approximation with the aid of the integral method of the turned flow for supersonic flows.

The essence of this method consists in the fact that along with the field of perturbation rates, which appears during the flow of uniform flow about the assigned body, which has speed V_1, the Mach number M_1, density ρ_1, is examined the field of the perturbation rates about this or allied body (both bodies have identical planform) during its flow in opposite direction, i.e., by the flow, which has the parameters $-V_1$, M_1, ρ_1. The first flow he is called straight

line, and the values, which relate to it, are noted by index F; the second flow he is called that which was turned; this flow parameters have index R.

For a triangular plate as the turned flow, is taken the flow about triangular plate with arbitrary angle Λ_R of the taper of leading edge (Fig. 78).

(If in direct/straight motion angle of attack is δ_F, then in the turned motion it can be different: δ_R.)

Let us construct now Mach cones for the points of edges O_1P, O_1O_2 in direct/straight motion and Mach cones for the points of edge O_2P in the turned motion; their envelopes together with the surface of plate limit certain volume T; its surface let us designate by letter S, internal standard to S - by letter n.

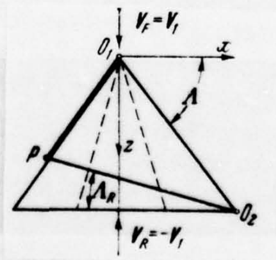


Fig. 78.

Page 252.

It is more precise, is examined the range, limited by the projection of wing on plane z = 0 and of the envelope of Mach cones, which emerge from the edges of this projection.

Let now V = i•u + j•v + k•w, W = u•i + v•j-m²₁w•k, where V is a dimensionless velocity vector of disturbance/perturbations in direct/straight or reverse motion; then the correctly following integral identity:

$$\int_S (V_F W_R \cdot n + V_R W_F \cdot n - V_F \cdot W_R n)\, dS =$$
$$= -\int_T [V_F \nabla \cdot W_R + V_R \nabla \cdot W_F - W_R \times (\nabla \times V_F) -$$
$$- W_F \times (\nabla \times V_R)]\, dT, \quad (16.10)$$

where $\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$ is an operator of Hamilton, which is the corollary of Gaussa - Ostrogradskiy.

If the velocity potentials of disturbance/perturbation φ in direct/straight and turned motions are represented in the form of the

expansions

$$\varphi = \varphi_1(x, y, z) + \varphi_2(x, y, z) + \ldots,$$

then let us hearth V_F understand the second approach/approximation for the velocity vector of disturbance/perturbation, i.e., $V_F = \nabla(\varphi_1 + \varphi_2)_F$, and hearth V_R - first approximation, i.e., $V_R = \nabla(\varphi_1)_R$.

If we as the streamlined body during the turned motion take plate at an angle of attack δ_R, then

$$u_R = -\delta_R n_R/(1-n_R^2)^{1/2},\ v_R = \delta_R,\ w_R = \frac{\delta_R}{m_1}(1-n_R^2)^{1/2},$$
$$n_R = \operatorname{tg}\Lambda_R \frac{1}{m_1}, \quad (\nabla \times V_R = 0,\ \nabla \cdot W_R = 0).$$

Function ϕ_1 is known from the usual linear solution of problem; $\nabla \times V_F = 0, \nabla \cdot W_F = Q$, where

$$Q = M_1^2[m_1^2(N-1)\varphi_{1z}^2 + \varphi_{1y}^2 + \varphi_{1x}^2]_z,$$

$N = (\gamma + 1)M^2{}_1/2m^2{}_1$; index F is here lowered.

Page 253.

Let us designate by S_F and S_R those parts of surface S, which are the envelopes of Mach cones in direct/straight and turned

motions. The flows before S_F and S_R are not agitated; therefore there respectively $V_F = 0,\ V_R = 0.$ Analyzing boundary conditions on S_F and S_R. Clark and Wallace come to the conclusion that the integrals in terms of S_F and S_R in identity (16.10) can be placed equal to zero. (This - one of the central places of method). Then equality (16.10) after scalar multiplication by V_1 can be converted to the form

$$\int_{-\frac{1}{n}}^{\frac{1}{n}} \frac{(\varphi_{1z}+\varphi_{2z})}{(1-n_R t)^2}\,dt + \frac{1}{m_1(1-n_R^2)^{1/2}} \int_{-\frac{1}{n}}^{\frac{1}{n}} \frac{(\varphi_{1y}+\varphi_{2y})}{(1-n_R^2 t)^2}\,dt =$$
$$= \frac{-2}{(1-n_{R/n})^2(1-n_R^2)^{1/2}} \int_T Q\,dT, \qquad (16.11)$$

where n = tg Λ_F; index F in equation (16.11) is lowered. Since the condition of the nonseparated flow of plate they are record/written in the form $\varphi_{1y} = \delta,\ \varphi_{2y} = \delta\varphi_{1z} - \delta z \varphi_{1yy}$ with y = 0, in relationship/ratio (16.11) enters only one unknown function φ_{2z}, through which it is expressed pressure coefficient on wing in the second approach/approximation:

$$C_p = -2(\varphi_{1z}+\varphi_{2z}) - \varphi_{1x}^2 - \varphi_{1y}^2 + m_1^2\varphi_{1z}^2.$$

If we consider n_R as independent variable, then relationship/ratio (16.11) it is singular integral equation for φ_{2z}. Without going into particulars of the solution to this equation, let us note that for D. Clark and D. Wallace, in the final analysis it was necessary to

search for the particular integral of equation for ϕ_2, that the equivalently to the solution to corresponding equation of Poisson. This particular integral was found by the methods of the theory of complex variable functions and, as note the authors, earlier it was obtained by F. Moore [123]. Thus, actually integral equation was used for the determination of the "supplementary" solution to the already homogeneous equation which made it possible to satisfy the conditions of flow on wing.

Page 254.

The obtained analytical expression for φ_{2z} on wing possessed on the Mach cone of undisturbed flow the special feature/peculiarity of square root, since method PLG was not applied. However, after φ_{2z} was found, was carried out the correction of solution by the method of PLG. Numerical results for C_p on wing with M_1 = 3.0; γ = 1.4; n = 0.35; δ = 4° render/showed in good agreement with the calculations of L. Fauell [187] for pressure side of wing.

The integral method of the turned flow was also used in work [194] for calculation in the second approach/approximation of interference between the triangular plate and the fuselage in the form of semicone. In this case, it was assumed that are previously known the fields of the disturbed parameters in the second

approach/approximation for the isolated/insulated body and a wing.

Besides the perturbation method, in which as main flow is accepted the uniform undisturbed flow, are applied methods of the type of method of "linearized characteristics". In work [195], where was examined flow near the lower part of the triangular plate, equation of motion they were linearized relative to the uniform flow, close to flow after the step shock of packing/seal on leading edge.

Besides the mentioned analytical methods in which is maintain/withstood the determined principle during obtaining of solution (for example, during the determination of the second approach/approximation by perturbation method hold smalls of the second order and disregard smalls of higher orders), there is another series of the methods which bear semi-empirical character. In these methods during the solution of problem, are made several assumptions which knowingly accurately are not fulfilled, but which on the average reflect the picture of flow about this concrete/specific/actual body. In works [196, 8], is examined the flow about the lower surface of triangular plate at angle of attack $\delta < \delta_d$.

In work [196] is utilized the polar coordinate system R, θ, Φ, depicted on Fig. 79. Of the wing surfaces corresponds θ = to 0. Main

assumptions consist in the fact that flow parameters are decompose/expanded in series of form $f(\theta, \phi) = f_1(\phi) + \theta f_1(\phi) + \theta^2 f_2(\phi) + \ldots$, and the equation of the shock layer above the range of the effect of the apex/vertex of wing is approximated by the parabola of form $\theta = \theta_0 + a\phi^2 + b\phi^4$.

Page 255.

Of course, these assumptions, the especially first, bear the very approximate character, but the pressure on the surface of plate, designed according to this method, corresponds rather well to the numerical solution of problem and to the results of experiments.

In work [8] is utilized the spherical coordinates R, θ, ϕ, depicted on Fig. 80, and is examined the flow of gas on wing and in the plane of the symmetry of flow ($\phi = 0$, $\pi/2$, $-\pi/2$). Here $w = 0$, entropy S is constant and equations of motion take the form

$$u\left(2-\frac{v^2}{a^2}\right)-v\,\mathrm{ctg}\,\theta+v_\theta\left(1-\frac{v^2}{a^2}\right)+\frac{1}{\sin\theta}w_\varphi=0,$$
$$v=u_\theta.$$

these equations are distinguished from the equations of axisymmetric flows only as term, which contains w_φ. If we assign by any form w_φ, then the determination of pressure on wing it is reduced to the integration of ordinary differential equation. In work [8] main

effort/forces are applied to to give approximation for w_φ.

However, it must be noted that this approach to the solution of problem is to a considerable degree empirical, since the value of term with w_φ can be agreed to only by solving of boundary-value problem for the zone of flow, subjected to the effect of the apex/vertex of wing.

16.3. Numerical methods of solution of problems of flow about conical bodies. It was historically obtained so that all the numerical methods, existing for the solution of the problems of section B, were developed for the solution of the problem of triangular plate (delta wing), (see [186, 187, 185, 192, 176, 177]).

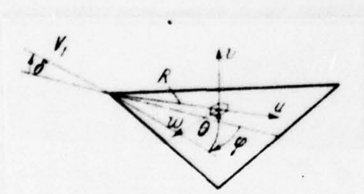


Fig. 79.

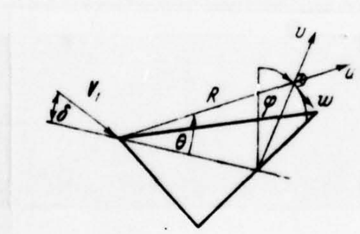


Fig. 80.

Page 256.

This does not mean that these methods are used only to the problem of triangular plate, but are conveniently set forth them on this problem, that also is made subsequently. Let us examine first case $\delta < \delta_d$, and then case $\delta > \delta_d$.

The numerical methods of the solution of the problem of triangular plate for $\delta < \delta_d$ are the methods successive being of. Most ideal of them was proposed to D. Babaev [185], [192]; it was applicable with any $\delta < \delta_d$ and $M_1 > 1$.

Is presented the first irrotational version of Babaev's method. For a wide range, the flow about the upper part of the triangular plate can be considered as irrotational (see [185, 188]); therefore for this flow there is the conical potential F, which satisfies equation (1.29).

If all values, which have the dimensionality of speed, are related to the modulus of velocity of undisturbed flow V_1 and are introduced the conical velocity potential of disturbance/perturbation f, then, retaining for dimensionless quantities the same designations, as for dimensional, we will obtain

$$\left.\begin{aligned}
&Af_{\xi\xi} + 2Bf_{\xi\eta} + Cf_{\eta\eta} = 0,\\
&A = a^2(1+\xi^2) - (u - \xi w)^2,\\
&B = a^2\xi\eta - (u - \xi w)(v - \eta w),\\
&C = a^2(1+\eta^2) - (v - \eta w)^2,\\
&a^2 = \frac{1}{M_1^2} + \frac{\gamma - 1}{2}(1 - u^2 - v^2 - w^2),\\
&u = f_\xi,\ v = f_\eta,\ w = 1 + f - \xi f_\xi - \eta f_\eta.
\end{aligned}\right\} \quad (16.12)$$

let us assume now that during the flow about the upper part of the plate is realized schematic a of figure 64. The calculation of the ranges of simple wave and uniform flow, the determination of characteristic 2-5-5-4 do not represent large work and here be examined they will not be. Let us turn on the calculation of conical flow in the range of 0-1-2-10-0 figures 64a.

Recall the boundary conditions of this problem.

Page 257.

On cut 0-10 $v = f_\eta = 0;$ in the plane of the symmetry of flow 0-1 $u = f_\xi = 0;$ on Mach cone 1-2 $u = f_\xi = 0, v = f_\eta = \sin\delta,$ $w = 1 + f - \xi f_\xi - \eta f_\eta = \cos\delta:$ on shock wave 2-10 [see relationship/ratios (1.47)]:

$$\left.\begin{aligned} & u = f_\xi = u_+ - \eta_s' P, \; v = f_\eta = v_+ + P, \\ & w = 1 + f - \xi f_\xi - \eta f_\eta = w_+ + (\xi\eta_s' - \eta_s) P, \; \eta_s' = \frac{d\eta_s}{d\xi}, \\ & P = \frac{2}{\gamma+1}\left[\frac{a_+^2}{v_+ + \eta_s' u_+ + w_+(\xi\eta_s' - \eta_s)} - \frac{v_+ - \eta_s' u_+ + w_+(\xi\eta_s' - \eta_s)}{1 + \eta_s'^2 + (\xi\eta_s' - \eta_s)^2}\right]. \end{aligned}\right\} \tag{16.13}$$

u_+, v_+, w_+, a_+ - the components of vector of speed and the speed of sound during before the shock wave 2-10.

Assignment u, v, w on curve 2-10 is equivalent to assignment f and its normal derivative, since functions f, f_ξ, f_η, determined by the formulas

$$f_\xi = u, \quad f_\eta = v, \quad f = w + \eta v + \xi u - 1 \qquad (16.14)$$

satisfy the condition of strip on shock wave (see point/item 1.9).

The arc of Mach cone 1-2 is simultaneously parabolic line and the characteristic of equation (1.29); in its vicinity F, it is determined by expansion (2.103), that contain the arbitrary function which has high value, at least for the flows in which the velocity vector deviates to small angle. Because of this the values of the derivatives of F in the vicinity of Mach cone considerably differ from their values by Mach cone. Therefore during the solution of problem by finite-difference method it is expedient to assign only f on Mach cone 1-2, satisfying equation (16.12) in internal mesh points. In this case, of course, the parts of flow near Mach cone are not considered in solution. Alternative to the aforesaid is the use of a low pitch of grid in the vicinity of arc 1-2.

It is presented now the idea of D. Babaev's method. Let be known certain approximate shock position 2-10 in Fig. 64a, given by equation $\eta = \eta_s^0(\xi)$. Then according to formulas (16.13), (16.14) we determine values of f and f_ξ when $\eta = \eta_s^0: f = \psi^0(\xi), f_\xi = u^0(\xi)$;

Page 258.

Let us find further the solution to equation (16.12) f^0 in range with the fixed/recorded boundary, that satisfies the boundary condition

$$f^0 = \psi^0(\xi) \quad \text{with } \eta = \eta_s^0(\xi),$$

retaining on the remaining parts of the boundary previous boundary conditions. If the shock position is assigned by equation $\eta = \eta_s^0$ it is correct, then when $\eta = \eta_s^0$ $f_\xi^0 = u^0(\xi)$. If this condition is not satisfied, then is selected the new shock configuration, determined by equation $\eta = \eta_s^1(\xi)$ of the condition that $f_\xi^0|_{\eta=\eta_s^0} = u^1(\xi)$, where $u^1(\xi)$ is calculated from formulas (16.13) for $\eta=\eta_s^1(\xi)$. Then is located $\psi^1(\xi)$ through formulas (16.13), (16.14), it is determined $f^1(\xi, \eta)$, again is checked condition $f_s^1|_{\eta=\eta_s^1} = u^1(\xi)$ so forth. Process

continues until within the limits of the assigned accuracy is made equality $\eta_s^n = \eta_s^{n+1}$.

The virtually described process is fulfilled as follows. Is assigned jump 2-10 (Fig. 64a), i.e., is assigned function $\eta = \eta_s^0(\xi)$. Range 0-1-2-10-0 is cover/coated with square network. Equation (16.12) he is record/written in finite-difference form. In this case, the derivatives are approximated by central differences. If h - the space of network, then for a point with numbers i = ξ/h and $k = \frac{\eta}{h}$ we have

$$f_\xi \approx \frac{1}{2h}(f_{i+1,\,k} - f_{i-1,\,k}),\quad f_\eta \approx \frac{1}{2h}(f_{i,\,k+1} - f_{i,\,k-1}),$$
$$f_{\xi\xi} \approx \frac{1}{h^2}(f_{i+1,\,k} - 2f_{i,\,k} + f_{i-1,\,k}),$$
$$f_{\xi\eta} \approx \frac{1}{4h^2}(f_{i+1,\,k+1} - f_{i+1,\,k-1} - f_{i-1,\,k+1} + f_{i-1,\,k-1}),$$
$$f_{\eta\eta} \approx \frac{1}{h^2}(f_{i,\,k+1} - 2f_{ik} + f_{i,\,k+1}).$$

Equation (16.12) accepts then the form

$$A_{i,\,k}(f_{i+1,\,k} + f_{i-1,\,k}) + B_{i,\,k}(f_{i,\,k+1} + f_{i,\,k-1}) - 2(A_{ik} + C_{i,\,k})f_{ik} + \frac{1}{2}B_{i,\,k}(f_{i+1,\,k+1} - f_{i+1,\,k-1} - f_{i-1,\,k+1} + f_{i-1,\,i-1}) = 0.$$

Page 259.

For solution by iteration the latter is converted to the form

$$f_{i,k} = \frac{1}{2(A_{i,k} + C_{i,k})} [A_{i,k}(f_{i+1,k} + f_{i-1,k}) + C_{i,k}(f_{i,k+1} + f_{i,k-1}) + \frac{1}{2} B_{i,k}(f_{i+1,k+1} - f_{i+1,k-1} - f_{i-1,k+1} + f_{i-1,k-1})]. \quad (16.15)$$

If we write equation (16.15) in all internal node/units of the finite-difference range, which corresponds to range 0-1-2-10-0, to express in finite-difference form boundary conditions (after assuming $f = \eta$ sinδ + cosδ-1 on the arc of Mach cone 1-2), then will be obtained the system of nonlinear (cubic) algebraic equations for determining values $f^0_{i,k}$ in all mesh points. This system is solved by the method of iterations.

[Substituting in the right sides of the equation (16.15) of value of n of approach/approximation, they obtain as a result of value n + 1 of approach/approximation]. After determination $f^0_{i,k}$ is more precisely formulated the shock position 2-10. For this purpose is determined value f^0_ξ on the boundary of grid range, which corresponds $\eta^0_s(\xi)$, i.e., when $\eta = kh$, and they compile an equation

$$f^0_\xi = u_+ - \eta_s' \frac{2}{\gamma+1} \left[\frac{a_+^2}{v_+ - \eta_s' u_+ + w_+(\xi\eta_s' - \eta_s)} - \frac{v_+ - \eta_s' u_+ + w_+(\xi\eta_s' - \eta_s)}{1 + \eta_s'^2 + (\xi\eta_s' - \eta_s)^2} \right]. \quad (16.16)$$

in which derived η_s' it is replaced by its finite-difference

expression

$$\eta_k' = \frac{\eta_{k+1} - \eta_k}{\xi_{k+1} - \xi_k} = \frac{h}{\xi_{k+1} - \xi_k},$$

a the values, noted by cross, are calculated on the boundary of grid range.

In equation (16.16) when $\eta = kh$ enter two unknown values, ξ_k and ξ_{k+1}. If we begin the refinement of the form of jump from point 2 during Fig. 64a whose position is known, and for which $k+1 = k_2 + 1 = \eta_2/h$, then into equation (16.16), written when $\eta = k_2h$, will enter only one unknown value ξ_{k_2}.

Page 260.

Solving equation (16.16) relatively ξ_{k_2}, we find more accurately position of jump when $\eta = k_2h$.

Knowing the position of point $\xi_{k_2}, \eta_{k_2} = k_2h$ on shock wave, we find in a described manner the position of neighboring point on jump, and so to the wing surface. Then we find f¹ so forth.

Before transfer/converting to the discussion of the results of calculations, let us make one observation of fundamental order of

relatively this method. As show calculations for functions f^0, f^1 and so forth, equation (16.12) is mixed, elliptical-hyperbolic type equation. For this reason the boundary-value problems for the determination of functions f^0, f^1, ... are, generally speaking, incorrectly placed, since the duct on which are given the boundary values of function (or to its normal derivative), it is the locked duct. (To this fact focused attention A. Nikol'skiy). Consequently, if we are unlimitedly decrease the space of network h, then as a result we will not obtain solution for f^0, f^1 so forth. However, as show the calculations of D. Babaev, in all examined by it cases the process converged. The possible explanation of this fact is such. To each amount of deflection the approximate shock position from the true corresponds value h, less which it is not possible to select the space of network during determination f^0, f^1, ... With a decrease in the deviation of the approximate position of jump from the true, this space decreases. Consequently, D. Babaev's method must be considered as empirical finite-difference algorithm of the solution to initial boundary-value problem, without introducing as intermediate stage boundary-value problems for f^0, f^1 so forth.

Figures 81a, b depicts the results of calculations for $M_1 = 4.0$; $\Lambda = 60°$; $\delta = 5°$; $h = 0.02$ [185].

Figures 81b as solid line depicts the results of the

calculations of pressure coefficient C_p in Babaev's method, by broken line in linear theory. Broken line in Fig. 81a is parabolic (conical-sonic) line of equation (16.12).

Page 261.

Shock wave is only near the wing surface, degenerating at point 2' into characteristic. The defect of solution is that the part of the boundary of the region of conical flow is the cut of rectilinear characteristic 2'-6. On the appearing in connection with this difficulties see point/item 14.2. It is possible that this defect was the consequence of the disregard of the rupture of accelerations, which was being spread inside range 0-1-2-6-2'-10-0 along the characteristic, emerging from point 6. It is not excluded, that with the given parameters of undisturbed flow and wing is realized circuit b, while not a Figure 81.

Let us examine now D. Babaev's method for vortex flow in an example of the flow about the lower surface of triangular plate (see Fig. 65). As the unknown functions are accepted the components of velocity u, v, w along the axes of the Cartesian system and specific enthropy S. These functions they satisfy system of equations (1.17)-(1.20). We assume that gas ideal. If all values, which have the dimensionality of speed, are related to the modulus of velocity

of undisturbed flow V_1, and instead of S are introduced the function

$$s = \frac{1}{\gamma(\gamma-1)} \ln\left[\frac{p/p_1}{(\rho/\rho_1)^{\gamma}}\right],$$

that, by retaining for dimensionless quantities the same designations, that also for dimensional, system of equations (1.17)-(1.20) after some conversions can be led to this form:

$$\left.\begin{aligned} &(u-\xi w)\, w_\xi + (v-\eta w)\, w_\eta + \xi\left(\frac{u^2+v^2+w^2}{2}\right)_\xi + \\ &\qquad + \eta\left(\frac{u^2+v^2+w^2}{2}\right)_\eta + a^2(\xi s_\xi + \eta s_\eta) = 0, \\ &w(\eta v_\eta + \xi v_\xi + w_\eta) + u(u_\eta - v_\xi) + a^2 s_\eta = 0, \\ &w(\eta u_\eta + \xi u_\xi + w_\xi) + v(v_\xi - u_\eta) + a^2 s_\xi = 0, \\ &(u-\xi w)\left(\frac{u^2+v^2+w^2}{2}\right)_\xi + (v-\eta w)\left(\frac{u^2+v^2+w^2}{2}\right)_\eta + \\ &\qquad + a^2(\xi w_\xi + \eta w_\eta - u_\xi - v_\eta) = 0, \\ &a^2 = \frac{1}{M_1^2} + \frac{\gamma-1}{2}(1 - u^2 - v^2 - w^2). \end{aligned}\right\} \quad (16.17)$$

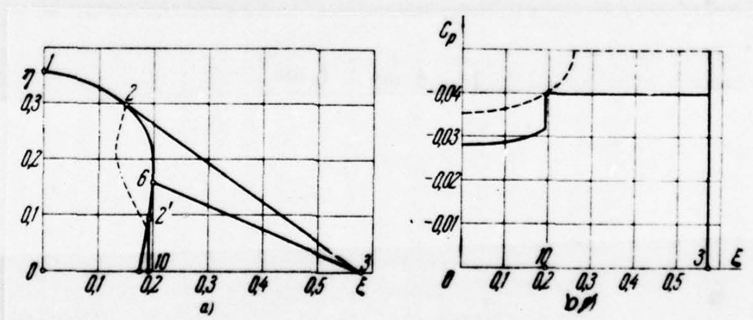


Fig. 81.

Page 262.

Conditions on shock wave 14-16 in Fig. 66 are given by formulas (1.47)-(1.49). The essence of method for vortex flow remains the same, as for an irrotational flow, only all operations become bulkier. Therefore primary attention let us pay those to the supplementary facts which are connected with the eddying of flow.

If we assign approximately shock wave 14-16 Fig. 66, by equation $\eta=\eta_*^0(\xi)$, then from formulas (1.47)-(1.49) they are determined by u, v, w, s when $\eta = \eta_*^0(\xi)$:

$$u = u^0(\xi), \quad v = v^0(\xi), \quad w = w^0(\xi), \quad s = s^0(\xi). \quad (16.18)$$

Assuming that at the points of the shock front are satisfied the equations (16.17), with the aid of (16.18) let us find here

$$u_\eta = \chi_1^0(\xi), \quad v_\eta = \chi_2^0(\xi), \quad w_\eta = \chi_3^0(\xi),$$
$$s_\eta = \chi_4^0(\xi). \qquad (16.19)$$

This always can be made, since line 14-16 is not system performance of equations (16.17).

Let us emphasize, that even so equality (16.18), (16.19) are written in the form u = $u^0(\xi)$, $u_\eta = \chi_1^0(\xi)$ and so forth, their right sides are known functions ξ, η_*, η_*'.

Page 263.

Just as in irrotational problem, range 0-15-14-16-0 is cover/coated with rectangular network and system of equations (16.17) is replaced by their finite-difference equivalent. Further is solved problem in range with fixed boundary moreover boundary conditions on arc 14-16 are assigned by relationship/ratios (16.18). The solution of the matching system of nonlinear algebraic equations is conducted by the method of steepest descent. In this case, it is considered that the flow line 0-14 is line of discontinuity derived u, v, w, s with the aid of one-sided difference formulas. After the determination of zero approximation (u^0, v^0, w^0, s^0) the refinement of the form of jump is conducted in the same way as this it was made in irrotational problem, only instead of equation (16.16) here is utilized the equation

$$(u_\eta^0 - \chi_1)^2 + (v_\eta^0 - \chi_2)^2 + (w_\eta^0 - \chi_3)^2 + (s_\eta^0 - \chi_4)^2 = 0, \tag{16.20}$$

where $u_\eta^0, v_\eta^0, w_\eta^0, s_\eta^0$ are calculated on the boundary of grid range, which corresponds $\eta = \eta_s^0(\xi)$, and for functions χ_j (j = 1, 2, 3, 4) are written their explicit expressions through ξ, η_s, η_s'; moreover derivative η_s' is replaced by its finite-difference expression

$$\eta_s' = \frac{\eta_{k+1} - \eta_k}{\xi_{k+1} - \xi_k} = \frac{\eta_{k+1} - \eta_k}{h}.$$

The refinement of the shock configuration is conducted from known point 14 in Fig. 66 in the direction of the plane of the symmetry of flow 0-16. In view of the approximate character of solution may not exist value η_k, which satisfies equation (16.20); therefore more precise value η_k is located by the minimization of the left side of the equality (16.20).

Sending away the reader after details in [192], let us note just in all the checked by the author of this work cases this method was led to the unique solution of task with very distinguished initial data for the shock configuration and functions u, v, w, s.

Page 264.

Relative to the fundamental side of matter it is possible to repeat the same as was said for an irrotational problem, only here the overdetermination of problem with the fixed/recorded boundary was obvious and there is no need to turn to the properties of mixed type equations.

Thus, during the use of Babaev's method it is necessary to exhibit precaution when selecting the space of network for problems with the fixed/recorded boundary.

Figure 82, undertaken work [192], depicts the designed shock configuration with $M_1 = 4$; $\Lambda = 60°$; $\delta = 10°$.

Figures as 83 solid lines depicts distribution of coefficient of pressure C_p on the wing surface, calculated according to Babaev's method for $M_1 = 4$; $\Lambda = 50°$; $\delta = 5°, 10°, 15°$; broken line designated the results of linear theory.

For the calculation of flow about the lower surface of triangular plate when $\delta > \delta_d$ A. Bazzhin [176, 177] used the method of the integral relationship/ratios of first approximation, described in point/item 11.2. Was utilized the flow spherical coordinates, depicted on Fig. 84.

The system of equations of first approximation coincides with system (11.13). Since the flow about pressure side of wing occurs with flow choking, for determining two unknown parameters of the problem $\theta^0(0)$, $u^*(0)$ [$d\theta^0/d\Phi = w^* = 0$ with $\theta = 0$] they are advanced the following conditions: 1) at point 3 in Fig. 67, 68 speed of cross flow is equal to the speed of sound; 2) solution is regular at the point where the denominator of the expression, which stands in right part one equation of system (11.13), turns into zero.

In all cases, examined by the author [176], was one such singular point.

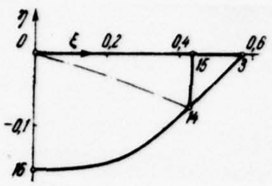


Fig. 82.

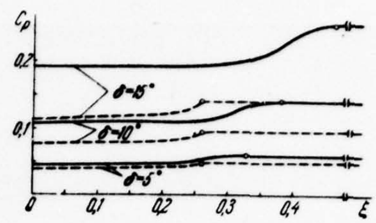


Fig. 83.

Page 265.

The formulated conditions made it possible to determine first approximation in an only manner. Since on the wing surface are two lines of the spreading of flow, value of entropy here also was selected in the process of calculations. Subsequently Bazzhin improved the method of calculation (see [177]) and were conducted systematic calculations in ranges M_1 = 4-10, $\Lambda = 70^\circ \div 85^\circ$, δ = 30°-60°. Figures 85 depicts the dependence of the coefficient of the normal force of plate C_n on angles Λ and δ with M_1 = 6 [177].

The method of the integral relationship/ratios of first approximation, as it was already noted earlier, makes it possible to

solve problem with good accuracy only at large values M_1, so that this method is related faster to supersonic regime of the flow about the body, than to supersonic mode/conditions.

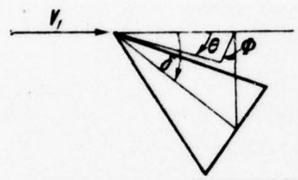


Fig. 84.

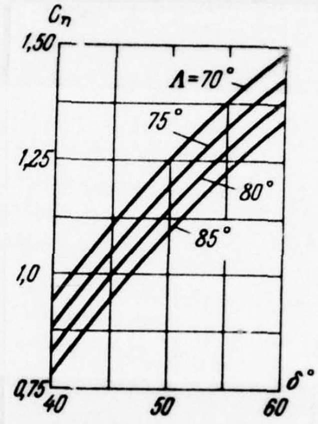


Fig. 85.

End section.

Page 266.

Chapter 3.

Hypersonic conical flows of gas.

A. The flow about the conical bodies in cases when bow shocks are connected only to the apex/vertexes of bodies.

§17. General characteristic of the properties of hypersonic conical flows and methods of the solution of the problems of the flow about the conical bodies.

17.1. Classification of problems of flow. In the third chapter are examined the cases of the flow of flows about the conical bodies of gas, which possess large Mach numbers, $M_1 \gg 1$. The common properties of such flows are well known, see [45, 5]. Furthermore, in p. 2.1 are examined in detail those special feature/peculiarities of

the flow about the round cones (at zero angle of attack) which appear at hypersonic speeds. This - nearness of leading shock wave to the surface of the streamlined body (in consequence of which gas moves in the shock layer where its density is considerably greater than density in undisturbed flow); the complication of the equation of state of gas, since with high temperatures are excited oscillatory degrees are free the molecules of gas, occurs their dissociation and ionization.

Were noted also those factors, which one should consider during the use of a conical flow theory for the flow-field analyses of real bodies. This is a possible flow irregularity, the powerful effect of small blunting of the leading edge/nose of body and thick viscous boundary layer on the parameters of inviscid flow about slender cones. All these facts occur, also, during the flow about other conical bodies.

Page 267.

Subsequently we will set forth the theory of the hypersonic conical flows of gas with the basic of the assumption that $M_1 \gg 1$, the disturbance/perturbations of the parameters of gas are of the order of the values of these parameters in undisturbed flow, and that there is equation of state of gas. Value M_1, beginning with which

flow can be considered hypersonic, it is determined by the specific conditions among which one of the main places belongs to the characteristic angle of the slope of the cell/elements of the surface of the streamlined body to direction of undisturbed flow. (The greater this angle, as those at smaller values M_1 flow has properties of hypersonic flows; see [5].).

Since when $M_1 \gg 1$ the streamlined body almost always partially or completely emerges the Mach cone of the undisturbed flow, constructed for the apex/vertex of body downstream, the classification, accepted for supersonic flows, is here unsuitable. The different cases of the flow about the conical bodies when $M_1 \gg 1$ it is logical to divide into two groups. The first group, A, compose those cases in which the bow shock is connected only to the apex/vertex of the streamlined body. To second group, B, let us relate the cases in which the bow shocks are connected to the sharp leading edges of the streamlined bodies. In the group A of body, they possess chamfered edges (if it is possible to speak about edges) or by sharp edges, but the flow about the bodies occurs at high angles of attack, so that bow shock cannot be that which was connected to the edges of body.

17.2. General characteristic of methods of solution of problems of flow about conical bodies. In the third chapter will not be

examined the theory of the slight disturbances for hypersonic speeds in which for main flow is taken the uniform undisturbed flow, since the results of this theory are most interesting for three-dimensional conical flows. The theory of the slight disturbances, in particular the laws of hypersonic similitude, they are set forth in the known books of G. Cherny [46], of U. Hayes and D. Probstin [5].

Also will not be considered Newton's theory, based on the concept of infinitely thin shock layer, for two reasons. First, the results of this theory for conical bodies are obtained from the more common/general/total theory of the shock layer of the final thickness which will be examined further.

Page 268.

In the Second place, to Newton's theory is allotted much place in the monograph of U. Hayes and D. Probstin [5], and also in Zh. Guiraud's book [162].

The complication of the equation of state of gas at hypersonic speeds makes more laborious the process of the solution of the problems of the flow about the conical bodies. (Fundamental difficulties in this case do not appear).

But, on the other hand, appears a number of factors which substantially facilitate the solution of the problems of flow. with hypersonic speeds about the windward face of body gas moves in the comparatively thin shock layer where its density is much greater than the density of undisturbed flow, moreover the parameters of gas are little affected across shock layer. Exception is fine/thin region in immediate proximity to the surface of the streamlined body - the vortex/eddy (entropic) layer where certain the parameters of gas, for example, the specific enthropy, can substantially change. However, at hypersonic speeds vorticity layers, as a rule, are very fine/thin and are located within viscous boundary layers, so that basic interest are of the flows of outside these layers. The solution of the problem of flow in vorticity layers is required mainly for the substantiation of the solutions of outside vorticity layers and estimating their thickness, and also for the check of numerical methods. That part of the body surface, which is located in aerodynamic shadow, does not in practice affect the aerodynamic characteristics of body and does not require calculation. For example, with calculation of the lift of triangular plate under the hum of attack pressure on its upper surface can be placed equal to zero. Furthermore, the distribution of pressure on body surface can be designed on the basis of theory for an ideal gas, since the account of the validity of gases little changes results.

As a result of the enumerated special feature/peculiarities of the flow of hypersonic flows about the conical bodies of gas simple analytical methods can give good results, moreover frequently in the locked form. For example, in the hypersonic variation of the theory of Stone for a round cone, developed by Kh. Chzhen [148], pressure in field of flow is expressed in elementary functions; to method of "linearized characteristics" for hypersonic speeds also it is possible to give completely analytical form; see work [197].

Page 269.

The accuracy of these methods is obtained acceptable for practice. Important analytical method for hypersonic speeds was proposed by A. Gonor [168.7]. In this method of shock layer the low parameter is the density ratio in undisturbed flow to characteristic density in shock layer, and the unknown values are decompose/expanded in series according to the degrees of the low parameter. (Details, and also observations about history of this method, see in §18.).

The majority of numerical methods at hypersonic speeds gives more accurate results, than at supersonic speeds. Among them - the method of the integral relationship/ratios of first approximation, the method of straight lines, the reverse/inverse method of two functions of current. It is possible to expect that with $M_1 \gg 1$

method of establishment will give less accurate results, than at supersonic speeds, since at hypersonic speeds vorticity layers are more are sharply pronounced, that it is led to a reduction in the accuracy of the approximation of the derived unknown values by finite differences in vorticity layers.

Among semi-empirical methods let us note the method of the "equivalent cone", [154, 155, 199], which gives excellent results for pressure on the surface of simple bodies with convex cross section.

Subsequently primary attention will be allotted to analytical methods, since the numerical methods were already dismantle/selected in the second chapter.

§18. Method of shock layer for a round cone.

18.1. Lead-in observations. The problem of the flow about the cone at an angle of attack by the supersonic flow of gas is one of the most important tasks of gas dynamics. At the moderate values of the Mach numbers of undisturbed flow, it was solved by perturbation method on angle of attack δ with the accuracy for values $O(\delta^2)$ inclusively by A. Stone. However, this solution is cumbersome and is

inconvenient for practical use even in the presence of tables of Z. Kopala.

Page 270.

Because of this natural was the wish to simplify the solution of Stone for hypersonic speeds, after using this specific property of hypersonic flows as smallness of the density ratio in undisturbed flow to characteristic density in shock layer. For an ideal gas with constant heat capacities, this relation and thickness of shock layer vanish, when $\gamma \longrightarrow 1$, and the Mach number of undisturbed flow $M_1 \longrightarrow \infty$ (see [46]).

For this reason in the case of ideal gas, the unknown values in the system of spherical coordinates, connected with body, are decompose/expanded in series not only according to degrees δ, but also according to the degrees $\sigma = \frac{\gamma - 1}{\gamma + 1}$ (OT) the analogous parameters. In this case, product $(\gamma - 1) M_1^2$ (or the allied value) is set/assumed by constant.

Without being stopped on earlier works (for example, see [202], let us turn to Kh. Chzhen's works [148] and Ya. Sapunkov [203-205] in which is obtained the most complete solution of a problem of this type. Work [148] in elementary functions gives the solution of

problem with an accuracy $O(\sqrt{\sigma^2 + \delta^2})$ inclusively in an entire region between the surface of leading shock wave and the surface of the streamlined cone. Furthermore, pressure and the circular/neighboring velocity component are found with an accuracy to values inclusively in field of flow. In works [203-205] in the locked form given complete solution of the problem of round cone with an accuracy to values 0 $(\delta^2 + \sigma^2)$ inclusively, suitable in all shock layer, with the exception of small vicinity of Ferry's special feature/peculiarity. (In this case were corrected the inaccuracies in the computational character, available in work [148].).

Work [205] gives solution for the case of the inadequate gas. the low parameter σ is here the relation of the density of undisturbed flow to characteristic density in shock layer.

The basic idea of method Ya. Sapunkov is the introduction of the auxiliary function ζ, which possesses the main properties of the function of the current of conical flow, but at the same time it makes it possible to obtain for it uniform approach/approximation everywhere in field of flow. First are located approximate solutions for the parameters of the flows in which enters unknown function ζ, and then are constructed successive constructing for ζ, evenly approximating function ζ in vorticity layer.

Page 271.

Thus, method Ya. Sapunkov is the combination of the method of expansion in terms of the small parameter and of the method successive being of.

In Stone's theory, angle of the attack of cone δ is considered small ($\delta \ll \varepsilon$, where ε — a half-angle of cone). Virtually good results are obtained according to Stone's theory when $\delta < \frac{\varepsilon}{2}$. This fact limits the field of application of solution of Stone (and his hypersonic version). Furthermore, approach itself to the solution of problem befits only for a round cone or the bodies, which differ little from round cone. New approach to the solution to the tasks in which are not superimposed substantial limitations on the value of angle of attack and the form of the cross section of cone, proposed A. Gonor [168.7]. In A. Gonor's method, the unknown values in shock layer are decompose/expanded according to the degrees of parameter $\varepsilon = \frac{\gamma-1}{\gamma+1}$, and as independent variables they are taken Φ and Ψ, where Φ there is angular coordinate in the spherical coordinates, depicted on Fig. 47, and Ψ - the function of current, which satisfies equation (10.2):

$$v\Psi_\theta + w \operatorname{cosec} \theta \Psi_\Phi = 0.$$

As a result A. Gonor succeeded in obtaining the first terms of the corresponding expansions in quadratures; following terms also are

expressed in quadratures from elementary functions (at least in principle).

It must be noted that A. Gonor's success is explained mainly by selection as the independent alternating/variable function of current Ψ. Previously analogous method was proposed by G. Chern (see [69]) for the solution to two-dimensional and axisymmetric problems of the flow about the pointed bodies. However, transition from flat/plane and axisymmetric flows to conical flows is qualitatively new space. Was somewhat later explained that the expansions in terms of degrees $\sigma = \frac{\gamma-1}{\gamma+1}$ do not represent the solution of problem in all shock layer. For low angles of attack δ, this directly follows from the hypersonic variation of the theory of Stone, which in its initial form does not give solution in vorticity layer about body surface.

Page 272.

The understanding of that fact that at hypersonic speeds there is new type vorticity layer, connected not with the smallness of angle of attack, but with the smallness of the density ratio in undisturbed flow to characteristic density in shock layer $\left(\sigma = \frac{\gamma-1}{\gamma+1} \ll 1\right)$, was reached in the works of R. Mel'nik and R. Sheing [206], of Kh. Chzhen [148] and B. Bulakh [207]. In work [7] A. Gonor found in quadratures the first terms of expansions in terms of degrees $\sigma = \frac{\gamma-1}{\gamma+1}$ for the

bodies, having "arbitrary" cross section. This solution possesses the same deficiency/lack, as solution for a round cone. A number of the authors was occupied by the correction of the solution of A. Gonor in vorticity layers about the surfaces of streamlined conical bodies [206, 204, 208-210]. Since the process of obtaining the first terms of expansions degrees σ for bodies with "arbitrary" cross section is repeated such for a round cone, and account of the inadequacy of gas is not introduced fundamental changes into this process, below we will disassemble in detail in an example of the round cone, streamlined with ideal gas, the different facts, connected with the method of shock layer, but in §19 it is presented him for bodies with the "arbitrary" cross section, streamlined with the inadequate gas.

18.2. Solution of problem of flow about round cone without taking into account of vorticity layer. Let us examine round cone with half-angle ε, streamlined at an angle of attack δ with the hypersonic uniform flow of ideal gas, which has speed V_1, number Maxa M_1, density ρ_1, and so forth.

To the system of spherical coordinates R, Θ, Φ, connected with body (see Fig. 47), the equations of motion of gas can be written in the form [see equations (1.1) - (1.5), (1.8)]

$$
\left.\begin{array}{ll}
(\rho v \sin\theta)_\theta + (\rho w)_\Phi + 2\rho u \sin\theta = 0, & \text{(а)} \\
v u_\theta + w \operatorname{cosec}\theta \cdot u_\Phi - v^2 - w^2 = 0, & \text{(б)} \\
\dfrac{u^2+v^2+w^2}{2V_1^2} + \dfrac{\gamma}{\gamma-1}\,\dfrac{p}{\rho V_1^2} = \dfrac{1}{2} + \dfrac{1}{(\gamma-1)M_1^2}, & \text{(в)} \\
v(p\rho^{-\gamma})_\theta + w\cdot\operatorname{cosec}\theta\,(p\rho^{-\gamma})_\Phi = 0, & \text{(г)} \\
v v_\theta + w\cdot\operatorname{cosec}\theta\cdot v_\Phi + u\cdot v - w^2\operatorname{ctg}\theta = -\dfrac{1}{\rho}\,p_\theta. & \text{(д)}
\end{array}\right\} \quad (18.1)
$$

Page 273.

On the surface of cone $\theta = \varepsilon,\ v = 0$ (condition of nonseparated flow).

Let us assign the surface of bow shock equation $\theta = \theta^*(\Phi)$ and designate through f^- and f^+ the value of function f directly do shock wave. Then for an undisturbed flow they occur of the equality

$$
\left.\begin{array}{l}
u^- = V_1(\cos\delta\cos\theta^* + \sin\delta\sin\theta^*\cos\Phi), \\
v^- = -V_1(\cos\delta\sin\theta^* - \sin\delta\cos\theta^*\cos\Phi), \\
w^- = -V_1\sin\delta\sin\Phi.
\end{array}\right\} \quad (18.2)
$$

The equations of consistency on shock wave let us write in the form

$$\left.\begin{aligned}
&V_n^+\rho^+ = V_n^-\rho^-, \ V_{\tau_1}^+ = V_{\tau_1}^-, \ V_{\tau_2}^+ = V_{\tau_2}^-, \\
&\frac{\rho^+}{\rho^-} = \frac{V_n^-}{V_n^+} = \frac{\frac{\gamma+1}{\gamma-1}}{1+\frac{2}{\gamma-1}\left(\frac{a^-}{V_n^-}\right)^2}, \\
&\frac{p^+}{p^-} = \frac{2\gamma}{\gamma+1}\left(\frac{V_n^-}{a^-}\right)^2 - \frac{\gamma-1}{\gamma+1}, \\
&(p^- = p_1, \ \rho^- = \rho_1, \ a^- = a_1),
\end{aligned}\right\} \quad (18.3)$$

where the value with indices n τ_1, τ_2 represent with respect to the projection of velocity vector on standard even two mutually perpendicular tangents to the surface of shock wave; they are located through the formulas

$$\left.\begin{aligned}
&V_n = \left[1+\left(\frac{\theta^{*\prime}}{\sin\theta^*}\right)^2\right]^{-1/2} \cdot \left(v - w\frac{\theta^{*\prime}}{\sin\theta^*}\right), \\
&V_{\tau_1} = \left[1+\left(\frac{\theta^{*\prime}}{\sin\theta^*}\right)^2\right]^{-1/2} \left(v \cdot \frac{\theta^{*\prime}}{\sin\theta^*} + w\right), \\
&V_{\tau_2} = u, \quad \theta^{*\prime} = \frac{d\theta^*}{d\Phi}.
\end{aligned}\right\} \quad (18.4)$$

Page 274.

Following A. Gonor, let us pass in system of equations (18.1) to

independent alternating/variable Φ and Ψ, where the function of current Ψ satisfies the equation

$$v\Psi_\theta + w \operatorname{cosec} \theta \Psi_\Phi = 0. \qquad (18.5)$$

Taking into account that $\Psi = \Psi[\theta(\Psi, \Phi), \Phi]$,

$$\Psi_\theta = \frac{1}{\theta_\Psi}, \quad \Psi_\Phi = -\frac{\theta_\Phi}{\theta_\Psi},$$

we will obtain formulas for transition to the new variables:

$$\left.\frac{\partial f(\Phi, \Psi)}{\partial \Phi}\right|_{\theta=\text{const}} = \frac{\partial f}{\partial \Phi} - \frac{\theta_\Phi}{\theta_\Psi}\frac{\partial f}{\partial \Psi} = f_\Phi - \frac{\theta_\Phi}{\theta_\Psi} f_\Psi,$$
$$\frac{\partial f(\Phi, \Psi)}{\partial \theta} = \frac{1}{\theta_\Psi}\frac{\partial f}{\partial \Psi} = \frac{1}{\theta_\Psi} f_\Psi.$$

After the replacement of alternating/variable and some transformations, the system of equations (18.1) will take the form

$$\left.\begin{array}{ll}
[\ln(\rho w \theta_\Psi)]_\Phi + 2\frac{u}{w}\sin\theta = 0, & (\text{а}) \\
w \operatorname{cosec} \theta u_\Phi - v^2 - w^2 = 0, & (\text{б}) \\
\frac{u^2+v^2+w^2}{2V_1^2} + \frac{\gamma}{\gamma-1}\frac{p}{\rho V_1^2} = \frac{1+2\beta}{2}, & (\text{в}) \\
(p\rho^{-\gamma})_\Phi = 0, & (\text{г}) \\
w \operatorname{cosec} \theta v_\Phi + uv - w^2 \operatorname{ctg}\theta = \frac{1}{\rho\theta_\Psi} p_\Psi, & (\text{д}) \\
w \operatorname{cosec} \theta \theta_\Phi = v, & (\text{е}) \\
\beta = [M_1^2(\gamma-1)]^{-1}. &
\end{array}\right\} \quad (18.6)$$

Equation (18.6e) is the consequence of equation (18.5) and serves for determining unknown function $\theta = (\Phi, \Psi)$.

as can be seen from relationship/ratios (18.3), density behind shock wave ρ^+ is of the order $\rho^- \frac{\gamma+1}{\gamma-1}$, i.e. it increases unlimitedly when $\gamma \to 1, M_1^2(\gamma-1) = \text{const}$ and is very great at values γ, close to unity.

Page 275.

The flow of gas under these conditions occurs in thin shock layer, which follows from the preservation/retention/maintaining of the mass flow rate through any tube of flow. Estimating with the aid of formulas (18.3) order of magnitude behind shock wave, we will obtain that the unknown functions one should search for in the form of the series

$$\left.\begin{aligned} &u = u_{(0)} + \sigma u_{(1)} + \ldots, \quad \rho = \frac{1}{\sigma}\rho_{(0)} + \rho_{(1)} + \ldots, \\ &v = \sigma v_{(0)} + \sigma^2 v_{(1)} + \ldots, \quad \theta = \varepsilon + \sigma\theta_{(0)} + \sigma^2\theta_{(1)} + \ldots, \\ &w = w_{(0)} + \sigma w_{(1)} + \ldots, \quad \sigma = \frac{\gamma-1}{\gamma+1}, \\ &p = p_{(0)} + \sigma p_{(1)} + \ldots, \end{aligned}\right\} \quad (18.7)$$

where the function f_k they depend on Φ and Ψ.

Subsequently for simplification in the recording of the formulas of function $f_{(0)}$ we will designate by symbol f_0.

Substituting expansions (18.7) in system (18.6) and assuming that

$$\beta = \frac{1}{(\gamma - 1) M_1^2} = \text{const},$$

we will obtain system of equations for functions with index zero in the form

$$\left.\begin{array}{ll} [\ln(\rho_0 w_0 \theta_{0\Psi})]_\Phi + 2\frac{u_0}{w_0}\sin\varepsilon = 0, & \text{(a)} \\ w_0 = \operatorname{cosec}\varepsilon\, u_{0\Phi}, & \text{(б)} \\ u_0^2 + w_0^2 = V_1^2(1 + 2\beta) - \frac{p_0}{\rho_0}, & \text{(в)} \\ \left(\frac{p_0}{\rho_0}\right)_\Phi = 0, & \text{(г)} \\ w_0^2 \operatorname{ctg}\varepsilon = \frac{1}{\rho_0 \theta_{0\Psi}} p_{0\Psi}, & \text{(д)} \\ w_0 \theta_{0\Phi} = v_0 \sin\varepsilon. & \text{(е)} \end{array}\right\} \quad (18.8)$$

The equation of leading shock wave let us present in the form $\theta^* = \varepsilon + \sigma\theta_0^*(\Phi) + \ldots$ when $\sigma \to 0$ $\theta^* \to \varepsilon$, but in variables Φ and Ψ on shock wave $\Psi = \Psi^*(\Phi)$, since conical stream surfaces, on which Ψ retains

constant values, they enter in shock layer at different values Φ.

Page 276.

After the substitution of expansions (18.7) into relationship/ratios (18.3) we obtain (taking into account (18.4)) boundary conditions on shock wave in the form

$$\left.\begin{aligned}
&u_0^+ = u_0^-,\ v_0^+ = v_0^-\left[1 + 2\beta\left(\frac{V_1}{v_0^-}\right)^2\right] + w_0^- \frac{\theta_0^{*\prime}}{\sin\varepsilon},\\
&w_0^+ = w_0^-,\ p_0^+ = \rho_1 (v_0^-)^2,\\
&\rho_0^+ = \frac{\rho_1}{1 + 2\beta\left(\frac{V_1}{v_0^-}\right)^2},\\
&\beta = \frac{1}{(\gamma - 1) M_1^2} = \mathrm{const.}
\end{aligned}\right\} \qquad (18.9)$$

Here values $u_0^-,\ v_0^-,\ w_0^-$ are determined from the formulas (18.2), in which θ^* is replaced by ε. From relationship/ratios (18.9) $u_0^+,\ w_0^+,\ p_0^+,\ \rho_0^+$ are defined as functions Φ (not depending on the form of head shock wave). Only unknown previously value is $\theta_0^{*\prime}$, and, consequently, v_0^+.

Let us find now the solution of system of equations (18.8), that

satisfies conditions (18.9) on shock wave, i.e., when $\Psi = \Psi^*(\Phi)$, and to condition $v_0 = 0$ when $\theta = \varepsilon$. The function of current Ψ is determined by equation (18.5) ambiguously, since any function $\psi(\Psi)$ is also the solution to this equation and can be accepted as the new function of current. We will use this arbitrariness and let us assume that on shock wave $\Psi^* = \Phi$, i.e. the value of the function of current Ψ equal to value that angle Φ, with which this flow line enters in shock layer. [Then, in particular, $\theta_0^{*\prime} = \frac{d}{d\Phi}\theta_0(\Phi, \Psi = \Phi) = \theta_{0\Phi} + \theta_{0\Psi}$.] From equation (18.8d) it follows that

$$\frac{p_0}{\rho_0} = \frac{p_0^+}{\rho_0^+} = \chi_0(\Psi). \qquad (18.10)$$

Function $\chi_0(\Psi)$ according to (18.9) is determined from the formula

$$\chi_0(\Psi) = \frac{p_0^+}{\rho_0^+} = \frac{(v_0^-)^2}{1 + 2\beta (V_1/v_0^-)^2}, \qquad (18.11)$$

in which Φ necessary to replace by Ψ.

Page 277.

Equation (18.8c) will take now the form

$$u_0^2 + w_0^2 = \Delta_0^2(\Psi) = V_1^2(1 + 2\beta) - \chi_0(\Psi). \qquad (18.12)$$

Hence it follows that u_0, w_0 one should search for in the form

$$u_0 = \Delta_0(\Psi)\sin z, \qquad w_0 = \Delta_0(\Psi)\cos z. \qquad (18.13)$$

Substituting (18.13) into equation (18.8b), we will obtain that $z_\Phi = \sin\varepsilon$, whence we find

$$z = \sin\varepsilon\,\Phi + \alpha_0(\Psi).$$

Substituting z in (18.13), we obtain

$$\left.\begin{aligned} u_0 &= \Delta_0(\Psi)\sin[\sin\varepsilon\Phi + \alpha(\Psi)],\\ w_0 &= \Delta_0(\Psi)\cos[\sin\varepsilon\Phi + \alpha(\Psi)].\end{aligned}\right\} \quad (18.14)$$

Function $\alpha_0(\Psi)$ is determined according to (18.9) from the formula

$$\alpha_0(\Psi) = \left(\operatorname{arctg}\frac{u_0^+}{w_0^+} - \sin\varepsilon\,\Phi\right)_{\Phi=\Psi} = \left(\operatorname{arctg}\frac{u_0^-}{w_0^-} - \sin\varepsilon\,\Phi\right)_{\Phi=\Psi}. \quad (18.15)$$

Thus, functions $u_0, w_0, p_0/\rho_0$ are determined independent of other unknown values.

Substituting u_0, w_0 by formulas (18.14) in equation (18.8a), after obvious transformations we will obtain

$$[\ln(\rho_0\theta_{0\Psi})]_\Phi + \operatorname{tg}[\sin\varepsilon\,\Phi + \alpha_0(\Psi)]\sin\varepsilon = 0,$$

whence, integrating by Φ, we find

$$\frac{\rho_0\theta_{0\Psi}}{\cos[\sin\varepsilon\Phi + \alpha_0(\Psi)]} = \Omega_0(\Psi). \quad (18.16)$$

For determining function $\Omega_0(\Psi)$ we will use the equality, which occurs on shock wave, i.e., when $\Psi=\Phi$, $\theta_0^{*} = \theta_{0\Phi} + \theta_{0\Psi}$, from which with the aid of (18.9) and (18.8f), is obtained the formula

$$(\theta_{0\Psi})_{\Phi=\Psi} = \frac{\sin\varepsilon}{w_0^-}\left\{v_0^+ - v_0^-\left[1+2\beta\left(\frac{V_1}{v_0^-}\right)^2\right]\right\} - \frac{v_0^+\sin\varepsilon}{w_0^+} = -\left\{\sin\varepsilon\frac{v_0^-}{w_0^-}\left[1+2\beta\left(\frac{V_1}{v_0^-}\right)^2\right]\right\}_{\Phi=\Psi}. \qquad (18.17)$$

Page 278.

Then from formula (18.7), (18.9) it follows

$$\Omega_0(\Psi) = \left[\frac{\rho_0\theta_{0\Psi}}{\cos[\sin\varepsilon\,\Phi + \alpha_0(\Psi)]}\right]_{\Phi=\Psi} = -\rho_1\sin\varepsilon\left\{\frac{v_0^-}{w_0^-\cos[\sin\varepsilon\,\Phi+\alpha_0(\Psi)]}\right\}_{\Phi=\Psi}. \qquad (18.18)$$

Equation (18.8e) with the aid of formulas (18.16), (18.14), (18.18) is converted to the form

$$p_{\Psi} = w_0^2\Omega_0(\Psi)\cos[\sin\varepsilon\,\Phi + \alpha_0(\Psi)]\,\mathrm{ctg}\,\varepsilon = -\rho_1\cos\varepsilon w_0^2\,\frac{\cos[\sin\varepsilon\,\Phi+\alpha_0(\Psi)]}{\cos[\sin\varepsilon\,\Psi+\alpha_0(\Psi)]}\left(\frac{v_0^-}{w_0^-}\right)_{\Phi=\Psi} = -\rho_1\cos\varepsilon w_0^2\left[\frac{v_0^-}{(w_0^-)^2}\right]_{\Phi=\Psi}. \qquad (18.19)$$

Integrating equation (18.19) for Ψ from the surface of shock wave, let us find expression for p in field of flow in the form

$$p_0 = p_0^+ - \rho_1 \cos\varepsilon \int_{\Phi}^{\Psi} w_0^3 \left[\frac{v_0^-}{(w_0^-)^2}\right]_{\Phi=\Psi} d\Psi. \qquad (18.20)$$

First term in formula (18.20) depends only on Φ and corresponds to Newton's theory, second term considers centrifugal forces in shock layer.

From equation (18.16) with the aid of formulas (18.10), (18.18) we obtain the relationship/ratio

$$\theta_{0\Psi} = \Omega_0(\Psi)\chi_0(\Psi)p_0^{-1}\operatorname{sco}[\sin\varepsilon\Phi + \alpha_0(\Psi)] = = -\rho_1 \sin\varepsilon \frac{w_0}{p_0}\left\{\frac{(v_0^-)^2}{(w_0^-)^2\left[1+2\beta\left(\frac{V_1}{v_0^-}\right)^2\right]}\right\}_{\Phi=\Psi},$$

whence, after integration for Ψ from shock wave, we find

$$\theta_0 = \theta_0^*(\Phi) - \rho_1 \sin\varepsilon \int_{\Phi}^{\Psi} \frac{w_0}{p_0}\left\{\frac{(v_0^-)^2}{(w_0^-)^2[1+2\beta(V_1/v_0^-)^2]}\right\}_{\Phi=\Psi} d\Psi. \qquad (18.21)$$

Page 279.

Unknown function $\theta_0^*(\Phi)$ must be determined from the condition of the flow

$$\theta_0 = 0,\ v_0 = 0.$$

According to indisputably the correct diagram of A. Ferri's flow about, the surface of cone is the flow line, on which $\Psi = \pi$ (see Fig. 48). If expansions (18.7) represent the solution of problem in all shock layer, then, after assuming in formulas (18.20), (18.21) $\Psi = \pi$, we will obtain respectively p_0 and θ_0 on the surface of cone as functions Φ. However, this it is not possible to make, since integral in formula (18.20) diverges when $\Psi = \pi, \Phi \neq \pi$ and, therefore, formula (18.21) becomes meaningless.

[Actually, $w_0(\Phi, \pi) = \Delta_0(\pi) \sin[\sin \varepsilon(\pi - \Phi)] \neq 0$, if $\Phi \neq \pi$, but $(\bar{w_0})_{\Phi=\Psi} = -V_1 \sin\delta \sin\Psi \to 0$, $(\bar{v_0})_{\Phi=\Psi} \to -V_1 \cos(\varepsilon + \delta) \neq 0$, when $\Psi \to \pi$.] It hence immediately follows that expansions (18.7) do not represent the solution of problem in

vorticity layer near body surface. As we will see further, the thickness of vorticity layer is very small with small σ, therefore with high accuracy conditions $v_0 = \theta_0 = 0$ it is possible to satisfy on its outer edge where $\Psi = \Psi_b(\Phi)$ [Ψ_b — is certain function Φ, to be determined]. Substituting $\Psi = \Psi_b(\Phi)$ in formula (18.21) and equalizing the expression, which stands in right side, to zero, we determine $\theta_0^*(\Phi)$:

$$\theta_0^*(\Phi) = \rho_1 \sin \varepsilon \int_{\Phi}^{\Psi_b(\Phi)} \frac{w_0}{p_0} \left\{ \frac{(v_0^-)^2}{(w_0^-)^2 [1 + 2\beta (V_1/v_0^-)^2]} \right\}_{\Phi=\Psi} d\Psi, \tag{18.22}$$

after which to expression for θ_0 it is possible to give the form

$$\theta_0 = \rho_1 \sin \varepsilon \int_{\Psi}^{\Psi_b(\Phi)} \frac{w_0}{p_0} \left\{ \frac{(v_0^-)^2}{(w_0^-)^2 [1 + 2\beta (V_1/v_0^-)^2]} \right\}_{\Phi=\Psi} d\Psi. \tag{18.23}$$

Function $\Psi_b(\Phi)$ is located by the satisfaction of relationship/ratio (18.8?) when $\Psi = \Psi_b(\Phi)$, where we set/assume that $v_0 = 0$.

Page 280.

Since

$$(\theta_{0\Phi})_{\Psi=\Psi_b} = \rho_1 \sin \varepsilon \left[\frac{w_0}{p_0} \left\{ \frac{(v_0^-)^2}{(w_0^-)^2 [1 + 2\beta (V_1/v_0^-)^2]} \right\}_{\Phi=\Psi} \right]_{\Psi=\Psi_b} \times \\ \times \frac{d\Psi_b}{d\Phi},$$

that satisfaction of relationship/ratio (18.8f), is possible only on the condition that

$$(w_0)_{\Psi=\Psi_b} = 0. \qquad (18.24)$$

From (18.24), (18.14) then we obtain equation for $\Psi_b(\Phi)$: $\sin\varepsilon\,\Phi + \alpha_0(\Psi_b) = \frac{\pi}{2}$, which according to (18.15) can be presented in the form

$$\Phi = \Psi_b + \frac{1}{\sin\varepsilon}\operatorname{arctg}\left[\left(\frac{w_0^-}{u_0^-}\right)_{\Phi=\Psi_b}\right]. \qquad (18.25)$$

Thus, the solution of problem of outside vorticity layer is obtained in quadratures. Since the vorticity layer is fine, pressure is changed across layer very little, and formula (18.20) when $\Psi = \Psi_b(\Phi)$ is determined p_0 on the surface of cone.

For pressure coefficient on the surface of cone, is obtained formula [168]

$$C_p = \frac{p_0}{\rho_1 V_1^2/2} = 2\{(\cos\delta\sin\varepsilon - \sin\delta\cos\varepsilon\cos\Phi)^2 - \\ -\cos\varepsilon\int\limits_{\Phi}^{\Psi_b}\left[1+\left(\frac{\cos\delta\cos\varepsilon+\sin\delta\sin\varepsilon\cos\Psi}{\sin\delta\sin\Psi}\right)^2\right]^{3/2}\sin\delta\sin\Psi\times \\ \times\cos^3\left[\sin\varepsilon(\Phi-\Psi) - \operatorname{arctg}\left(\frac{\cos\delta\cos\varepsilon+\sin\delta\sin\varepsilon\cos\Psi}{\sin\delta\sin\Psi}\right)\right]\times \\ \times(\cos\delta\sin\varepsilon - \sin\delta\cos\varepsilon\cos\Psi)\,d\Psi\}, \qquad (18.26)$$

which it predicts sufficiently well distribution C_p on body; see [168]. The position of leading shock wave is also determined well by the solution of ambition; see [211, 212].

Page 281.

If we disregard thickness of vorticity layer, i.e., to consider that surface of cone corresponds the value $\Psi = \Psi_b(\Phi)$, that with the aid of formula (18.23) it is possible to establish/install [168] that the flow lines $(\Psi = \text{const})$ concern the surface of cone, so that last/latter it is their envelope (what in actuality no) (Fig. 86).

The values of the radial component of speed u_0 and of density ρ_0 when $\Psi = \Psi_0(\Phi)$ also cannot be identified with the values of these quantities on the surface of cone. This fact it is easy to establish/install, since values u, w on the surface of cone can be found with the aid of simple calculations [207].

Let us present $\theta^*(\pi)$ in the form

$$\left.\begin{aligned}
&\theta^*(\pi) = \varepsilon + \sigma\vartheta^* + \ldots,\\
&\vartheta^* = \theta_0^*(\pi) = \frac{\sin^3\varepsilon\cos(\varepsilon+\delta)\,[\sin^2(\delta+\varepsilon)+2\beta]}{\sin^2\delta\sin(\delta+\varepsilon)}\times\\
&\times\left\{\left[1+\frac{\sin\delta}{\sin\varepsilon\cos(\delta+\varepsilon)}\right]\ln\left[1+\frac{\sin\delta}{\sin\varepsilon\cos(\delta+\varepsilon)}\right]-\right.\\
&\qquad\left.-\frac{\sin\delta}{\sin\varepsilon\cos(\delta+\varepsilon)}\right\}.
\end{aligned}\right\} \quad (18.27)$$

[Formula (18.27) is obtained from formula (18.22) by passage to the limit when $\Phi = \pi$ with the aid of elementary, but cumbersome calculations.]

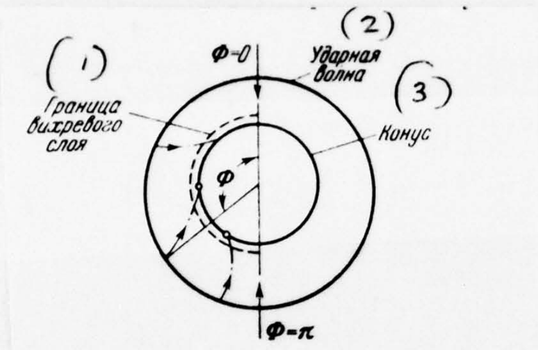


Fig. 86.

Key: (1). Boundary of vorticity layer. (2). Shock wave. (3). Cone.

Page 282.

If we write the conditions on shock wave (18.3) when $\Phi = \pi$, to consider that $p\rho^{-\gamma} = \mathrm{const}$ on the flow line in question and to exclude ρ from equation (18.1c), then when $\theta = \varepsilon$ we will obtain the relationship/ratio

$$u^2 + w^2 + \frac{2\gamma}{\gamma+1}\left(\frac{p}{\rho_1}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{V_1^2}{\gamma}\right)^{\frac{1}{\gamma}}\left\{\frac{2\gamma}{\gamma+1}\sin^2[\delta+\theta^*(\pi)] - \sigma^2\beta(\gamma+1)\right\}^{\frac{1}{\gamma}}\left\{1+\frac{2\beta}{\sin^2[\delta+\theta^*(\pi)]}\right\} = (1+2\beta)V_1^2,$$

$$\left(\beta = \frac{1}{M_1^2(\gamma-1)},\ \sigma = \frac{\gamma-1}{\gamma+1}\right). \qquad (18.28)$$

Equation (18.1b) when $\theta = \varepsilon$ (taking into account the fact that here $v = 0,\ w \neq 0$), is record/written in the form

$$w = \frac{1}{\sin \varepsilon} u_{\Phi}. \qquad (18.29)$$

Let us assume now that on the very surface of cone u, w, p it is possible to present in the form

$$\left.\begin{array}{l} u = u_0^{\times} + \sigma u_1^{\times} + \ldots, w = w_0^{\times} + \sigma w_1^{\times} + \ldots, \\ p = p_0^{\times} + \sigma p_1^{\times} + \ldots \end{array}\right\} \qquad (18.30)$$

Substituting (18.30) and $\theta^* (\pi) = \varepsilon + \sigma\vartheta^* + \ldots$ in (18.28), decompose/expanding result according to degrees σ and equalizing the coefficients with identical degrees σ, we obtain

$$u_0^{\times 2} + w_0^{\times 2} = V_1^2 \cos^2 (\delta + \varepsilon), \qquad (18.31)$$

$$\left.\begin{array}{l} w_0^{\times} w_1^{\times} + u_0^{\times} u_1^{\times} + V_1^2 \{\vartheta^* \cos (\delta + \varepsilon) \sin (\delta + \varepsilon) + \\ \quad + [\sin^2 (\delta + \varepsilon) + 2\beta] \ln \dfrac{p_0^{\times}}{\rho_1 V_1^2 \sin^2 (\delta + \varepsilon)} \end{array}\right\} = 0. \qquad (18.32)$$

Substituting (18.30) in (18.29), analogously we obtain

$$w_0^{\times} = \frac{1}{\sin\varepsilon}\, u_{0\Phi}^{\times}, \qquad w_1^{\times} = \frac{1}{\sin\varepsilon}\, u_{1\Phi}^{\times}. \qquad (18.33)$$

Substituting (18.33) in (18.31), we find differential equation for $u_0^{\times}$:

$$u^{\times 2} + \frac{1}{\sin^2\varepsilon}\left(\frac{du_0^{\times}}{d\Phi}\right)^2 = V_1^2\cos^2(\delta+\varepsilon). \qquad (18.34)$$

Page 283.

By unique solution of equation (18.34), to the satisfying conditions of the symmetry of the flow

$$w_0^{\times} = \frac{1}{\sin\varepsilon}\, u_{0\Phi}^{\times} = 0 \quad \overset{(1)}{\text{при}} \quad \Phi = 0, \pi,$$

Key: (1). with

it is

$$\left.\begin{array}{l} u_0^{\times} = V_1\cos(\delta+\varepsilon) = \text{const}, \\ w_0^{\times} \equiv 0. \end{array}\right\} \qquad (18.35)$$

From relationship/ratios (18.32), (18.33), (18.35) then we obtain

$$\left.\begin{aligned} u_1^{\times} &= V_1\left[-\vartheta^*\sin(\delta+\varepsilon)+\frac{\sin^2(\varepsilon+\delta)+2\beta}{\cos(\delta+\varepsilon)}\times\right.\\ &\qquad\left.\times\ln\frac{\rho_1 V_1^2\sin^2(\delta+\varepsilon)}{p_0^{\times}}\right],\\ w_1^{\times} &= -V_1\frac{\sin^2(\delta+\varepsilon)+2\beta}{\sin\varepsilon\cos(\delta+\varepsilon)}\frac{1}{p^{\times}}\frac{dp_0^{\times}}{d\Phi}.\end{aligned}\right\} \quad (18.36)$$

Formulas (18.30), (18.35), (18.36) are determined u and w on the surface of cone with an accuracy to values $O(\sigma)$ inclusively, if known pressure p on cone with an accuracy to values $O(1)$ (values $p_0^{\times}$ are located from the solution of A. Gonor).

Since on the outer edge of vorticity layer according to (18.14)

$$u_0 = u_0(\Phi) = \Delta_0[\Psi_b(\Phi)] \neq \text{const}, \qquad w_0 = 0$$

[specifically, $u_0(0)=V_1\cos(\varepsilon-\delta),\ u_0(\pi)=V_1\cos(\varepsilon+\delta)$], that u substantially changes within vorticity layer, and $w = O(\sigma)$, i.e., is really/actually here low value.

value.

18.3. Solution of problem of flow about round cone taking into account vorticity layer. Let us now, solving the problem of cone, to take into consideration vorticity layer in that measure which is necessary to evaluate its thickness and confirmation of that fact that the solution of A. Gonor is correct outside vorticity layer and correctly determines pressure on body.

Since $v = O(\sigma)$ in shock layer, equation (18.6b) can be written in the form

$$w\,(u_\Phi - \sin\theta \cdot w) = O\,(\sigma^2). \qquad (18.37)$$

Page 284.

Let us present p and ρ as follows:

$$\left.\begin{aligned} p &= p_{(0)} + \sigma p_{(1)} + O(\sigma^2), \\ \rho &= \frac{1}{\sigma}\left[\rho_{(0)} + \sigma\rho_{(1)} + O\,(\sigma^2)\right]. \end{aligned}\right\} \qquad (18.38)$$

Here values $p_{(k)}, \rho_{(k)}$ are of the order 1, but, unlike expansions (18.7), they can depend on σ.

Equation (18.6d) expresses the constancy of entropy along flow lines, and its solution he is record/written in the form

$$p\rho^{-\gamma} = p^+ (\rho^+)^{-\gamma} = \chi(\Psi). \qquad (18.39)$$

Here the superscript "+ "means that the function is calculated at the intersection of shock wave from flow lines $\Psi = \text{const}$. Eliminating ρ with the aid of (18.39) from equation (18.6c), substituting here (18.38), as a result we obtain

$$u^2 + w^2 + \frac{p^+_{(0)}}{\rho^+_{(0)}} \left[1 + \sigma \left(1 + 2 \ln \frac{p_{(0)}}{\rho^+_{(0)}} + \frac{p^+_{(1)}}{p^+_{(0)}} - \frac{\rho^+_{(1)}}{\rho^+_{(1)}} \right) \right] + \\ + O(\sigma^2) = V_1^2 (1 + 2\beta). \qquad (18.40)$$

Let us introduce the designations:

$$\left.\begin{aligned} & V_1^2 (1 + 2\beta) - \frac{p^+_{(0)}}{\rho^+_{(0)}} = \Delta_0^2 (\Psi), \\ & \frac{p^+_{(0)}}{\rho^+_{(0)}} \left(1 + 2 \ln \frac{p_{(0)}}{\rho^+_{(0)}} + \frac{p^+_{(1)}}{p^+_{(0)}} - \frac{\rho^+_{(1)}}{\rho^+_{(0)}} \right) = -2\Delta_0 (\Psi) G(\Phi, \Psi), \end{aligned}\right\} (18.41)$$

then relationship/ratio (18.40) of signs the form

$$u^2 + w^2 = \Delta_0^2 (\Psi) + 2\sigma \Delta_0 (\Psi) G(\Phi, \Psi) + O(\sigma^2). \qquad (18.42)$$

From (18.42) it follows that

$$\left.\begin{aligned} u &= [\Delta_0 (\Psi) + \sigma G(\Phi, \Psi)] \sin z (\Phi, \Psi, \sigma) + O(\sigma)^2, \\ w &= [\Delta_0 (\Psi) + \sigma G(\Phi, \Psi)] \cos z (\Phi, \Psi, \sigma) + O(\sigma^2). \end{aligned}\right\} \qquad (18.43)$$

Substituting expressions (18.43) in equation (18.37), we obtain equation for function z (Φ, Ψ, σ)

$$[(\Delta_0 + \sigma G)\cos z + O(\sigma^2)]\{\sigma G_\Phi \sin z + (\Delta_0 + \sigma G)\cos z\,(z_\Phi - \sin\theta)\} + O(\sigma^2) = 0, \quad (18.44)$$

where

$$G_\Phi = \frac{\Delta_0^2 - V_1^2(1+2\beta)}{\Delta_0}\frac{[p_{(0)}]_\Phi}{p_{(0)}} \text{ [see (18.41)]}.$$

Page 285.

In the field where values z differ from $\frac{\pi}{2}$ by finite quantities, the solution to equation (18.44) it is possible to search for as expansion according to degrees σ

$$z = z_0 + \sigma z_1 + \ldots \quad (18.45)$$

Substituting (18.45), and also $\theta = \varepsilon + \sigma\theta_{(0)} + \ldots$, in (18.44),

equalizing zero-order terms on σ, we will obtain equation for z_0 in the form

$$z_{0\Phi} - \sin \varepsilon = 0.$$

Its solution, $z_0 = \sin \varepsilon\Phi + \alpha_0(\Psi)$, coincides with previously obtained expression for z. Let us examine now the solution to equation (18.44) in the field where z is close to $\pi/2$. Let us present z in the form $z = (\pi/2) - \sigma y$. Then $\cos z = \sigma y + O(\sigma^3)$; $\sin z = 1 + O(\sigma^2)$, and equation (18.44) after reduction on σ^2 will take the form

$$\begin{aligned}&[(\Delta_0 + \sigma G)\, y + O(\sigma)]\, \{G_\Phi[1 + O(\sigma^2)] - (\Delta_0 + \sigma G)\times\\ &\times[y + O(\sigma^2)]\,[\sin \varepsilon + \sigma y_\Phi + O(\sigma)]\} + O(\sigma) = 0. \quad (18.46)\end{aligned}$$

If we exclude the vicinities of the plane of the symmetry of the flow where $\Phi = 0$, π, a $G_\Phi = \infty$, that dominant term for y is determined without integration by means of the equating of zero-order members on σ in equation (18.46):

$$y = \frac{G_\Phi}{\sin \varepsilon \Delta_0} + O(\sigma) = \frac{\Delta_0^2 - V_1^2(1+2\beta)}{\Delta_0^2 \sin \varepsilon} \frac{[p_{(0)}]_\Phi}{p_{(0)}} + O(\sigma). \quad (18.47)$$

Then in the field in question, according to equations (18.43) we obtain

$$\left.\begin{aligned}u &= \Delta_0(\Psi) + \sigma G(\Phi, \Psi) + O(\sigma^2),\\ w &= \sigma \frac{\Delta_0^2(\Psi) - V_1^2(1+2\beta)}{\Delta_0(\Psi)\sin \varepsilon} \frac{[p_{(0)}]_\Phi}{p_{(0)}} + O(\sigma^2).\end{aligned}\right\} \quad (18.48)$$

Page 286.

[From formula (18.47) when $\Psi = \pi$ follow previously obtained values u, w on the surface of cone; see (18.35), (18.36)]. If we are interested by the dominant terms of solution in shock layer, then from formulas (18.43), we obtain, that for u, w they are determined in the form

$$\left.\begin{array}{l} u = \Delta_0(\Psi)\sin z_0 + O(\sigma), \\ w = \Delta_1(\Psi)\cos z_0 + O(\sigma\cos z_0), \end{array}\right\} \quad (18.49)$$

where function z_0 satisfies the equation

$$\frac{1}{\Delta_0}\sigma G_\Phi + \left(\frac{\pi}{2} - z_0\right)(z_{0\Phi} - \sin\varepsilon) = 0 \quad (18.50)$$

and the boundary condition

$$\Phi = \Psi, \quad z_0 = \sin\varepsilon\,\Psi + \alpha_0(\Psi). \quad (18.51)$$

Equation (18.50) [or equation (18.44)] can be solved with the aid of the method of PLG, but we by this be occupied will not be, but let us focus attention on the fact that from equation (18.50) it follows that in the range where

$$z_1 = \frac{\pi}{2} + O(\sigma^m), m < 1,$$
$$z_{0\Phi} - \sin\varepsilon = o(1).$$

This means that the solution of A. Gonor is correct outside the range where according to (18.49) w = O(σ).

Now it is not difficult to rate/estimate the thickness of

vorticity layer and change in the pressure across it.

Let us integrate equation (18.6a) for Φ from shock wave where $\Phi = \Psi$) and result let us present in the form

$$\rho\theta_\Psi = \rho^+ \theta_\Psi^+ \frac{w^+}{w} \exp\left[\int_\Phi^\Psi \frac{u \sin\theta}{w} d\Phi\right]. \quad (18.52)$$

In field of flow on any flow line $\Psi = \text{const}$ when $0 < \Phi < \pi$ are fulfilled the inequalities $\Phi < \Psi$ and $w < 0$. When we are moved along flow line $\Psi = \text{const}$ from shock wave and fall into vorticity layer, then w becomes the value of order $O(\sigma)$.

Page 287.

Then from formula (18.52) we obtain, that here

$$\rho\theta_\Psi = O\left[\frac{1}{\sigma} \exp\left(-\frac{c}{\sigma}\right)\right], \quad c > 0, \quad (18.53)$$
$$\theta_\Psi = O\left[\exp\left(-\frac{c}{\sigma}\right)\right],$$
$$\theta = \int_\pi^{\Psi} \theta_\Psi d\Psi = O\left[\exp\left(-\frac{c}{\sigma}\right)\right]. \quad (18.54)$$

From equations (18.6d) (18.53) it follows that in vorticity layer $p_\Psi = O[\exp(-\frac{c}{\sigma})]$, and the pressure across vorticity layer also changes by value O [exp(-c/б)], since the change Ψ in vorticity layer is value O(1).

The given estimations for a vorticity layer are valid outside

some vicinities of the stagnation points on the single sphere where $w = v = 0$, and bear asymptotic character.

18.4. Comparison of theoretical and experimental results. The calculation of the second approach/approximation, i.e., the determination of coefficients of $p_{(1)}$, $\theta_{(1)}$ so forth in expansions (18.7), is laborious operation. However, A. Gonor [213-214] knew how to find formula for $p_{(1)}$ on the surface of cone for $\Phi = 0, \pi$. The formula from which is calculated the correction for coefficient of pressure C_p in meridian planes $\Phi = 0$, $\Phi = \pi$ on the surface of cone, takes the form

$$\Delta C_p = \left[\frac{2p_{(1)}\sigma}{\rho_1 V_1^2}\right]_{\Phi=\pi,0} = \frac{\gamma-1}{\gamma+1}\left[2\left\{2\sin^2(\varepsilon\pm\delta)\,t^{-2}\times\right.\right.$$
$$\times[(1+t)\ln(1+t)-t]\left[1+\frac{2}{(\gamma-1)M_1^2\sin^2(\varepsilon\pm\delta)}\right]-$$
$$\left.-\sin^2(\varepsilon\pm\delta)-\frac{1}{M_1^2}\right\}+2t^{-4}\frac{\sin^2(\varepsilon\pm\delta)}{1+t}\left\{t^4+3t^2(1+t)\times\right.$$
$$\times[\ln(1+t)-t]+3t(1+t^2)\left[t+\frac{t}{1+t}-2\ln(1+t)\right]-$$
$$\left.\left.-(1+t)^3\left[t+\frac{2t}{1+t}\left(1+\frac{t}{4(1+t)}\right)-3\ln(1+t)\right]\right\}\right], \quad (18.55)$$

where

$$t=\pm\frac{\sin\delta}{\sin\varepsilon\cos(\varepsilon\pm\delta)}.$$

Page 288.

Sign "+" is related to the windward face of cone ($\Phi = \pi$), sign "-" to shadow side ($\Phi = 0$). Figure 87, a, b, c [213-214], gives the comparison of theoretical and experimental results with $M_1 = 4$. By

solid lines are shown the theoretical curves, designed by formula (18.26). Broken lines are carried out taking into account values of the correction of the second approach/approximation with $\Phi = 0, \pi$ and linear interpolation of the correction between these values for intermediate angles of Φ. Experimental data are shown by marks ⊙, ⊡.

Curves in Fig. 87a, b, c are constructed for values ε, equal to with respect 20°, 25°, 30°.

From curve/graphs it is evident that in windward part of the surface of the cone, up to angles $\Phi \approx 90°$, average experimental values do not differ from theoretical data first approximation more than by 5-6o/o. In the shadow side of surface, the experimental data will move away from the theoretical. However, taking the correction into account of the second approach/approximation, coincidence everywhere distinct.

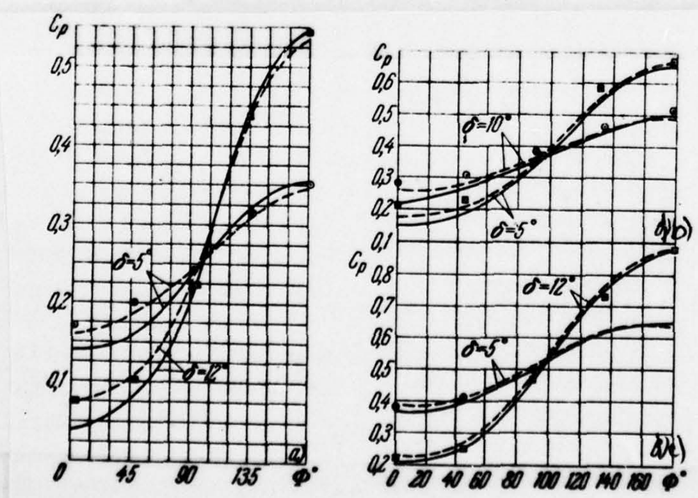


Fig 87.

Page 289.

It must be noted that than more ε, the lesser the disagreement of first approximation with the results of experiment that and understandably since good results for an entire surface of cone it is possible to expect with $M^2_1 \sin^2 (\varepsilon \pm \delta) \gg 1$.

The shock position in the plane of the symmetry of flow can be easily determined by shadow method, and therefore is of interest to compare theoretical and experimental results for $\theta^*(0)$ and $\theta^*(\pi)$. If we designate $\theta^*(0) = \theta^*_-$, $\theta^*(\pi) = \theta^*_+$, then for these angles occurs formula [211]

$$\left.\begin{aligned} \theta^*_\pm &= \varepsilon + \frac{\gamma-1}{\gamma+1}\frac{\operatorname{tg}(\varepsilon\pm\delta)}{t^2}\left[(1+t)\ln(1+t)-t\right]\times \\ &\qquad\times\left[1+\frac{}{(\gamma-1)M_1^2\sin^2(\varepsilon\pm\delta)}\right], \\ t &= \pm\frac{\sin\delta}{\sin\varepsilon\cos(\varepsilon\pm\delta)}. \end{aligned}\right\} \quad (18.56)$$

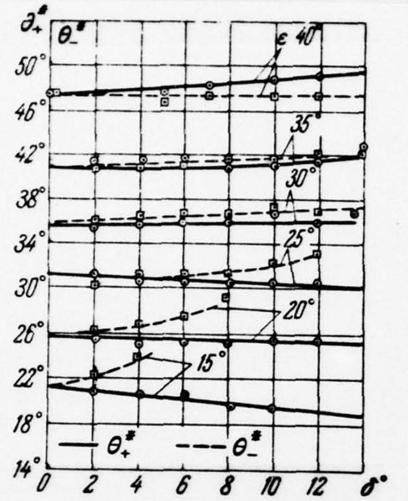


Fig. 88.

Page 290.

In Fig. 88, taken from work [211], solid line plotted/applied curve/graph θ^*_+ for γ = 1.4, by the broken line of curve/graphs θ^*_-; experimental points with M_1 = 4 for the windward face of cone are designated in sign ⊙, for shadow - by mark ⊡.

As is evident Fig. 88, the coincidence of theoretical and experimental data good. Certain refinement of formula (18.56) is obtained in work [212].

§19. Method of shock layer in the general case.

19.1. Lead-in observations. In §19 will be examined the method of shock layer in connection with conical bodies with the cross section of "arbitrary" form. The duct of the cross section of body by plane z = 1 is assumed to be that having continuously variable chamber (exception/elimination they can compose leading edges of body), gas - by that having equation of state, for example, i = i (p, S), where i is the specific enthalpy $\left(\frac{1}{\rho}=\frac{\partial i}{\partial p}\Big|_S\right)$.

So as into §18, us they will interest dominant terms for the parameters of gas in shock layer.

Role σ ($\sigma = \gamma - 1/\gamma + 1$ for an ideal gas) will here play the density ratio in undisturbed flow, ρ_1, to characteristic density in shock layer, $\rho^{\times}$, i.e.

then in the shock layer $\sigma=\frac{\rho_1}{\rho^{\times}}=\text{const};$

$$\rho=\frac{1}{\sigma}\rho_{(0)}+\rho_{(1)}+\ldots, \tag{19.1}$$

where $\rho_{(0)}$, $\rho_{(1)}$, ... - the value of the order of one.

Problem let us solve in the system of the generalized spherical coordinates R, θ, Φ, depicted on Fig. 89, and we will use the equations of motion of gas (1.21)-(1.25).

Since the vorticity layer at hypersonic speeds is very fine, solution outside it determines pressure on body surface and gives boundary conditions for the calculation of viscous boundary layer on body. (Thickness of vorticity layer is less than the boundary layer thickness under normal conditions in hypersonic flight).

Page 291.

The set-forth further solution belongs to A. Gonor [7]. (in basic work [7] is examined the case of ideal gas, but generalization to the case of the inadequate gas is conducted by elementary form: see [215]) . Solution of problem for the inadequate gas taking into account vorticity layer is given in the works of R. Mel'nik and R. Sheing [206] and Ya. Sapunkov [205, 209]. There it is possible to find the information about the different anomalies, which appear because of the degree of approximation of the solution of problem: see also [162].

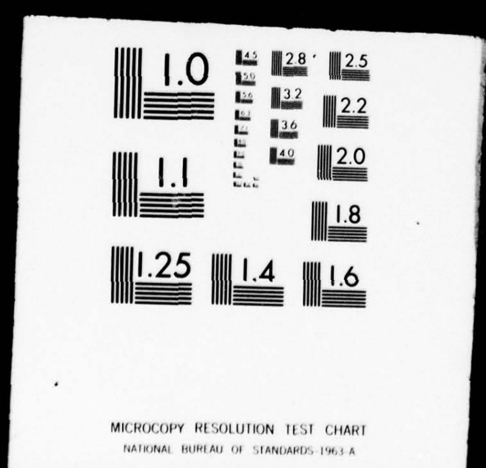
19.2. Solution of problem of dominant terms in shock layer. let us examine the conical body, streamlined at an angle of attack δ with

AD-A066 205 FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO F/G 20/4
NONLINEAR CONICAL FLOWS OF GAS.(U)
MAR 78 B M BULAKH
UNCLASSIFIED FTD-ID(RS)T-0266-78 NL

6 OF 7
AD
A066205

6 OF 7

AD A066205



the hypersonic uniform flow of gas, which has speed V_1, Mach number M_1, the density of gas ρ_1 and of so forth (see Fig. 89). In the system of the generalized orthogonal spherical coordinates R, θ, Φ of the equation of motion of gas (1.21)-(1.25) they are record/written in this form:

$$\left.\begin{aligned}
&2\rho u + \frac{v}{A_1}\rho_\theta + \frac{w}{A_2}\rho_\Phi + \frac{\rho}{A_1A_2}[(vA_2)_\theta + (wA_1)_\Phi] = 0, && \text{(а)}\\
&\frac{v}{A_1}u_\theta + \frac{w}{A_2}u_\Phi - v^2 - w^2 = 0, && \text{(б)}\\
&\frac{v}{A_1}\left(i + \frac{u^2+v^2+w^2}{2}\right)_\theta + \frac{w}{A_2}\left(i + \frac{u^2+v^2+w^2}{2}\right)_\Phi = 0, && \text{(в)}\\
&\frac{v}{A_1}S_\theta + \frac{w}{A_2}S_\Phi = 0, && \text{(г)}\\
&\frac{v}{A_1}v_\theta + \frac{w}{A_2}v_\Phi + uv + vw\frac{(\ln A_1)_\Phi}{A_2} - \\
&\qquad - w^2\frac{(\ln A_2)_\theta}{A_1} = -\frac{1}{\rho A_1}p_\theta, && \text{(д)}\\
&i = i(p, S), \quad \frac{1}{\rho} = \frac{\partial i}{\partial p}\Big|_S. && \text{(е)}
\end{aligned}\right\} \quad (19.2)$$

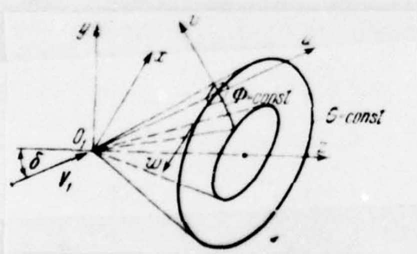


Fig. 89.

Page 292.

(Recall that here u, v, w are components of vector of the speed in

the direction of an increase with respect to R, θ, Φ; S, i - specific enthropy and enthalpy; p, ρ - pressure and density; A_1, A_2 - the coefficient of the Lames, who are known functions θ and Φ.)

We convert system of equations (19.2) to independent alternating/variable Φ, Ψ, where the function of current Ψ. satisfies the equation

$$\frac{v}{A_1}\Psi_\theta + \frac{w}{A_2}\Psi_\Phi = 0. \tag{19.3}$$

As a result we will obtain following system of equations:

$$\left.\begin{aligned}
&\left[\ln(\rho A_1 w \theta_\Psi)\right]_\Phi + 2\frac{u}{w}A_2 = 0, && \text{(a)}\\
&\frac{w}{A_2}u_\Phi - v^2 - w^2 = 0, && \text{(b)}\\
&\left(i + \frac{u^2+v^2+w^2}{2}\right)_\Phi = 0, && \text{(c)}\\
&i_\Phi - \frac{1}{\rho}p_\Phi = 0, && \text{(d)}\\
&\frac{w}{A_2}v_\Phi + uv + vw\,\frac{[(\ln A_1)_\Phi]_{\theta=\mathrm{const}}}{A_2} - \\
&- w^2\frac{(\ln A_2)_\theta}{A_1} = -\frac{1}{\rho A_1 \theta_\Psi}\cdot p_\Psi, && \text{(e)}\\
&\rho = \rho(p, i), && \text{(f)}\\
&\frac{w}{A_2}\theta_\Phi = \frac{v}{A_1} && \text{(g)}
\end{aligned}\right\} \tag{19.4}$$

[When deriving the equation (19.4d) we used the known equality T dS = di - dp/ρ, where T is absolute temperature]. On body surface $\theta = \varepsilon = \mathrm{const}$, v = 0. Let us assign the surface of leading shock wave by equation $\theta = \theta^*(\Phi)$ ($\Psi = \Phi$ in alternating/variable Φ and Ψ) let us designate by f^- and f^+ of the value of function f directly to shock wave.

Page 293.

Then for an undisturbed flow they occur of the equality

$$\left.\begin{aligned} u^- &= V_1 \frac{(\sin\delta\, R_y + \cos\delta\, R_z)}{|\operatorname{grad} R|}\bigg|_{\theta=\theta^*}, \\ v^- &= V_1 \frac{(\sin\delta\, \theta_y + \cos\delta\, \theta_z)}{|\operatorname{grad} \theta|}\bigg|_{\theta=\theta^*}, \\ w &= V_1 \frac{(\sin\delta\, \Phi_y + \cos\delta\, \Phi_z)}{|\operatorname{grad} \Phi|}\bigg|_{\theta=\theta^*}, \end{aligned}\right\} \quad (19.5)$$

which are valid, if vector V_1 lie/rests at plane yO_1z and it composes angle δ with axis O_1z. The conditions of consistency on shock wave let us write in the form

$$\left.\begin{aligned} & V_n^+\rho^+ = V_n^-\rho^-, \quad V_{\tau_1}^+ = V_{\tau_1}^-, \quad V_{\tau_2}^+ = V_{\tau_2}^-, \\ & p^+ + \rho^+(V_n^+)^2 = p^- + \rho^-(V_n^-)^2, \\ & \frac{(u^+)^2 + (v^+)^2 + (w^+)^2}{2} + i^+ = \frac{V_1^2}{2} + i_1, \\ & (\rho^- = \rho_1, \quad p^- = p_1), \end{aligned}\right\} \quad (19.6)$$

where the value with indices n, τ_1, τ_2 represent with respect to the projection of velocity vector on standard even two mutually perpendicular tangents to the surface of shock wave; they are located through the formulas:

$$\left.\begin{aligned} & V_n = [A_2^2 + A_1^2(\theta^{*\prime})^2]^{-1/2}(vA_2 - A_1\theta^{*\prime}w), \\ & V_{\tau_1} = [A_2^2 + A_1^2(\theta^{*\prime})^2]^{-1/2}(vA_1\theta^{*\prime} + wA_2), \\ & V_{\tau_2} = u, \quad \theta^{*\prime} = \frac{d\theta^*}{d\Phi}, \quad \theta = \theta^*. \end{aligned}\right\} \quad (19.7)$$

Let us search for the solution of problem in the form

$$\left.\begin{aligned} &u = u_0 + O(\sigma), \quad \rho = \frac{1}{\sigma}\rho_0 + O(1), \\ &v = \sigma v_0 + O(\sigma^2), \quad i = i_0 + O(\sigma), \\ &w = w_0 + O(\sigma), \quad \theta = \varepsilon + \sigma\theta_0 + O(\sigma^2), \\ &p = p_0 + O(\sigma); \quad \sigma = \frac{\rho_1}{\rho^\times} = \text{const}. \end{aligned}\right\} \tag{19.8}$$

Substituting expansions (19.8) in system (19.4), we will obtain system of equations for the dominant terms of the solution

$$\left.\begin{aligned} &\left[\ln(\rho_0 A_1 w_0 \theta_{0\Psi})\right]_\Phi + 2\frac{u_0}{w_0}A_2 = 0, && \text{(a)} \\ &u_{0\Phi} - A_2 w_0 = 0, && \text{(b)} \\ &(u_0^2 + w_0^2)_\Phi = 0, && \text{(c)} \\ &i_{0\Phi} = 0, && \text{(d)} \\ &w_0^2(\ln A_2)_\theta = \frac{1}{\rho_0\theta_{0\Psi}} \cdot p_{0\Psi}, && \text{(e)} \\ &\rho_0 = \sigma\rho(p_0, i_0), && \text{(f)} \\ &\frac{w_0}{A_2}\theta_{0\Phi} = \frac{v_0}{A_1}. && \text{(g)} \end{aligned}\right\} \tag{19.9}$$

Page 294.

Here and throughout coefficients A_1, A_2 and $A_{2\theta}$ are calculated when $\theta = \varepsilon$ and are functions only of Φ. On the shock wave where $\Phi = \Psi$, from relationship/ratios (19.6), (19.7) we will obtain

$$\left.\begin{aligned} &u_0^+ = u_0^-, \quad w_0^+ = w_0^-, && \text{(a)} \\ &p_0^+ = p_1 + \rho_1(v_0^-)^2, && \text{(b)} \\ &v_0^+ = \frac{A_1}{A_2}w_0^-\theta_0^{*\prime} + \frac{\rho_1}{\rho_0^+}v_0^-, && \text{(c)} \\ &i_0^+ = \frac{V_1^2}{2} + i_1 - \frac{(u_0^-)^2 + (w_0^-)^2}{2}, && \text{(d)} \\ &\rho_0^+ = \sigma\rho(p_0^+, i_0^+). && \text{(e)} \end{aligned}\right\} \tag{19.10}$$

Here values u_0^-, v_0^-, w_0^- are determined from the formulas (19.5), in which θ^* is replaced by ε. Formulas (19.10b, d) give p_0^+, i_0^+ as

functions Φ (or Ψ, since on shock wave $\Phi = \Psi$). Equation (19.10e) is the equation of state of gas, written in the form $\rho = \rho$ (p, i). Thus, only unknown previously value in relationship/ratios (19.10) it is

$$\theta^{*\prime} = \frac{d}{d\Phi} \theta_0 [\Phi, \Psi = \Phi] = (\theta_{0\Phi} + \theta_{0\Psi})_{\Phi=\Psi}.$$

For the ideal gas of relationship/ratio (19.10) they take the form

$$\left.\begin{aligned} & u_0^+ = u_0^-, \quad w_0^+ = w_0^-, \\ & p^+ = p_1 + \rho_1 (v_0^-)^2, \\ & v_0^+ = \frac{A_1}{A_2} w_0^- \theta_0^{*\prime} + v_0^- \left[1 + \frac{2a_1^2}{(\gamma-1)(v_0^-)^2}\right], \\ & \rho_0^+ = \rho_1 \left[1 + \frac{2a_1^2}{(\gamma-1)(v_0^-)^2}\right]^{-1}. \end{aligned}\right\} \quad (19.11)$$

On the surface of the streamlined body

$$\theta_0 = \varepsilon, \quad v_0 = 0. \quad (19.12)$$

Page 295.

The solution of system of equations (19.9) with boundary conditions (19.10), (19.12) is conducted of analogous with the case round cone. From equation (19.9c) we obtain

$$u_0^2 + w_0^2 = \Delta_0^2 (\Psi), \quad (19.13)$$

where

$$\Delta_0(\Psi) = \pm \sqrt{(u_0)^2 + (w_0)^2} = \pm \sqrt{(u_2^-)^2 + (w_0^-)^2}\,\Big|_{\Phi=\Psi},$$

and further

$$\left.\begin{aligned} u_0 &= \Delta_0(\Psi) \sin z_0 (\Phi, \Psi), \\ w_0 &= \Delta_0(\Psi) \cos z_0 (\Phi, \Psi). \end{aligned}\right\} \quad (19.14)$$

Substituting (19.14) into the equation (19.9b), we find equation for

z_0 in the form $z_{0\Phi} - A_2 = 0$;

solution of which it will be $z_0 = \int_{\Psi}^{\Phi} A_2 d\Phi + \alpha_0(\Psi)$. Substituting z_0 in (19.14), we obtain

$$\left.\begin{aligned} u_0 &= \Delta_0(\Psi)\sin\left[\int_{\Psi}^{\Phi} A_2 d\Phi + \alpha_0(\Psi)\right], \\ w_0 &= \Delta_0(\Psi)\cos\left[\int_{\Psi}^{\Phi} \right]. \end{aligned}\right\} \quad (19.15)$$

Function α_0 (Ψ) is determined according to (19.10) from the formula

$$\alpha_0(\Psi) = \left(\operatorname{arctg}\frac{u_0^-}{w_0^-}\right)_{\Phi=\Psi}. \qquad (19.16)$$

Now one should select sign Δ_0 (Ψ) from the condition that

$$w_0^+ = w_0^-.$$

Page 296.

Substituting u_0, w_0 by formulas (19.15) into equation (19.9a), after obvious conversions we will obtain

$$[\ln(\rho_0 A_1 \theta_{0\Psi})]_{\Phi} + \operatorname{tg}\left[\int_{\Psi}^{\Phi} A_2 d\Phi + \alpha_0(\Psi)\right] A_2 = 0,$$

whence, integrating by Φ, we find

$$\frac{\rho_0 A_1 \theta_{0\Psi}}{\cos\left[\int_{\Psi}^{\Phi} A_2 d\Phi + \alpha_0(\Psi)\right]} = \Omega_0(\Psi). \qquad (19.17)$$

For determining function Ω_0 (Ψ) we will use the equality, which occurs on shock wave, i.e., when $\Phi = \Psi$,

$$\theta_0^{\bullet'} = \theta_{0\Phi} + \theta_{0\Psi},$$

from which with the aid of (19.10), (19.9g) is obtained the formula

$$(\theta_{0\Psi})_{\Phi=\Psi} = \left\{ \frac{A_2}{A_1 w_0^-} \left(v_0^+ - \frac{\rho_1}{\rho_0^+} v_0^- \right) - \right.$$
$$\left. - \frac{v_0^+ A_2}{A_1 w_0^-} \right\}_{\Phi=\Psi} = -\left(\frac{A_2}{A_1} \cdot \frac{v_0^-}{w_0^-} \frac{\rho_1}{\rho_0^+} \right)_{\Phi=\Psi}. \quad (19.18)$$

Then from formulas (19.17), (19.18) it follows

$$\Omega_0(\Psi) = \left\{ \frac{\rho_0 A_1 \theta_{0\Psi}}{\cos\left[\int\limits_{\Psi}^{\Phi} A_2\, d\Phi + \alpha_0(\Psi) \right]} \right\}_{\Phi=\Psi} =$$
$$= -\left[A_2 \frac{v_0^-}{w_0^-} \frac{\rho_1}{\cos \alpha_0(\Psi)} \right]_{\Phi=\Psi}. \quad (19.19)$$

Equation (19.e) with the aid of formulas (19.17), (19.15) and (19.19) is converted to the form

$$p_{0\Psi} = \frac{w_0^2 (\ln A_2)_\theta}{A_1} \Omega_0(\Psi) \cos\left[\int\limits_{\Psi}^{\Phi} A_2\, d\Phi + \alpha_0(\Psi) \right] =$$
$$= -\rho_1 w_0^2 \frac{(\ln A_2)_\theta}{A_1} \frac{\cos\left[\int\limits_{\Psi}^{\Phi} A_2\, d\Phi + \alpha_0(\Psi) \right]}{\cos \alpha_0(\Psi)} \left[A_2 \frac{v_0^-}{w_0^-} \right]_{\Phi=\Psi} =$$
$$= \rho_1 w_0^3 \frac{(\ln A_2)_\theta}{A_1} \left[A_2 \frac{v_0^-}{(w_0^-)^2} \right]_{\Phi=\Psi}. \quad (19.20)$$

Page 297.

Integrating equation (19.20) for Ψ from the surface of shock wave, let us find expression for p in the form

$$p_0 = p_0^- - \rho_1 \frac{(\ln A_2)_\theta}{A_1} \int\limits_{\Psi}^{\Phi} w_0^3 \left[A_2 \frac{v_0^-}{(w_0^-)^2} \right]_{\Phi=\Psi} d\Psi. \quad (19.21)$$

First term in formula (19.21) depends only on Φ and corresponds

to Newton's theory; second term considers centrifugal forces in shock layer.

Equation (19.9d) has the solution

$$i_0 = i_0(\Psi) = (i_0^+)_{\Phi=\Psi} = \left[\frac{V_1^2}{2} + i_1 - \frac{(u_0^-)^2 + (w_0^-)^2}{2}\right]_{\Phi=\Psi}. \quad (19.22)$$

From the equation of state of gas ρ_0, is defined as function Φ and Ψ

$$\rho_0 = \sigma\rho(p_0, i_0). \quad (19.23)$$

With the aid of formulas (19.19), (19.23), (19.15) equation (19.17) he is record/written in the form

$$\theta_{0\Psi} = \frac{\Omega_0(\Psi)}{A_1\rho_0}\cos\left[\int_{\Psi}^{\Phi} A_2\, d\Phi + \alpha_0(\Psi)\right] = -\frac{\rho_1 w_0}{\rho_0 A_1}\cdot\left[\frac{v_0^- A_2}{(w_0^-)^2}\right]_{\Phi=\Psi}. \quad (19.24)$$

Integrating equation (19.24) for Ψ from shock wave, we find

$$\theta_0 = \theta_0^*(\Phi) - \frac{\rho_1}{A_1}\int_{\Phi}^{\Psi}\frac{w_0}{\rho_0}\left[\frac{v_0^- A_2}{(w_0^-)^2}\right]_{\Phi=\Psi} d\Psi. \quad (19.25)$$

The unknown function $\theta^*_0(\Phi)$ will be determined from the condition of flow $\theta_0 = 0$, v_0 = of 0. In the exact solution of problem, the body surface is flow line on single sphere and there Ψ is piecewise constant function.

Page 298.

Approximate solution of problem, determined by expansions (19.8) can be unsuitable in vorticity layer, near body surface; then conditions $\theta_0 = 0$, $v_0 = 0$ we are, as in the case of round cone, satisfy on the outer edge of the vorticity layer where $\Psi = \Psi_b(\Phi)$. Thus, we take, that on body surface (or on the boundary of vorticity layer)

$$\Psi = \Psi_b(\Phi). \qquad (19.26)$$

If vorticity layer isabsent, then $\Psi_b = const$, otherwise $\Psi_b = \Psi_b(\Phi)$. Substituting $\Psi = \Psi_b$ in formula (19.25) we set/assume $\theta_0 = 0$, we will obtain formula for $\theta^*_0(\Phi)$ in the form

$$\theta_0^* = \frac{\rho_1}{A_1} \int_{\Phi}^{\Psi_b} \frac{w_0}{\rho_0} \left[\frac{v_0^- A_2}{(w_0^-)^2} \right]_{\Phi=\Psi} d\Psi. \qquad (19.27)$$

Formula (19.25) can be now presented in the form

$$\theta_0 = \frac{\rho_1}{A_1} \int_{\Psi}^{\Psi_b(\Phi)} \frac{w_0}{\rho_0} \left[\frac{v_0^- A_2}{(w_0^-)^2} \right]_{\Phi=\Psi} d\Psi. \qquad (19.28)$$

Of functions $\Psi_b(\Phi)$ is located by the satisfaction of equation (19.9g) when $\Psi = \Psi_b(\Phi)$, where placed $v_0 = 0$. After substitution (19.28) in (19.9g), we will obtain

$$\left[\frac{w_0^2 \rho_1}{A_1 \rho_0} \frac{v_0^- A}{(w_0^-)^2} \right]_{\Phi=\Psi_b} \frac{d\Psi_b}{d\Phi} = 0. \qquad (19.29)$$

Equation (19.29) allows for two solutions:

1) $\frac{d\Psi_b}{d\Phi} = 0,$

2) $(w_0)_{\Phi=\Psi_b} = 0.$

To the first of them corresponds the case when in field of flow there is no vorticity layer. This is possible for the flows where $w_0 \neq 0$.

Such flows appear during the flow about the bodies with the pointed edges where leading shock wave is connected to the edges of body.

Page 299.

For bodies with chamfered edges Ψ_b must be determined from equation $(w_0)_{\Phi=\Psi_b}=0,$ which taking into account (19.15), (19.16) can be written in the form

$$\int\limits_{\Psi_b^*}^{\Phi} A_2\,d\Phi = \left(\operatorname{arctg}\frac{w_0^-}{u_0^-}\right)_{\Phi=\Psi_b}. \qquad (19.30)$$

Pressure on body surface is located through the formula

$$p_0 = p_0^+ - \rho_1\frac{(\ln A_2)_0}{A_1}\int\limits_{\Psi_b^*(\Phi)}^{\Phi} w_0^3\left[A_2\frac{v_0^-}{(w_0^-)^2}\right]_{\Phi=\Psi} d\Psi. \qquad (19.31)$$

Let us note that the selection of the concrete/specific/actual equation of state of gas does not affect, according to (19.31), the distribution of pressure on body. [This effect is included in terms O(σ)]. The remaining parameters of flow depend on the selection of the equation of state of gas.

For the bodies whose surface is convex, $A_{20}>0,$ and pressure on body surface p_0 is less than behind shock wave, p_0^+; for concave surfaces relationship/ratio reverse/inverse. Second term in the form (19.31) characterizes the effect of the centrifugal forces, caused by transverse overflow of gas in shock layer.

The value of the solution of A. Gonor consists not only in the fact that it determines pressure on body surface (this can be made and is simpler than), but mainly in the fact that it gives all parameters of flow in shock layer.

19.3. Comparison of theoretical and experimental results for elliptical cones. Let us examine the conical body, by cross section of which plane z = 1 is ellipse (Fig. 90). The equation of this ellipse in plane $\xi\eta$ takes the form

$$\frac{\xi^2}{a^2} + \frac{\eta^2}{b^2} = 1.$$

Instead of assigning of semi-axis a and c, usually are assigned the parameters

$$m = \frac{a}{b} \quad \text{and} \quad h = \frac{1}{\sqrt{\pi ab}}.$$

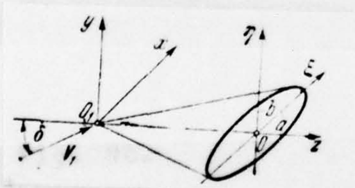


Fig. 90.

Page 300.

In work [213] on formula (19.31) was produced the calculation of pressure on the surface of elliptical cones with different values of m, h, δ. (See also [7]). Without being stopped on the details of these calculations, let us make a comparison of theoretical and experimental results. In Fig. 91a, b, taken from work [214], solid lines plotted/applied theoretical the distribution curves of pressure coefficient C_p on the surface of elliptical cones when $M_1 = \infty$. Figure 91a corresponds to the case δ = 0. Experimental points with $M_1 = 4$ are designated as follows:

$$\square - m = 2,\ \ h = 1{,}16;\quad \diamond - m = \frac{3}{2},\ \ h = 1{,}16;$$
$$\triangle - m = 2,\ \ h = 3{,}1;\quad \bigcirc - m = 5,\ \ h = 1{,}16.$$

Fig. 91b, corresponds to case of m = 5, h = 1.16. Experimental points are obtained with δ = 6° and δ = 12°, $M_1 = 4$; they are designated in marks $\bigcirc - \delta = 6°, \square - \delta = 12°$.

In work [216] produced the comparison of the results, given by theoretical methods, including the method of shock layer, with experimental data for elliptical cones.

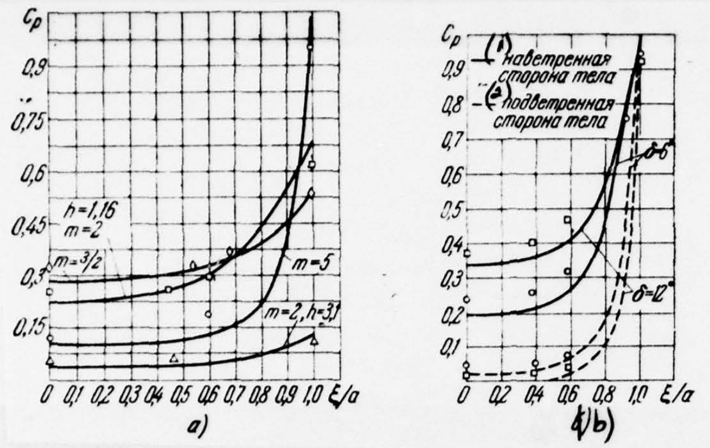


Fig. 91.

Key: (1). the windward face of body. (2). the lee side of body.

Page 301.

§20. Method of shock layer for delta wings.

20.1. Lead-in observations. Let us examine fine/thin conical triangular wing with sharp leading edges and sweep angle Λ, placed without slip into hypersonic flow of gas, which has the Mach numbers $M_1 \gg 1$ and of so forth, at an angle of attack δ (Fig. 92a, b).

Wing thickness we assume small in comparison with its spread/scope $2b = 2$ ctg Λ, angle of attack δ - by such, that the leading shock wave is connected only to the apex/vertex of body. At these angles of attack, the flow about pressure side of wing occurs with flow choking, and pressure on the upper part of the wing is virtually equal to zero. We will use the method of shock layer for the solution of the problem of the flow about pressure side of wing. However, immediately it is clear that the method of shock layer in that form in which it is presented into §19, is here inapplicable. Actually, if we fix angles δ, Λ, a $\sigma = \frac{\rho_1}{\rho^x}$. to fix to zero ($\rho^x$ - characteristic density in shock layer), then bow shock at certain value σ will become connected to leading edges (for example, see the book [46]).

Mach angle after the oblique shock wave, deviating flow to angle δ, is of the order $\delta^{1/2}$ tg δ.

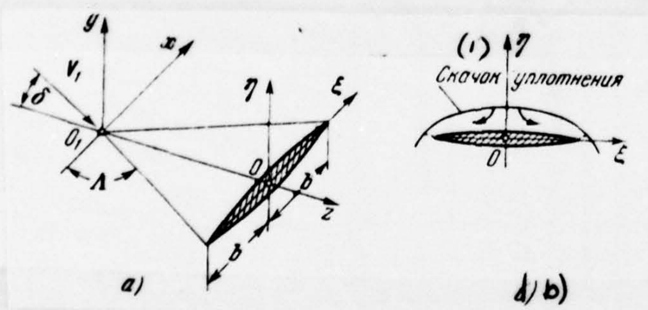


Fig. 92.

Key: (1). shock wave.

Page 302.

So that the bow shock would be connected only to the apex/vertex of body, ratio $\frac{b}{\sigma^{1/2}\operatorname{tg}\delta} = \frac{\operatorname{ctg}\Lambda}{\sigma^{1/2}\operatorname{tg}\delta} = \Omega$ must be that which was limited with $\delta \to 0$. More precise examination for a triangular plate and the ideal gas where $\sigma = \frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)M_1^2\sin^2\delta}$, shows [217] that in this case of $0 < \Omega < 2$.

Therefore in asymptotic theory it is necessary to make the supplementary assumption that $b \to 0$, when $\sigma \to 0$. Depending on the speed of tendency b toward zero with $\sigma \to 0$, are obtained the asymptotic theories, which cover the different intervals of angles of attack δ.

In the work of Zh. Koul and Zh. Braynerd [218] is examined the problem of the flow about the flat/plane delta low-aspect-ratio wing at angle of attack $\delta = 90°$. The surface of plate is divide/marked off into the bands, arrange/located on the wingspan, and it is considered that the velocity component along the chord of wing is negligible. Then the problem of the flow about the mentioned band stops to equivalent two-dimensional problem of the flow about the plate,

establish/installed perpendicular to unperturbed flow, which is solved by the method of shock layer. When $\delta = 90°$ flow cannot be conical, but for values δ, close to 90°, this is possible, if σ sufficiently little (for example, see [219]).

For this reason the results of Zh. Koul and Zh. Braynerda can be considered as maximum for conical flows with $\Omega = 0$. The more detailed study of problem [217] shows that the spread/scope of wing 2b must be $o(\sigma)$ with $\sigma \longrightarrow 0$, if we want to remain within the framework of conical flow theory, since with $b = O(\sigma)$ of equations, valid near leading shock wave, they are the same as in work [218], but near the surface of the streamlined body, must be taken into account a change of the flow parameter chord wise.

In A. Messiter's work [218] is examined the case when the spread/scope of triangular plate 2b is value $O(\sigma^{1/2})$ i.e.

$$\Omega = \frac{\operatorname{ctg} \Lambda}{\sigma^{1/2} \operatorname{tg} \delta} = \text{const},$$

at $\sigma \longrightarrow 0$, moreover $0 < \Omega < 2$.

The most important part of this work are the laws of the similarities about which it will be said further.

Page 303.

K. Hyde in work [220] propagated the theory of Messiter to the case of the delta wings of final thickness and proposed the simple method of the solution of the reverse problem (was assigned the form of the leading shock wave, unknown it is the form of the streamlined body). Specifically, was given approximate solution of the problem for a delta plate and wings with a cross section of the type of lens and rhomb. However, in note [221] it was establish/installed that the account of those terms in the solution to the problems which were reject/thrown in work [220], substantially makes theoretical results worse for interval of $1 < \Omega < 2$. On the other hand, the results K. Hydes for a plate will agree well with experiment with $1 < \Omega < 2$. Therefore reverse/inverse method the Hydes for values Ω, which lie at interval of $1 < \Omega < 2$, one should consider as semi-empirical. As we will see further, the results of Messiter for interval of $0.2 < \Omega < 0.5$ are distant from average experimental data (possible reasons for this phenomenon will be discussed below). Numerical solution of problem is given by Messiter only for interval of $0 < \Omega < 0.5$. Thus, at present there are no reliable quantitative data on the theory of Messiter for interval of $0.5 < \Omega < 2$. Because of this we will be restricted to setting and the solution of the problem of dominant terms for flow parameters in shock layer, and also by the conclusion/derivation of the important laws of similarity for the case of triangular plate (it is more precise, for the wing whose lower surface is plane triangle or differs little from plane).

Discussion of the limits of the applicability of the solution of Messiter is conducted in point/item 20.2.

20.2. Method of shock layer for triangular of plate. In point/item 20.2, is given the setting and solution of the problem of dominant terms for the parameters of the flow of gas in shock layer about the lower surface of triangular plate in cases when values $\Omega=\frac{b}{\sigma^{1/2}\operatorname{tg}\delta}=\frac{\operatorname{ctg}\Lambda}{\sigma^{1/2}\operatorname{tg}\sigma}$ with are to interval of 0 < Ω < 2 (more accurately this interval it is determined below). Gas let us consider ideal. (This assumption is unessential.

Page 304.

Furthermore, pressure on the surface of plate does not in practice depend on the equation of state of gas). The low parameter of the problem it is

$$\sigma=\frac{\gamma-1}{\gamma+1}+\frac{2}{(\gamma+1)M_1^2\sin^2\delta}=\frac{\gamma-1}{\gamma+1}(1+2\beta), \qquad (20.1)$$

where

$$\beta=\frac{1}{(\gamma-1)M_1^2\sin^2\delta}=\text{const}.$$

When $\sigma \longrightarrow 0$ thickness of shock layer, at least in the plane of the symmetry of flow (ξ = 0), is of the order O (σ), and the spread/scope of wing O($\sigma^{1/2}$). Therefore within limit when $\sigma \longrightarrow 0$ (β = const) the

parameters of flow in the range between the surface of leading shock wave and the surface of wing approach values of these parameters after the step shock of packing/seal, deviating flow to angle δ, i.e.,

$$\left.\begin{aligned}&\frac{\mathbf{V}}{V_1}\to \mathbf{k}\cos\delta,\\&\frac{p-p_1}{\rho_1 V_1^2}\to \sin^2\delta,\\&\sigma\left(\frac{\rho}{\rho_1}\right)\to 1,\qquad \sigma\to 0,\end{aligned}\right\}\tag{20.2}$$

where **k** - an unit vector of axis O_1z (see Fig. 92).

Following A. Messiter [217], let us search for the parameters of the flow of gas in the which interests us range in the form of the expansions

$$\left.\begin{aligned}&\frac{w}{V_1}=\cos\delta+\sigma\,\frac{\sin^2\delta}{\cos\delta}\,w^0(\xi^0,\eta^0)+\dots,\\&\frac{v}{V_1}=\sigma\sin\delta\,v^0(\xi^0,\ \eta^0)+\dots,\\&\frac{u}{V}=\sigma^{1/2}\sin\delta\,u^0(\xi^0,\eta^0)+\dots,\\&\frac{p-p_1}{\rho_1V_1^2}=\sin^2\delta+\sigma\sin^2\delta\,p^0(\xi^0,\eta^0)+\dots,\\&\frac{\rho_1}{\rho}=\sigma-\sigma^2\left(1+p^0+\frac{u^{02}+2w^0}{1+2\beta}\right)+\dots,\\&\xi^0=\frac{\xi}{\sigma^{1/2}\,\mathrm{tg}\,\delta},\qquad \eta^0=\frac{\eta}{\sigma\,\mathrm{tg}\,\delta},\\&\left[\beta=\frac{1}{(\gamma-1)M_1^2\sin^2\delta},\quad \sigma=\frac{\gamma-1}{\gamma+1}(1+2\beta)\right].\end{aligned}\right\}\tag{20.3}$$

Here u, v, w - the components of vector of speed of V along the axes Cartesian system, depicted on Fig. 92; in each formula in dots are designated the terms of higher order on σ, than given. Last/latter equation (20.3) is a consequence of Bernoulli's integral (1.8):

$$\frac{u^2+v^2+w^2}{2}+\frac{\gamma}{\gamma-1}\,\frac{p}{\rho}=\frac{V_1^2}{2}\left[1+\frac{2}{(\gamma-1)M_1^2}\right].$$

Page 305.

It is assumed that the values with the zero and their derived in terms of ξ^0, η^0 are value O (1) at $\varepsilon \longrightarrow$ 0. The equations of motion of gas let us write, according to point/item 1.3, in the form

$$\left.\begin{aligned}
&\rho(u_\xi + v_\eta - \xi w_\xi - \eta w_\eta) + (u - \xi w)\rho_\xi + (v - \eta w)\rho_\eta = 0,\\
&(u - \xi w)u_\xi + (v - \eta w)u_\eta = -\frac{1}{\rho}p_\xi,\\
&(u - \xi w)v_\xi + (v - \eta w)v_\eta = -\frac{1}{\rho}p_\eta,\\
&(u - \xi w)\left(\frac{p_\xi}{p} - \gamma\frac{\rho_\xi}{\rho}\right) + (v - \eta w)\left(\frac{p_\eta}{p} - \gamma\frac{\rho_\eta}{\rho}\right) = 0.
\end{aligned}\right\} \quad (20.4)$$

First equation (20.4) is an equation of continuity, the second and the third - the equation of momentum in projections on the axis O_1x, O_1y respectively; last/latter - the equation of energy, i.e., the preservation/retention/maintaining of entropy.

If the equation of leading shock wave is written in the form $\eta = \eta_s(\xi)$ and the parameters of gas immediately after shock wave is designated by index s below about the sign of function, then according to formulas (1.47) we will obtain

$$\left.\begin{aligned}
&u_s = u_1 - \eta_s' P,\ v_s = v_1 + P;\ w_s = w_1 + (\xi\eta_s' - \eta_s) P,\\
&\eta_s' = \frac{d\eta_s}{d\xi},\\
&P = \frac{2}{\gamma+1}\left[\frac{a_1^2}{v_1 - \eta_s' u_1 + w_1(\xi\eta_s' - \eta_s)} - \right.\\
&\qquad\left. - \frac{v_1 - \eta_s' u_1 + w_1(\xi\eta_s' - \eta_s)}{1 + \eta_s'^2 + (\xi\eta_s' - \eta_s)^2}\right].
\end{aligned}\right\} \quad (20.5)$$

Furthermore, from relationship/ratios (1.46) it follows

$$p_s = p_1 + \rho_1 V_{n1}(V_{n1} - V_{ns}), \quad (20.6)$$

where the normal to shock wave component of vector of speed, designated V_n, is determined from the formula

$$V_n = \frac{v - u\eta_s' + w(\xi\eta_s' - \eta_s)}{\sqrt{1 + \eta_s'^2 + (\xi\eta_s' - \eta_s)^2}}. \quad (20.7)$$

In the selected system of coordinates (see Fig. 92)

$$u_1 = 0,\ v_1 = -V_1 \sin\delta,\ w_1 = V_1 \cos\delta.$$

SUBJECT CODE 142D

Page 306.

The condition of the nonseparated flow of wing he is record/written in the form

$$\eta = 0, \ v = 0. \tag{20.8}$$

Substituting expansions (20.3) under equations (20.4) and boundary conditions (20.5) - (20.8), equalizing there the terms of the lowest order on σ, we will obtain as a result of equation and boundary conditions for coefficients with zero in this form:

$$\left.\begin{array}{ll}
u^0_{\xi^0} + v^0_{\eta^0} = 0, & \text{(a)} \\
(u^0 - \xi^0)\, u^0_{\xi^0} + (v^0 - \eta^0)\, u^0_{\eta^0} = 0, & \text{(б)} \\
(u^0 - \xi^0)\, v^0_{\xi^0} + (v^0 - \eta^0)\, v^0_{\eta^0} = -p^0_{\eta^0}, & \text{(в)} \\
(u^0 - \xi^0)\, w^0_{\xi^0} + (v^0 - \eta^0)\, w^0_{\eta^0} = 0. & \text{(г)}
\end{array}\right\} \tag{20.9}$$

With

$$\left.\begin{array}{ll}
\eta^0 = \eta^0_s(\xi^0), & \\
u^0_s = -\eta^{0'}_s, \quad \eta^{0'}_s = \dfrac{d\eta^0_s}{d\xi^0}, & \text{(a)} \\
v^0_s = \eta^0_s - \xi^0\eta^{0'}_s - (\eta^{0'}_s)^2 - 1, & \text{(б)} \\
p^0_s = 2\,(\eta^0_s - \xi^0\eta^{0'}_s) - (\eta^{0'}_s)^2 - 1. & \text{(в)}
\end{array}\right\} \tag{20.10}$$

(Formula for w_s^0 will not be required and is not here extracted.). With

$$\left.\begin{array}{ll} \eta^0=0 & -\Omega<\xi^0<\Omega, \\ v^0=0 & \left(\Omega=\dfrac{\operatorname{ctg}\Lambda}{\sigma^{1/2}\operatorname{tg}\delta}\right). \end{array}\right\} \quad (20.11)$$

Besides conditions (20.10), (20.11) it is necessary to still require so that the component of vector of the speed in the plane, perpendicular to leading edge, would stop the equal local velocity of sound on leading edge, if we move over the wing surface to leading edge.

Since on the surface of plate v = 0, this condition he is record/written as follows:

$$(u\sin\Lambda - w\cos\Lambda)^2 = a^2, \quad \xi\to b, \ \eta=0.$$

For an ideal gas $a^2=\gamma\dfrac{p}{\rho}$, and from the equation of Bernoulli it follows

$$a^2=(\gamma-1)\frac{V_1^2}{2}\left[1+2\beta\sin^2\delta-\frac{u^2+w^2}{V_1^2}\right].$$

Page 307.

Substituting expansions (10.3) under the condition

$$(u \sin \Lambda - w \cos \Lambda)^2 = (\gamma - 1)\frac{V_1^2}{2}\left[1 + 2\beta \sin^2 \delta + \frac{u^2 + w^2}{V_1^2}\right]$$

when $\xi \to b,\ \eta = 0,$ and by equalizing the terms of the smallest order from ε, we will obtain $(u^0 - \Omega)^2 = 1$ or

$$u^0 = \pm(1 + \Omega) \quad \text{with} \quad \eta^0 = 0,\ \xi^0 \to \pm\Omega. \tag{20.12}$$

On the strength of symmetry problem can be examined for $0 \leq \xi^0$; then with

$$\xi^0 = 0,\ u^0 = 0,\ \eta_s^{0'} = 0. \tag{20.13}$$

Of equalization (20.9a, b, c) with boundary conditions (20.10)-(20.13) they make it possible by only form to determine u^0, v^0, p^0, η_s^0 (After the determination of these values w^0 it is determined by integration. For the calculation of pressure on body w^0, it is not required and subsequently be determined will not be). Equations (20.9) possess two performance characteristics. The first of them is the family of flow lines $\Psi^0 = \text{const}$, where the function of current Ψ^0 satisfies the equation

$$(u^0 - \xi^0)\Psi^0_{\xi^0} + (v^0 - \eta^0)\Psi^0_{\eta} = 0. \tag{20.14}$$

This family exists of precise equations. The second family, as it is easy to verify that that consists of direct/straight $\xi^0 = \text{const}$. It appears because of the approximate formulation of the problem.

During the solution of system of equations (20.9) we drop temporarily the zero near the sign of the functions in equations (20.9)-(20.14), since this can not cause misunderstandings and substantially simplifies the recording of formulas.

Method of the solution of system (20.9) the same as into §19. As independent variables they are selected ξ and Ψ, in which regard on the shock wave we assume $\xi = \Psi$.

After transition to independent variables ξ, Ψ, taking into account the formulas

$$\frac{\partial f(\xi, \Psi)}{\partial \xi} = f_\xi - f_\Psi \frac{\eta_\xi}{\eta_\Psi}, \quad \left.\frac{\partial f(\xi, \Psi)}{\partial \eta}\right|_{\xi=\mathrm{const}} = f_\Psi \frac{1}{\eta_\Psi}$$

in equations (20.9a, b, c), we will obtain

$$\left.\begin{aligned} &\eta_\Psi u_\xi - u_\Psi \eta_\xi + v_\Psi = 0, && \text{(a)} \\ &u_\xi = 0, && \text{(b)} \\ &(u-\xi) v_\xi \eta_\Psi = -p_\Psi, && \text{(c)} \\ &\eta_\xi = \frac{v-\eta}{u-\xi}. && \text{(d)} \end{aligned}\right\} \quad (20.15)$$

Page 308.

When deriving the equation (20.15c) is used the formula (20.15d), which is the consequence of equation (20.14).

On the shock wave where $\Psi = \xi$, must be made conditions (20.10),

moreover

$$\eta_s' = \frac{d}{d\xi}\eta(\xi, \xi = \Psi) = \eta_\xi + \eta_\Psi.$$

In the exact solution of problem, the surface of plate is the conical flow line, on which $\Psi = \text{const.}$ If near fairing is a vorticity layer, in which expansions (20.3) do not represent the solution of problem, then the conditions of flow (20.11), (20.12) must be satisfied on the outer edge of the vorticity layer where $\Psi = \Psi_b(\xi)$ or $\xi_b = \xi_b(\Psi)$. [See, §§18, 19. Name "vorticity layer" bears here conditional character.]. Let us now look at the solution of system of equations (20.15). The solution to equation (20.15b) it is

$$u = u(\Psi). \qquad (20.16)$$

It differentiated equation (20.15d) for Ψ, taking into account equation (20.15a), we will obtain

$$(u - \xi)\eta_{\Psi\xi} + \eta_\Psi = 0,$$

whence let us find

$$\left.\begin{aligned} \eta_\Psi &= [u(\Psi) - \xi] f'(\Psi), \\ \eta(\xi, \Psi) &= \eta_s(\xi) + \int_\xi^\Psi [u(\Psi_1) - \xi] f'(\Psi_1)\, d\Psi_1, \end{aligned}\right\} \qquad (20.17)$$

where $f'(\Psi)$ is derivative from unknown function $f(\Psi)$. From equations (20.15d) and (20.17) for determining $v(\xi, \Psi)$ is obtained the

formula

$$v(\xi, \Psi) = \eta_s(\xi) + \int_{\xi}^{\Psi} [u(\Psi_1) - u(\Psi)] f'(\Psi_1) d\Psi_1 + \\ + [u(\Psi) - \xi] \eta_s'(\xi) - [u(\Psi) - \xi][u(\xi) - \xi] f'(\xi). \quad (20.18)$$

Substituting u, v by formulas (20.16), (20.18) under boundary conditions (20.10a, b) let us find

$$\left. \begin{aligned} u(\Psi) &= -\eta_s'(\Psi), \\ f'(\Psi) &= \frac{1}{[u(\Psi) - \Psi]^2}. \end{aligned} \right\} \quad (20.19)$$

With the aid of (20.19) formula (20.18) is reduced to the form

$$v(\xi, \Psi) = \eta(\xi, \Psi) + [u(\Psi) - \xi] \left\{ \int_{\Psi}^{\xi} \frac{d\Psi_1}{[u(\Psi_1) - \Psi_1]^2} - u(\xi) - \frac{1}{u(\xi) - \xi} \right\}; \quad (20.20)$$

it also it is possible to write as follows:

$$\eta_{\xi} = \frac{v - \eta}{u - \xi} = \int_{\Psi}^{\xi} \frac{d\Psi_1}{[u(\Psi_1) - \Psi_1]^2} - u(\xi) - \frac{1}{u(\xi) - \xi}. \quad (20.21)$$

Page 309.

Formula (20.17) will take the form

$$\eta(\xi, \Psi) = \eta_s(\xi) + \int_{\xi}^{\Psi} \frac{u(\Psi_1) - \xi}{[u(\Psi_1) - \Psi_1]^2} d\Psi_1. \quad (20.22)$$

Equation (20.15c) taking into account formulas (20.20)-(20.22) gives

$$p_{\Psi}=-\frac{[u(\Psi)-\xi]^3}{[u(\Psi)-\Psi]^2}u'(\xi)\left\{1-\frac{1}{[u(\xi)-\xi]^2}\right\}. \qquad (20.23)$$

Integrating (20.23) by ψ from shock wave and taking into account of formula (20.10c), (20.19), we will obtain

$$p(\xi,\Psi)=2\eta_s(\xi)+2\xi u(\xi)-u^2(\xi)-1+$$
$$+\left\{\frac{1}{[u(\xi)-\xi]^2}-1\right\}u'(\xi)\int\limits_{\xi}^{\Psi}\frac{[u(\Psi_1)-\xi]^3}{[u(\Psi_1)-\Psi_1]^2}d\Psi_1. \qquad (20.24)$$

In formulas (20.19), (20.20), (20.22), (20.24) only unknown functions is $\eta_s(\xi)$. It one should determine from the conditions of flow (20.11), (20.12). If the spreading of gas, for example, it occurs at point 0 (Fig. 92b) and vorticity layer near body is absent, then for determination $\eta_s(\xi)$ is necessary to assume $\eta=0$, $\psi=0$ in formula (20.22). However, this it is not possible to make, since integral in this formula when $\Psi=0$, $\xi\neq 0$ diverges, since with small Ψ $u(\Psi)=-\eta_s'(0)\Psi+\ldots$. Hence it follows that near body surface is a vorticity layer, and the conditions of the flow about the body one should satisfy on its outer edge where $\xi_b=\xi_b(\Psi)$ or $\Psi=\Psi_b(\xi)$. From relationship (20.22) we find

$$\eta_s=-\int\limits_{\xi}^{\xi_b(\Psi)}\frac{u(\Psi_1)-\xi}{[u(\Psi_1)-\Psi_1]^3}d\Psi_1, \qquad (20.25)$$

$$\eta(\xi,\Psi)=\int\limits_{\xi}^{\xi_b(\Psi)}\frac{u(\Psi_1)-\xi}{[u(\Psi_1)-\Psi_1]^3}d\Psi_1. \qquad (20.26)$$

Let us differentiate equality (20.26) on ξ and relationship/ratio (20.15d) let us present in the form

$$[u(\Psi)-\xi]\left\{\frac{1}{u(\xi)-\xi}+\int_{\xi}^{\xi_b(\Psi)}\frac{d\Psi_1}{[u(\Psi_1)-\Psi_1]^2}\right\}=\eta(\xi,\Psi)-v(\xi,\Psi). \quad (20.27$$

Page 310.

When $\xi=\xi_b$ $\eta\approx v\approx 0$, therefore equality (20.27) can be fulfilled only, if

$$u(\Psi)=\xi_b(\Psi). \quad (20.28)$$

The only unknown function, entering the solution of problem, is now $u(\Psi)$. In order to compose for it equation, let us substitute into formula (20.22) $\xi=\xi_b(\Psi)=u(\Psi)$ and let us differentiate result on Ψ. Taking into account that $\eta(\xi_b,\Psi)=0$ and $\frac{d\xi_b}{d\Psi}\neq 0$, we will obtain after simple calculations the unknown equation

$$u[u(\Psi)]+\frac{1}{u[u(\Psi)]-u(\Psi)}=\int_{\Psi}^{u(\Psi)}\frac{d\Psi_1}{[u(\Psi_1)-\Psi_1]^2}. \quad (20.29)$$

Equation (20.29) is functional equation. It must be solved under the boundary conditions

$$\left.\begin{aligned}&u(0)=0, && (a)\\ &u\to 1+\Omega,\ \xi\to\Omega. && (б)\end{aligned}\right\} \quad (20.30)$$

However, condition (20.30b) cannot be made. Actually, when we, moving

over the outer edge of vorticity layer, we approach a leading edge, then $\xi_b \to \Omega$ and according to (20.28) $u \to \Omega$ a not to $1 + \Omega$. Instead of (20.30b) A. Messiter advances the following condition:

$$u(\Omega) = 1 + \Omega, \tag{20.31}$$

physical sense of which lies in the fact that the component of vector of the speed in the plane, perpendicular to leading edge, reaches the local velocity of sound not on leading edge, but in the point of shock wave when $\xi = \Omega$. Thus when we move over straight line $\xi = \Omega$ from leading edge to shock wave, the component of speed u grow/rises from value Ω to value of $1 + \Omega$. In actuality the picture must be reverse/inverse, since during the flow about the sharp edge sonic line takes the form, depicted on Fig. 68.

Finally boundary conditions for u are taken in the form

$$u(0) = 0, \quad u(\Omega) = 1 + \Omega. \tag{20.32}$$

Equation (20.29) with conditions (20.32) determines unique solution $u = u(\Psi, \Omega)$. Differentiating equation (20.29) for Ψ, let us arrive at the equation

$$u'[u(\Psi)]\,u'(\Psi)\left\{\frac{1}{[u[u(\Psi)] - u(\Psi)]^2} - 1\right\} = \frac{1}{[u(\Psi) - \Psi]^2}, \tag{20.33}$$

which with boundary conditions (20.32) was solved by numerical methods by A. Messiter, [217] for Ω, belonging to interval of $0 < \Omega <$

0.5.

Page 311.

Furthermore, for Ω < 0.005 to them was obtained the analytical solution to equation (20-33) by the method of the expansion in terms of degrees Ω, which when Ω → 0 coincide with numerical solution.

After plotting the comparison of the results of the theory of Messiter with the results of other solutions and experiments for later period, we will try ourselves to determine those values Ω from interval of 0 < Ω < 2, at which one should expect the disagreement between the solution of Messiter and the exact solution of problem. On Fig. 93a, b, is depicted the typical picture of the distribution of the lines of flow $\Psi = \mathrm{const}$ in field of flow and the form of shock wave.

The solution of Messiter is unsuitable near the surface of plate in vorticity layer and in certain vicinity of leading edge. (This range on Fig. 93 is separated by broken line). Pressure across vorticity layer, undoubtedly it is little affected and here complications be it cannot. It is a different matter with range near front/leading be it cannot. It is a different matter with range near leading edge. If the value of the parameter Ω is low, then the solution of Messiter can prove to be unsuitable in the significant

part of the which interests us range. According to the analytical solution of A. Messiter when $\Omega \to 0$ the relation

$$\frac{\eta_s(0)}{b} = \frac{\eta_s^0(0)\,\sigma^{1/2}}{\Omega} = \sigma^{1/2}\left[2\ln\frac{1}{2\Omega} + O\ (1)\right].$$

With fixing σ and $b \to 0$, in consequence of which $\Omega \to 0, \frac{\eta_s(0)}{b} \to \infty$. This means that the range in which are valid formulas (20.3) with small, but final σ, is confined when $\Omega \to 0$ to the plane of the symmetry of flow $\xi = 0$, since the first terms in expansions (20.3) correspond approximately to the parameters of flow after the shock wave, deviating flow in the plane of the symmetry of flow to angle δ, and shock wave considerably differs in form from plane.

From the aforesaid it follows that the solution of Messiter with small Ω can give the quantitative results, distant from the true.

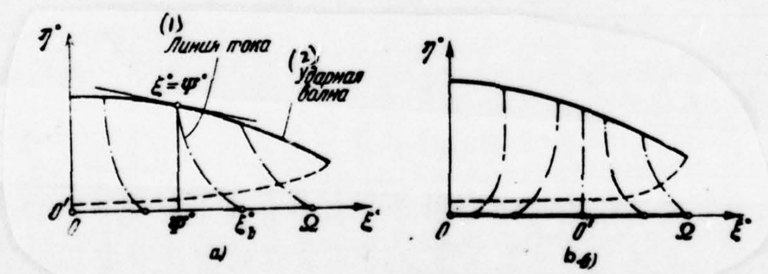


Fig. 93.

Key: (1). Flow line. (2). Impact wave.

Page 312.

Thus, the solution of Messiter is correct not in all interval of $0 < \Omega < 2$, but only with $\Omega > \Omega^{*} > 0$. (Judging by the experimental data given to Fig. 94, $\Omega^{*} \approx 0.7$-0.8./).

Let us note one additional fact, connected with the theory of Messiter. At values $\Omega < \Omega^{**}$ the point of the spreading of flow O', is point O on Fig. 93a, but if $\Omega > \Omega^{**}$, then this point is shift/sheared in the direction of leading edge (Fig. 93b). (With $\Omega \geqslant 2$ leading shock wave becomes connected to leading edge and the solution of Messiter already is unsuitable). According to the data [220], where obtained approximate solution for a plate by reverse/inverse method $\Omega^{**} \approx 1$.

20.3. Laws of similarity. Under equations (20.9) and boundary conditions (20.11), (20.12), [(20.30)], (20.13) enters the only parameter Ω; therefore u^0, v^0, w^0, p^0, ρ^0 are functions ξ^0, η^0, Ω, that also is actually, the expression of the law of similarity. Specifically, for pressure it is possible to write

$$\frac{2p}{\rho_1 V_1^2} = \frac{2}{\gamma M_1^2} + 2\sin^2\delta + \sigma 2\sin^2\delta\, p^0(\xi^0, \eta^0, \Omega) + \ldots \quad (20.34)$$

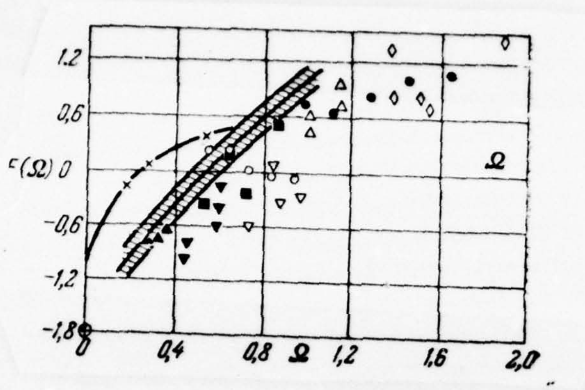


Fig. 94.

Page 313.

Set/assuming pressure on the upper part of the plate equal to zero and integrating (20.34) by the lower surface of plate, after division

into its area, we will obtain

$$C_y = \frac{2}{\gamma M_1^2} + 2\sin^2\delta + 2\sigma\sin^2\delta\,\frac{1}{b}\int_0^b p^0(\xi^0, 0, \Omega)\,d\xi = \\ = \frac{2}{\gamma M_1^2} + 2\sin^2\delta + 2\sigma\sin^2\delta\,\frac{1}{\Omega}\int_0^\Omega p^0(\xi^0, 0, \Omega)\,d\xi^0, \quad (20.35)$$

where C_y the coefficient of normal force C_n. (elementary area/sites on wing are taken in the form of triangles with single height/altitude and basis/base $d\xi$.).

Formula (20.35) can be rewritten in the form, which expresses the approximate law of similarity

$$\frac{C_n - 2\sin^2\delta - \frac{2}{\gamma M_1^2}}{\sigma\sin^2\delta} = F(\Omega), \qquad (20.36)$$

where

$$F(\Omega) = \frac{2}{\Omega}\int_0^\Omega p^0(\xi^0, 0, \Omega)\,d\xi^0.$$

From rule (20.36) it follows that for the calculation of the normal force of the delta wing whose lower surface differs little from triangular plate, it suffices to know only one function $F = F(\Omega)$

(if, of course, $0 < \Omega < 2$). This function can be determined by calculation or by processing the results of experiments. In exactly the same manner for the coefficient of pitching moment C_m (relative to axis O_1x) is obtained the law of similarity in the form

$$\frac{C_m - \frac{2}{3}\left(2\sin^2\delta + \frac{2}{\gamma M_1^2}\right)}{\sigma \sin^2\delta} = G(\Omega). \qquad (20.37)$$

To Fig. 94, undertaken from work [218], is given the comparison of the results of the experiments, processed taking into account law (20.36) for the range of parameters $0.2 < \sigma < 0.4$; $0.1 < b < 0.6$; $0.2 < \Omega < 2$, with the results of the calculations of Messiter [217]. Here is plotted/applied the shaded band, into which fall values $F(\Omega)$, obtained by Bazzhin [177], by processing the results of calculations according to the method of the integral relationship/ratios of the flow about the triangular plate in ranges $M_1 = 4-10$; $\Lambda = 70° - 85°$, $\delta = 30°-60°$.

Page 314.

For experimental values for Fig. 94, are accepted the following designations:

○ — $M_1 = 4{,}07$, $b = 0{,}33$,
● — $M_1 = 4{,}07$; $b = 0{,}58$,
◇ — $M_1 = 6{,}9$; $b = 0{,}36$,
▽ — $M_1 = 6{,}9$; $b = 0{,}231$,
▼ — $M_1 = 6{,}9$; $b = 0{,}140$,
△ — $M_1 = 3{,}36$; $b = 0{,}5$,
▲ — $M_1 = 3{,}3$; $b = 0{,}167$,
□ — $M_1 = 5{,}8$; $b = 0{,}158$,
■ — $M_1 = 5{,}8$; $b = 0{,}214$.

Numerical values, obtained by A. Messiter, are designated, — ×; result of Zh. Koul and Zh. Braynerda [218], [F(0) $\approx$ - 1.8] is plotted/applied by cross — ⊕.

Experimental points are obtained at values of the parameters, very not adequate/approaching for checking the law of similarity, but also they have a tendency to be grouped about one curve. More reliable material are the results of the calculations of A. Bazzhin. As can be seen from Fig. 94, width of shaded band does not exceed value of 0.4, which provides determination C_n with accuracy 1.5-2.0o/o, if $M_1 \sin \delta > 2$. (Inequality $M_1 \sin \delta > 2$ is equivalent to the condition of smallness σ).

Thus, is observed the situation which fairly often is encountered in different questions. Asymptotic theory gives mediocre/satisfactory quantitative results, but it leads to such laws

of the similarities which are fulfilled well even beyond the limits of its applicability.

§21. Survey/coverage of the results, obtained by other methods.

21.1. Lead-in observations. The majority of the methods, examined in the second chapter, is applied for the calculations of hypersonic flows. First of all this observation is related to numerical methods, to which are dedicated §§10, 11, point/item 16.3, where are given concrete/specific/actual data also for hypersonic speeds.

Page 315.

Therefore we will give here only survey/coverage of the results which follow from the specific character of the hypersonic flows of gas.

21.2. Method of "linearized characteristics". As already noted into §17, method of "linearized characteristics" at hypersonic speeds can be presented into analytical to form. This follows from that fact that for a round cone at zero angle of attack and $M_1 >> 1$ it is possible to obtain simple analytical solution, if we disregard ratios

v^2/a^2, v/a, where v - velocity component in the direction of an increase in the angle θ, (see Fig. 5), a - the local velocity of sound (see point/item 2.6). If we moreover to assume that the half-angle of cone ε is small, then solution is obtained in elementary form, respectively the solutions to equations for disturbance/perturbations are represented in elementary form.

In work [197] solution of such type obtained for elliptical cones, in work [222] - for a triangular plate with fuselage in the form of the half of round cone (leading edges of plate are assumed to be supersonic), while in work [223] - for conical bodies with rhombiform cross section.

21.3. Laws of similarity for extended and fine/thin conical bodies, streamlined at high angles of attack. V. Sichev in work [224] will derive the laws of similarity within the framework of the theory of the slight disturbances for the pointed bodies, streamlined with hypersonic flows of gas at high angles of attack. Conical bodies are a special case. Since us they interest conical flows, let us give the conclusion/derivation of these laws for conical bodies.

Let there be the conical body, streamlined with hypersonic flow of gas at an angle of attack δ (Fig. 95a).

Let us designate the spread/scope of wing (in plane $z = 1$) or the thickness ratio of the body by 2ε (Fig. 95b). Let us assume that the Mach number of undisturbed flow M_1, and angle of attack δ are such, that $M_1 \sin \delta >> 1$, and $\varepsilon \ll 1$. About the windward face of the streamlined body, there is a powerful shock wave after which the gas moves in shock layer, the flow after maximum characteristics (broken line on Fig. 95b) not affecting flow in basic part of shock layer.

Page 316.

(Flow about the body occurs with flow choking about the windward face of body.). On the shadow part of the body surface, the pressure is virtually equal to zero. The size/dimension of zone of flow, limited by maximum characteristics and leading shock wave (Fig. 95b), is of the order ε but the velocity component in the direction of axis O_1z here differs little from $V_1 \cos \delta$. If value ε unlimitedly is decreased, then flow in the which interests us range on plane ξ, η increasingly will be less and less characterized by from flow about the cylinder, which has the same cross section, as this conical body, and streamlined by flow with not disturbed speed $V_1 \sin \delta$, pressure p_1, density ρ_1 so forth.

Let us write the equation of the surface of the streamlined body

in the form

$$\eta^0 = \eta_b^0(\xi^0),$$

where

$$\eta^0 = \frac{\eta}{\varepsilon}, \quad \xi_0 = \frac{\xi}{\varepsilon}, \qquad (21.1)$$

$-1 < \xi^0 < 1$, for the case of symmetrical flow, which we will examine further for simplification in the recording of formulas.

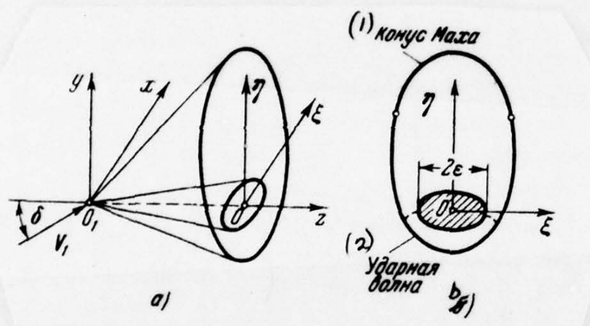


Fig. 95.

Key: (1). Mach cone. (2). Impact wave.

Page 317.

If we introduce the designations

$$K_1 = \varepsilon \operatorname{ctg} \delta, \; K_2 = M_1 \sin \delta,$$

then the parameters of flow in the part of the shock layer, limited by maximum characteristics, let us search for in the form of the asymptotic expansions:

$$\left.\begin{aligned}
p &= \sin^2\delta\rho_1 V_1^2 [p_{(0)}(\xi^0, \eta^0, K_2) + \\
&\quad + K_1 p_{(1)}(\xi^0, \eta^0, K_2) + O(\varepsilon^2)], \\
\rho &= \rho_1 [\rho_{(0)}(\xi^0, \eta^0, K_2) + K_1 \rho_{(1)}(\xi^0, \eta^0, K_2) + \\
&\quad + O(\varepsilon^2)], \\
u &= V_1 \sin\delta [u_{(0)}(\xi^0, \eta^0, K_2) + \\
&\quad + K_1 u_{(1)}(\xi^0, \eta^0, K_2) + O(\varepsilon^2)], \\
v &= V_1 \sin\delta [v_{(0)}(\xi^0, \eta^0, K_2) + \\
&\quad + K_1 v_{(1)}(\xi^0, \eta^0, K_2) + O(\varepsilon^2)], \\
w &= V_1 \cos\delta [1 + K_1 \operatorname{tg}^2\delta w_{(1)}(\xi^0, \eta^0, K_2) + \\
&\quad + O(\varepsilon^2)].
\end{aligned}\right\} \quad (21.2)$$

Furthermore, the equation of bow shock let us present in the form

$$\eta^0 = \eta^0_{s0}(\xi^0, \; K_2) + K_1 \eta^0_{s1}(\xi^0, \; K_2) + O(\varepsilon^2). \quad (21.3)$$

Substituting expressions (21.1)-(21.3) in the equations of motion of gas (20.4), Bernoulli's integral (1.8), condition on shock wave (20.5) -(20.7), the condition of the nonseparated flow of the body

$$\frac{d\eta_b}{d\xi} = \frac{v - \eta_b w}{u - \xi w}$$

and by equalizing there the terms of identical order for ε, it is possible to establish that equation and boundary conditions for functions $f_{(0)}$, $f_{(1)}$ (f = u, v, w, p, ρ) they contain the only

parameter K_2. (in these equations, of course, enters the specific heat ratio of γ).

Hence follows the validity of formulas (21.2). Specifically, it is possible to write

$$\frac{p}{\rho_1 V_1^2/2} = 2\sin^2\delta\,[p_{(0)}(\xi^0, \eta^0, K_2, \gamma) + K_1 p_1(\xi^0, \eta^0, K_2, \gamma)] + O(\varepsilon^2). \quad (21.4)$$

Page 318.

If we as characteristic area take the projected area of body surface on plane y = 0, equal to ε, then the coefficients of normal force C_n, of axial force C_t and of pitching moment C_m, according to (21.4), can be written in the form of the following expressions:

$$C_n = \frac{1}{2\varepsilon}\int_{-\varepsilon}^{+\varepsilon}\frac{2p_b}{\rho_1 V_1^2}\,d\xi = \sin^2\delta\int_{-1}^{+1}[p_{(0)} + K_1 p_{(1)}]_b\,d\xi^0,$$

$$C_t = \frac{1}{2\varepsilon}\int_{-\varepsilon}^{+\varepsilon}\frac{2p_b}{\rho_1 V_1^2}\,|\eta_b|\,d\xi = \varepsilon\sin^2\delta\int_{-1}^{+1}[p_{(0)} + K_1 p_{(1)}]_b\,|\eta_b^0|\,d\xi^0,$$

$$C_m = \frac{2}{3}C_n,$$

where $p_b = p|_{\eta=\eta_b(\xi)}$ pressure on body surface. Converting these expressions to the form in which usually are represented the laws of similarity, we will obtain the formulas

$$\left.\begin{aligned}
&\frac{C_n}{\sin^2\delta} = A_1(K_2, \gamma) + K_1 B_1(K_2, \gamma),\\
&\frac{C_t}{\varepsilon \sin^2\delta} = A_2(K_2, \gamma) + K_1 B_2(K_2, \gamma),\\
&\qquad C_m = \frac{2}{3} C_n,\\
&(K_1 = \varepsilon \operatorname{ctg}\delta, \quad K_2 = M_1 \sin\delta).
\end{aligned}\right\} \quad (21.5)$$

Functions $A_l, B_l \ (l = 1, 2)$ can be determined their calculations or by processing experimental data. During an increase in value K_2, the pressure rapidly approaches its limiting value, and the essential parameter is K_1. Thus, for $K_2 > 4$-5 formulas (21.5) are simplified, for example,

$$\frac{C_n}{\sin^2\delta} = A_1(\infty, \gamma) + K_1 B_1(\infty, \gamma).$$

In the case of large values M_1, the gas one should consider as inadequate, then instead of γ in formulas (21.5) appear ρ_1 and p_1; furthermore, it is necessary to require the constancy of the chemical composition of gas in undisturbed flow.

Page 319.

The laws of similarity (21.5) are derived with an accuracy to terms $O(\varepsilon^2)$, are more precise, $O(K_1^2)$; therefore they are used in the case when $K_1 \ll 1$, i.e., value ε must be much less than value of angle of attack δ.

To Fig. 96, undertaken work [177], gives the results of calculations according to the method of the integral relationship/ratios of the flow about the windward face of the triangular plate, processed in accordance with the laws of similarity (21.5). On Fig. 96, numbering of points from 1 to 6 corresponds to following values of the parameters

NN	K_2	M_1	$\delta°$
1	3,06	4	50
2	3,00	6	30
3	3,45	4	60
4	3,45	5,37	40
5	5,18	6	50
6	5,18	8,05	40

As can be seen from Fig. 95, the law of similarity (21.5) is fulfilled well for $K_1 < 1$. [Points, which possess identical values K_2, lie down to the single line, close to straight line. At wish hence it is possible to find functions $A_1(K_2, \gamma)$ and $B_1(K_2, \gamma)$.

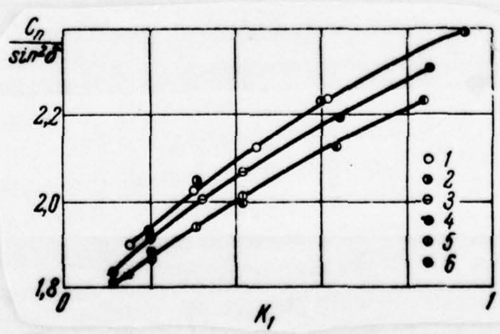


FIG. 96.

Fig. 96.

Page 320.

In conclusion let us discuss the viscosity effect of gas on the formulated laws of similarity. As shown in work [225], this effect is substantial, generally speaking, only at low angles of attack.

21.4. Method of "equivalent cone". Pressure on the surface of conical body p_b, can be approximately determined according to Newton's theory (without taking into account of centrifugal forces in shock layer), by the formula

$$p_b - p_1 = \rho_1 V_{n1}^2,$$

where p_1, ρ_1 - pressure and the density of undisturbed flow, V_{n1} is normal to the surface of the body of the component of the vector of the undisturbed speed in the point in question. V. Zakkey in small note [154] proposed the simple semi-empirical method of the "equivalent cone", which gives the best results for p_b, than Newton's theory. For the given point of body, is determined that V_{n1}, and then is located the semiangle of "equivalent cone" from the condition that for this cone, streamlined at zero angle of attack with undisturbed flow with the previous parameters, the component of the

vector of the undisturbed speed, normal to the surface of cone, is equal to V_{n1}. Pressure at the point of body in question, p_b, is taken as equal to pressure on the surface of "equivalent cone". In comparison with Newton's theory in the method of "equivalent cone" approximately is considered the finiteness of Mach number M_1 and supplementary compression during the particle motion of gas from surface of shock wave to the surface of the streamlined body. Centrifugal forces in shock layer here are not considered; therefore the method in question let us use in cases when the surface curvature of body is changed smoothly. The comparison of the distributions p_b, calculated according to the method of "equivalent cone", with experimental data for the elliptical cones, which refer the large and semiminor axes of the cross sections, equal to 1.39; 1.87 during M_1 = 6, angles of attack δ = 0.10, 20°, will show their excellent agreement. (Reynolds number in experiments, calculated according to free-stream conditions, is equal to 0.31×10^6).

The further development of the method of "equivalent cone" is made in works [155, 199].

Page 321.

B. the flow about the conical wings in cases when bow shocks are

connected to their leading edges.

§ 22. Survey/coverage of the results, obtained by different methods.

22.1. Numerical methods. during of conical flows by numerical methods one should distinguish ranges where flow conical-supersonic, and range where flow conical-transonic (subsonic). Let us examine, for a definition, the delta wing the circuit of flow about which is depicted on Figs. 69, 70.

In conical-supersonic ranges 2-3-4-5-2 and 3-11-15-3 (Fig. 70) the flow of gas can be designed to any degree of accuracy by method of characteristics (see point/items 12.2, 12.3). Since the problem here is completely analogous to two-dimensional problem of gas dynamics, it is possible to utilize the semi-empirical simplified version of method of characteristics, called the "method of rarefaction waves" (see [5]). Since on the shadow side of the surface of the streamlined body pressure little and it can be disregarded, basic difficulty for calculations is the range of a conical-transonic flow 12-13-15-11-12 (Fig. 70). The method of the solution of the problem of flow in this case they are Babaev's method [192] (see Section 163), which, however, designed only flow about the triangular

plate, where the flow in the interesting us range conical-subsonic, and the method of establishment [29]. To Fig. 97, undertaken work [226], depicts the distributions of pressure coefficient on triangular plate (windward face) for values of the parameters $M_1 = \infty$ (1000); $\Lambda = 45°$; $\delta = 10, 20, 30°$.

22.2. Met od of shock layer. The method of shock layer, described in §19, can successfully be applied for the flow-field analyses of conical wings (for example, see [227]), but at the same time it it possesses the defect of fundamental character. Let us turn again to the case of delta wing (Figs. 69, 70). When $\sigma \to 0$ size/dimensions of transonic range 12-13-15-11-12 will be $O(\sigma^{1/2}\ \mathrm{tg}\ \Delta)$, where Δ is a sum of the angle of attack δ and of the half of angular wing thickness; therefore transonic range is confined into point when $\sigma \to 0$.

Page 322.

In actuality this range can occupy the significant part of shock layer (see Fig. 97). The application/use of formulas §19 to an entire wing surface indicates the extrapolation of these formulas, valid in conical-supersonic ranges, to transonic range. Especially good these facts are visible during solution for a triangular plate in work [7], where it is obtained on two members in expansions (19.8) for an ideal

gas. In this solution the pressure is constant on an entire wing surface, and the surface of shock wave consists of two planes, which intersect the planes of the symmetry of flow, i.e., is actually found solution for uniform flows behind the step shocks of packing/seal, connected to the leading edges of plate. Since the pressure is little affected in transonic range (see Fig. 97), the lift coefficients and resistance for an entire plate will agree well with experimental data.

For the bent wings these facts are veiled, but they also occur. During the more exact solution of problem, it is necessary to utilize in the transonic range of such type expansion, as into §23, however, to do this is complex.

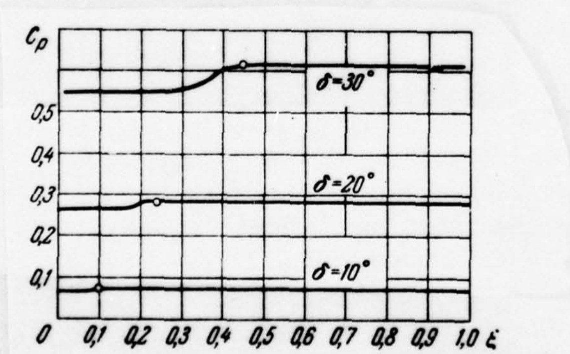


Fig. 97.

Page 323.

22.3. Method of slight disturbances. In perturbation method as main streams, are utilized, besides the undisturbed uniform flow, also the flows about other bodies whose flow about can be designed sufficiently simply and is accurate.

The simplest bodies are cone and wedge. If we as basic accept flow about round cone, then we will obtain method of "linearized characteristics". To flow disturbance about wedge are dedicated [228, 229, 195].

Work [228] examines the problem of conical flow about the windward face of two flat/plane quadrants, streamlined at an angle of attack δ, shifted additionally one relative to the other to angle $2\varepsilon\delta$. It is assumed that δ and ε are small, so that problem is reduced to the determination of disturbed flow about wedge, moreover for the determination of flow about wedge itself also is utilized perturbation method where as basic is accepted the incident to wedge uniform flow of gas. In work [229] the same method is applied for the solution of the problem of the flow about the windward face of triangular plate with supersonic edges. In work [195] the problem of the windward face of triangular plate is solved without assumption about the smallness of angle of attack, moreover is allow/assumed the

slip of wing. Since for main flow in these works is taken certain uniform flow, pressure in the range of the effect of the apex/vertex of wing is the harmonic function of the corresponding independent variables; but essential difference from usual linear theory is in the fact that the eddying of flow after leading shock wave cannot be disregarded, and that the form of the latter also must be determined in resolving problem.

22.4. Final observations. In the preceding/previous point/items are examined the conical flows of gas, appearing during the flow about solid bodies (or walls). However, conical flows appear also in some jet streams and in detached flows about nonconical bodies. If, for example, uniform supersonic flow of gas escape/ensues in the atmosphere through the nozzle, which is the duct with rectangular cross section. Cut off on the plane, component certain angle with its axis, then in the vicinities of the points of inflection of section/shear appear conical fields.

~~end section~~.

Page 324.

In work [230] is found the dominant term of all conical irrotational flows, which begin from sonic plane, i.e., the plane on which the velocity vector of flow is equal on the module/modulus of the local velocity of sound and it is directed along the normal to this plane.

Somewhat not expected is the fact of the emergence of conical flows during the flow about the nonconical bodies with flow breakaway. However, such flows were reveal/detected experimentally during the longitudinal flow about the cylinder with needle (for example, see [231]) and during the flow about the nozzle for the measurement of static pressure [232].

In conclusion let us note one additional fact, discovered during the study of conical flows and the being of interest for the common/general/total theory of the flow about the bodies at supersonic speeds.

If the about pointed body flows the uniform supersonic flow of

gas, then the leading shock wave, connected to its apex/vertex, is "weak", in the sense that in the second theoretically possible mode/conditions of the flow about the body with "powerful" shock pressure wave behind wave front is higher than in flow conditions with "weak" wave.

This point of view is conventional and it was repeatedly confirmed by experiments. In G. Chernyy's work [184], apparently, is reveal/detected for the first time theoretically (and confirmed by experiments) the fact that in a number of cases during the flow about the narrow delta flat/plane wings at high angles of attack the leading shock wave, connected only to the apex/vertex of body, is "powerful". This means that the history of the emergence of concrete/specific/actual supersonic flow, generally speaking, can have vital importance.

page 325.

APPENDICES.

Results of the calculations of the transonic axisymmetric flow about the round cone.

Table II. 1 Dependence between M_1 and M_1^* with γ = 1.4.

M_1	M_1^*	$\operatorname{ctg} \alpha_1$	$1-\frac{1}{M_1^*}$	α_1
1,00	1,00	0	0	90°
1,02	1,017	0,201	0,016	78,6
1,04	1,033	0,286	0,032	74,1
1,06	1,049	0,352	0,047	70,6
1,08	1,065	0,408	0,061	67,8
1,10	1,081	0,458	0,075	65,3
1,12	1,097	0,504	0,088	63,2
1,14	1,113	0,547	0,101	61,3
1,16	1,128	0,588	0,114	59,6
1,18	1,143	0,626	0,125	58,6
1,20	1,158	0,663	0,137	56,7

Page 326.

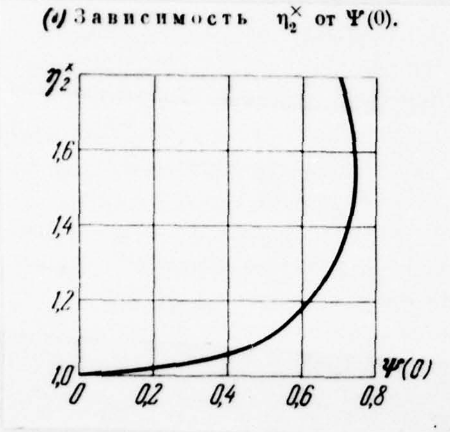


Fig. 98.

Key: (1). Dependence $\eta_2^\times$ on $\Psi(0)$.

Fig. 99.

Key: (1). "Apple-shaped" curve for transonic speeds. (2). Shock wave. (3). Linear theory, apple-like curve. (4). Sonic line. (5). "Apple-like" curve.

Pages 327-328.

Table II.2. Fields of flow during the transonic flow about the cones.

$\eta_i\times$	$\Psi_2=1{,}40$			$\Psi_2=1{,}20$		
	$-\chi$	Ψ	$-\chi/\Psi$	$-\chi$	Ψ	$-\chi/\Psi$
1,8258	2,2428	1,4000	1,6020	—	—	—
1,800	2,2247	1,3887	1,6020	—	—	—
1,700	2,1558	1,3446	1,6033	—	—	—
1,600	2,0889	1,3000	1,6068	—	—	—
1,581	—	—	—	1,7498	1,2000	1,4582
1,500	2,0244	1,2552	1,6128	1,7196	1,1790	1,4585
1,400	1,9627	1,2102	1,6218	1,6799	1,1501	1,4607
1,300	1,9042	1,1652	1,6343	1,6401	1,1195	1,4650
1,200	1,8491	1,1202	1,6507	1,6003	1,0870	1,4722
1,100	1,7980	1,0754	1,6719	1,5618	1,0532	1,4829
1,000	1,7514	1,0309	1,6989	1,5254	1,0185	1,4977
0,900	1,7097	0,9870	1,7722	1,4919	0,9831	1,5175
0,800	1,6736	0,9439	1,7731	1,4620	0,9475	1,5430
0,700	1,6437	0,9019	1,8225	1,4366	0,9119	1,5754
0,600	1,6200	0,8613	1,8816	1,4166	0,8768	1,6156
0,500	1,6051	0,8227	1,9510	1,4029	0,8427	1,6648
0,400	1,5978	0,7867	2,0310	1,3964	0,8103	1,7233
0,300	1,5994	0,7544	2,1210	1,3979	0,7806	1,7908
0,200	1,6099	0,7271	2,2141	1,4078	0,7549	1,8649
0,100	1,6281	0,7068	2,3035	1,4253	0,7353	1,9383
0,000	1,6470	0,7973	2,3620	1,4442	0,7258	1,9898
$\eta_i\times$	$\Psi_2=1{,}00$			$\Psi_2=0{,}80$		
	$-\chi$	Ψ	$-\chi/\Psi$	$-\chi$	Ψ	$-\chi/\Psi$
1,414	1,3466	1,0000	1,3466	—	—	—
1,400	1,3465	0,9999	1,3466	—	—	—
1,300	1,3395	0,9945	1,3469	—	—	—
1,2911	—	—	—	1,0043	0,8000	1,2554
1,200	1,3252	0,9828	1,3484	1,0286	0,8199	1,2545
1,100	1,3007	0,9665	1,3520	1,0386	0,8286	1,2534
1,000	1,2860	0,9468	1,3583	1,0388	0,8287	1,2535
0,900	1,2649	0,9245	1,3682	1,0334	0,8230	1,2557
0,800	1,2448	0,9003	1,3827	1,0249	0,8129	1,2611
0,700	1,2265	0,8747	1,4022	1,0154	0,7996	1,2699
0,600	1,2115	0,8483	1,4282	1,0064	0,7838	1,2840
0,500	1,2008	0,8216	1,4615	0,9994	0,7664	1,3040
0,400	1,1955	0,7954	1,5030	0,9957	0,7481	1,3310
0,300	1,1968	0,7707	1,5527	0,9967	0,7300	1,3653
0,200	1,2051	0,7487	1,6107	1,0032	0,7131	1,4069
0,100	1,2206	0,7315	1,6686	1,0156	0,6993	1,4523
0,000	1,2380	0,7228	1,7128	1,0300	0,6921	1,4882

DOC = [redacted] PAGE 1534

Table II. 2. (con't)

$\eta_i^{\times}$	$\Psi_z=0,60$			$\Psi_z=0,40$		
	$-\chi$	Ψ	$-\chi/\Psi$	$-\chi$	Ψ	$-\chi/\Psi$
1,1952	0,7070	0,6000	1,1783	—	—	—
1,1181	—	—	—	0,4446	0,4000	1,1115
1,100	0,7589	0,6454	1,1759	0,4639	0,4175	1,1111
1,0541	—	—	—	—	—	—
1,0260	—	—	—	—	—	—
1,000	0,7855	0,6706	1,1712	0,5275	0,4778	1,1040
0,900	0,7982	0,6840	1,1670	0,5594	0,5113	1,0491
0,800	0,8029	0,6897	1,1645	0,5768	0,5320	1,0842
0,700	0,8029	0,6895	1,1645	0,5854	0,5448	1,0745
0,600	0,8006	0,6854	1,1681	0,5894	0,5516	1,0685
0,500	0,7977	0,6781	1,1764	0,5904	0,5540	1,0657
0,400	0,7958	0,6686	1,1902	0,5902	0,5532	1,0669
0,300	0,7964	0,6578	1,2107	0,5904	0,5500	1,0735
0,200	0,8006	0,6468	1,2378	0,5922	0,5453	1,0860
0,100	0,8092	0,6372	1,2699	0,5967	0,5403	1,044
0,000	0,8199	0,6318	1,2977	0,6032	0,5370	1,1233

$\eta_i^{\times}$	$\Psi_z=0,20$			$\Psi_z=0,10$		
	$-\chi$	Ψ	$-\chi/\Psi$	$-\chi$	Ψ	$-\chi/\Psi$
1,0541	0,2106	0,2000	1,0530	—	—	—
1,0260	—	—	—	0,1026	0,1000	1,0260
1,000	0,2656	0,2534	1,0481	0,1333	0,1302	1,0233
0,900	0,3142	0,3043	1,0325	0,1847	0,1840	1,0038
0,800	0,3403	0,3353	1,0149	0,2093	0,2139	0,9308
0,700	0,3553	0,3562	0,9975	0,2243	0,2343	0,9573
0,600	0,3635	0,3706	0,9808	0,2327	0,2490	0,9345
0,500	0,3675	0,3803	0,9661	0,2369	0,2595	0,9129
0,400	0,3686	0,3863	0,9542	0,2384	0,2669	0,8932
0,300	0,3684	0,3894	0,9461	0,2381	0,2722	0,8747
0,200	0,3681	0,3903	0,9431	0,2371	0,2749	0,8625
0,100	0,3687	0,3897	0,9461	0,2362	0,2759	0,8561
0,000	0,3708	0,3887	0,9539	0,2364	0,2758	0,8571

Page 329.

REFERENCES.

1. A. Busemann. Druke auf kegelförmige Spitzen bei Bewegung mit Überschallgeschwindigkeit, Z angew. Math. Mech., 9 (6), 496-498 (1929).

2. A. Busemann. Infinitesimale Kegelige Überschallströmung, Schriften der Deutschen Akademie der Luftfahrforschung, 7B, H 3, 105-122 (1943). [Is a Russian translation/conversion in the collection "gas dynamics", IL, 1950].

3. F. I. Frankl' and Ye. A. Karpovich. Thin-body gas dynamics, the State Technical Press, 1948.

4. R. T. Jones, D. Cohen. Aerodynamics of wings at high velocities. Chapter I in the book of the "aerodynamicist of aircraft components at high velocities". IL, M., 1959.

5. W. D. Hays, R. F. Probstein. Hypersonic Flow Theory. Academic Press, New York - London, 1959. [Is the Russian translation/conversion: W. D. Hays, R. F. Probstein. Theory of the hypersonic flows. IL, M., 1962].

6. A. Ferri. Supersonic Flow Around Circular Cones at Angles of Attack, NACA, Report No. 1045, 1951.

7. A. L. Gonor. Flow about the conical bodies with motion of gas at high supersonic velocity, Izv. of the AS USSR, mechanic and machine-building, No 1, 34-40, 1959.

8. R. A. Scheuing. Outer inviscid hypersonic flow with attached shock wavex, ARS J, 31 (4), 486-505 (1961). [Is the Russian translation/conversion: "rocket engineering", No 4, 23-45, 1961].

9. B. M. Bulakh. To the theory of nonlinear conical flows, PMM, Vol. XIX, of iss. 4, 394-409 (1955).

10. M. J. Lighthill. The Shock Strength in Supersonic "Conical Fields", Phil. Mag., 40, 7 s, No 311, 1202-1223 (1949).

11. A. Busemann. Die achsensymmetrische kegelige Uberschallstromung, Luftfahrtforschung, 19 (4), 137-145 (1942). [Is a Russian translation/conversion in coll. "gas dynamics", IL, 1950].

12. J. W. Reyn. Differential-geometric Considerations on the Hodograph

Transformation for Irrotational Conical Flow, Archive Rat. Mech. Anal., 6 (4), 229-354 (1960).

13. R. Kurant and D. Gilbert. Methods of mathematical physics. Gostekhizdat M., l. 1951, Vol. I, chapter II, §8; T. II, chapter I, §6; chapter V, §§4-7; supplement to chapter V, §2.

Page 330.

14. B. M. Bulakh. Nonlinear conical flows of gas, PMM, Vol. XXII, NO 6, 781-788 (1958).

15. J. H. Giese, H. Cohn. Canonical Equations for Non-Linearized Steady Irrotational Conical Flow, Quart. Appl. Math., v 4, 351-360 (1955). [Is a Russian translation/conversion in the collection of "mechanic", 1956, 1, 22-32].

16. M. F. Bourquard. Ondes ballistiques planes obliques et ondes coniques. Application à l'etude de la résistance del'air Compte Rendus Ac. Sci. Paris, No 194 (7 Mars), 1932.

17. G. I. Taylor and J. W. Massoll. The Air Pressure on a Cone Moving at High Speeds, I-II, Proc. Roy. Soc., Ser., A, v 139, No A 838, 278-311 (1933).

18. J. W. Massolle. The Conical Shock Wave formed by a Cone moving at a High Speed, Proc. Roy. Soc., Ser. A, v 159, No 899, 459-472 (1937).

19. G. V. Lipman, A. Roshko. Cell/elements of gas dynamics, IL, M., 1960.

20. A. Ferri. Aerodynamics of supersonic flows, the State Technical Press of Moscow-Leningrad, 1952.

21. A Busemann. Aerodynamischer Auftrieb bei Ubershallgeschwindigkeit, Luftfahrtorschung, 12, 210-220 (1935). [Is a Russian translation/conversion in coll. "gas dynamics", GONTI, 1939].

22. Ya. B. Zeldovich and Yu. P. Rayzer. Physics of shock waves and high-temperature hydrodynamic phenomena, "science", M. 1966.

23. R. F. Riddell. Ed., ch. "Chemical Kinetic Effects" in Hypersonic Flow Research, Academic Press, New York - London, 141-317, 1962. [Is the Russian translation/conversion: the "study of hypersonic flows", "world", M., 104-230, 1964].

24. A. S. Predvoditelev et al. Tables of the thermodynamic functions of air, Vol. I, of the AS USSR, 1962; Vol. II, the AS USSR, 1957; Vol. III, the AS USSR, 1959.

25. S. I. Kuznetsov. Diagrams and the tables of the flow of the dissociating air about wedge, cone and convex surface, Oborongiz, 1962.

26. N. M. Kuznetsov. Thermodynamic functions and Hugoniot curves of air at high temperatures, "machine-building", 1965.

27. A Ferri. A Review of some recent developments in hypersonic flow, Advances in Aeronautical Sciences, vol. 2, 723-770, Pergamom Press, 1959. [Is Russian translation/conversion in the collection of the "problem of flight at high velocities", IL, M., 1960].

28. A. N. Krayko. Analytical representation of the thermodynamic functions of air, eng. journal, Vol. IV, No 3, 548-550, 1964.

29. K. I. Babenko, G. P. Voskresenskiy, A. N. Lyubimov, V. V. Rusanov. Three-dimensional/space flow of perfect gas about the smooth bodies. "science" M., 1964.

Page 331.

30. Yu. A. Kibardin, S. I. Kuznetsov, A. N. Lyubimov and B. Ya. Kuznetsov. Atlas of gas-dynamic functions at high velocities and the high temperatures of air flow, Gosenergoizdat, Moscow-Leningrad, 1961.

31. G.K. Inger, D.A. Meis. Shock polars for dissociating gasses, AIAA J, 1 (8), 1771-1775 (1963). [Is the Russian translation/conversion: "rocket engineering and astronautics", 1963, No 8, 13-19].

32. J.C. South and P.A. Newman. Application of the Method of Integral Relations to Real-Gas Flows Past Ponted Bodies, AIAA, J, v 3 (9), 1645-1652 (1965). [Is the Russian translation/conversion: "rocket engineering and astronautics", 1965, No 9, 100-110].

33. Oswatitsch K. und Sjödin L., Kegelige Überschallströming in Schallnähe, Österreichischen Ingenieur — Archiv, B. VIII, H. 4, 284—282 (1954).

34. J.A. Cole, G.E. Solomon and W.W. Willmarth. Transonic Flow Past Simple Bodies, JAS, 20 (9), 627-634 (1953). [Is a Russian translation/conversion in the collection of "mechanic", No 6, 56-68 (1954)].

35. G.W. Johnston. An Investigation of the Flow About Cones and Wedges at and Beyond the Critical Angle, JAS, 20 (6), 378-382 (1953). [Is a Russian translation/conversion in the collection of "mechanic", No 2,

20-26, 1954].

36. Zienkiewicz H. K., Flow about Cones at Very High Speeds, Aeron. Quart., v. VIII, 4, 384—394, 1957.

37. S.Feldman. Hypersonic Conical Shocks for Dissociated Air in Thermodynamic Equilibrum, Jet Propulsion, v 27 (12), 1253-1254 (1957).

38. M. F. Romig. Application of the hypersonic similarity rule to conical flow of dissociated air, Aero-Space Eng., v 18 (3), 56-59 (1959).

39. F. E. Ehlers. Equilibrium flow of an ideal dissociating gas over a cone, AIAA, J, v 3 (8), 1529-1530 (1965). [Is the Russian translation/conversion: "rocket engineering and astronautics", 1965, No 8, 213-214].

40. V. H. Blackman. Vibrational Relaxation in Oxygen and Nitrogen, J Fluid Mech., v 1 (1), 61-85 (1965).

41. R. Sedney, N. Gerber. Nonequilibrium Flow over a Cone, AIAA J, v 1 (11), 2482-2486 (1963). [Is the Russian translation/conversion: "rocket engineering and astronautics", 1963, No 11, 22-28].

42. R. E. Oliver. An experimental investigation of flow over simple blunt bodies at nominal Mach number of 5.8, JAS, 23 (2), 177-179

(1956). [Is a Russian translation/conversion in the collection of "mechanic", 1959, No 6, 39-42].

43. R.H.Johnson. The Cone-Sphere in Hypersonic Helium Above Mach 20, Aero/Space Eng., v 18 (2), 30-34, 53, 1959.

44. P. I. Chushkin. Study of the flow about the blunted bodies of revolution at hypersonic speeds, Zh. Vychisl. math. and math. Physicists, Vol. 2, No 2, 255-275 (1962).

Page 332.

45. P. I. Chushkin, N. P. Shulishina. Tables of supersonic flow about blunt-nosed cones. Publ. CC of the AS USSR, 1961.

46. G. G. Chernyy. Flows of gas at high supersonic velocity, Fizmatgiz, 19 9.

47. M.H.Bertram. Tip-bluntness effects on cone pressure at M = 6, 58, JAS, v 23 (9), 898-900 (1965).

48. H.K.Cheng. Similitude of Hypersonic Real-Gas Flows over Slender Bodies With Blunted Noses, JAS, v 26 (9), 575-588 (1959).

49. J. Lukasiewicz, J. D. Whitfield and R. Jackson. Aerodynamic Testing at Mach Numbers from 15 to 20, Nupersonic Flow Research, Academic Press, New York - London, 473-512, 1962. [Is the Russian translation/conversion: the study of hypersonic flows, world", M., 327-356, 1964].

50. J. K. Yakura. Theory of Entropu Layers and Nose Bluntness in Hypersonic Flow, Hypersonic Flow Research, Academic Press, New York - London, 421-457, 1962. [Is the Russian translation/conversion: the study of hypersonic flows, "world", M., 298-324, 1964].

51. B. Dayman Jr. Hypersonic Viscous Effects on Free - Flight Slender Cones, AIAA J, v 3 (8), 1391-1400 (1965). [Is the Russian translation/conversion: "rocket engineering and astronautics", 1965, No 8, 22-33].

52. W. C. Lyons Jr, J. J. Brady, Z. J. Levensteins. Hypersonic Drag, Stability and Wake Data for Cones and Spheres. AIAA J, v 2 (1), 1948-1956 (1964). [Is the Russian translation/conversion: "rocket engineering and astronautics", 1964, No 11, 74-84).

53. M. Van Dyke. Perturbation Methods in Fluid Mechanics, Academic Press, New York - London, 1964, Ch. IX, §9.6-9.8. [Is the Russian translation/conversion: M. Van Dyke, the perturbation methods in fluid mechanics, "world", M., 1967].

54. M. D. Van Dyke. First and Second-Order Theory of Supersonic Flow Past Bodies of Revolution, JAS, v 18 (3), 161-178 (1951).

55. J. B. Broderick. Supersonic Flow past Round Pointed Bodies of Revolution, Quart. Jour. Mech. Appl. Math., v 2 (1), 98-120 (1949).

56. D. N. Morris. A Summary of the Supersonic Pressure Drag of Bodies of Revolution, JAS, v 3 (7), 563-572 (1961).

57. H. E. Fettis. An Approximate Solution to Supersonic Conical Flow, JAS, 23 (12), 1122-1123 (1956).

58. L. Pottsepp. Inviscid Hypersonic Flow Over Unyawed Circular Cones, JAS, v 27 (7), 558-559 (1960).

59. O. P. Sidorov. On the calculation of the flow about cone, Izv. of the higher edcnl. institutions, avn. technician, No 2, 144-146, 1959.

60. N. Girishwar. Hypersonic Flow Past Unyawed Circular Cones, J Indian Math. Soc., v XXVIII (1), 7-24 (1964).

Page 333.

61. V. P. Ionov. Determination the flow parameters of gas of the surface of the cone, which moves at a high speed, taking into account gaseous dissociation (approximation methods), coll. "phased gas dynamics and heat exchange", the publ. of the AS USSR, M., 1961.

62. A. S. Mikheev. Analytical solution of the problem of the flow of supersonic flow about the cone of gas, the herald of L.G.U., No 7, 105-112, 1962.

63. L. Lees. Note on the hypersonic-similarity law for an unyawed cone, JAS, v 18 (19), 700-702 (1951).

64. A.G.Hammitt and K.R.A.Marthy. Approximate Solutions for Supersonic Flow over Wedges and Cones, JAS, v 27 (1), 71-73 (1960).

65. H.Williams. Hypersonic Conical Flow, ARS, J, v 29 (9), 46-48 (1959).

66. A. A. Il'yushin. Law of flat/plane sections in aerodynamics high supersonic velocities, PMM, Vol. 20, of iss. 6, 733-755 (1956).

67. G. M. Bam-Zelikovich, A. I. Bunimovich, M. P. Mikhaylov. Motion

of slender bodies at high supersonic velocities, in the collection "theoretical hydromechanics", No 4, Oborongiz, M., 1949.

68. M. D. Van Dyke. Applications of hypersonic small-disturbance theory, JAS., v of 21 (3), 179-186 (1954) [is a Russian translation/conversion in the collection of "mechanic", 1955, No 3, 49-63].

69. G. G. Chernyy. Flow of gas about the bodies with high supersonic velocity, PAS SSSR< 107, 2 (1956); Mitt. Forschungsinst. Phys. Strahlantriebe, Stuttgart, Intern. Tagung, 6-8, Febr. 1956.

70. K. Oswatitsch, S. B. Berndt. Aerodynamics similarity at axisimmetric transonic flow around slender bodies, KTN Aero., TN 15, Roy. Inst. Tech. Stockholm, 1950.

71. W. D. Hayes. On hypersonic similitude, Quart, Appl. Math., 5 (1), 105-106 (1947). [Is a Russian translation/conversion in the collection of "mechanic", 1950, N 2, 26-27].

72. M. D. Van Dyke. The combined supersonic-hypersonic similarity rule, JAS., v 18 (7), 499-500 (1951). [Is a Russian translation/conversion in the collection of "mechanic", 1952, No 2, 64-66].

73. N. F. Krasnov. Aerodynamics of bodies of revolution, Oborongiz, Moscow, 1958.

74. W. E. Simon, L. A. Alter. Approximations for supersonic flow over cones, AIAA, J, v 1 (7), 1696-1698 (1963). [Is the Russian translation/conversion: "rocket engineering and astronautics", 1963, No 7, 244-246].

75. R. D. Linell and J. Z. Baily. Similarity-Rule Estimation Method for Cones and Parabolic Noses, JAS., v 23 (8), 796-797 (1956). [Is a Russian translation/conversion in the collection of "mechanic", 1957, No 2].

76. J. W. Cleary. Wadge and cone theory for $M_1 = \infty$, AIAA J, v 3 (10), 1983-1984 (1965). [Is the Russian translation/conversion: "rocket engineering and astronautics", 1965, No 10, 262-263].

Page 334.

77. G. W. Zumwalt, H. H. Tang. Mach Number Independence of the Conical Shock Pressure Coefficient, AIAA J, v 1 (10), 2389-2391 (1963). [Is the Russian translation/conversion: "rocket engineering and astronautics", 1963, No 10, 190-191].

78. R. A. Hord. Conical Shock-Wave Angle, AIAA, J, v 2 (7), 1359-1360 (1964). [Is the Russian translation/conversion: "rocket engineering and astronautics", 1964, No 7, 248].

79. S. S. Kvashina, G.G. black. Establish/installed flow of flow about the cone of the detonating gas, PMM, Vol. XXIII, of iss. 1, 182-186 (1959).

80. G. G. Chernyy. Self-similar problems of the flow of combustible mixture about the bodies of gases. Proceedings of the AS USSR, the "mechanic of liquid and gas", No 6, 10-24, 1966.

81. A. A. Nikol'skiy. Conical axially symmetric supersonic gas flows of evacuation/rarefaction, coll. of theoretical works on aerodynamics, M., Oborongiz, 43-55, 1957.

82. G. L. Grodzovskiy. Supersonic axially symmetric conical flows with oblique shock, which border to parallel steady flow, PMM, Vol. XXIII, iss. 2, 379-383 (1959).

83. G. I. Maykapar. On the wave impedance of axially nonsymmetric bodies in supersonic flow, PMM, Vol. XXIII, issue 2, 376-378 (1959).

84. T. Nnonweiler. Aerodynamic Problems of Manned Space Vehicles, J. Roy. Aeron. Soc., v 63 (585), 521-528 (1959).

85. T. Nonweiler. Delta wings of Shapes Amenable to Exact Shock-Wave Theory, J Roy. Aeron. Soc., v 67 (625), 39-40 (1963).

86. V. V. Keldysh. Exact solutions for carrier systems with one and two step shocks of packing/seal, eng. journal, Vol. I, iss. 3, 22-39 (1961).

87. D.H.Peckham. On Three-Dimensional Bodies of Delta Planform wich can Support Plane Attached Shock Waves, A.R.S.C.P., No 640, 1963.

88. A. L. Gonor. Conical bodies of smallest resistance in hypersonic flow of gas, PMM, Vol. XXVIII, of iss. 2, 383-386 (1964).

89. A. L. Gonor. The exact solution of the task of the flow of supersonic flow about some three-dimensional bodies of gas, PMM, Vol. XXVIII, of iss. 5, 974-976 (1964).

90. G. I. Maykapar. On the construction of supersonic flow about solid bodies with the aid of the step shocks of packing/seal, Izv. of the AS USSR, mech- and machinebuilding. No 5, 142-144, 1964.

91. A. L. Gonor. Some three-dimensional/space flows with the Mach

interaction of the shock waves. Izv. of the AS USSR, mechanic of liquid and gas, No 6, 135-140, 1966.

92. J. W. Flower. Configurations for High Supersonic Speeds Derived for Simple Shock-Wave and Expansions, J. Roy. Aeron. Soc., v 67 (632), 476-480 (1963).

Page 335.

93. A. L. Gonor, A. I. Shvets. Study of the system of jumps during the flow about the star-shaped bodies. Izv. of the AS USSR, mechanic of liquid and gas, No 3, 98-102, 1966.

94. G. G. Chernyy. To the study of the bodies of smallest resistance high supersonic velocities. PMM. Vol. XXVIII, iss. 2, 387-388 (1964).

95. L.C.Squire. Pressure Distributions and Flow Patterns at M = 4.0 on Some Delta Wings, ARC RM, No 3373, 1964.

96. V. V. Keldysh. Study of flow in the vicinity of the V-shaped wings, formed by stream surfaces after the step shock of packing/seal. Proceedings of the AS USSR, the "mechanic of liquid and gas", No 4, 50-55, 1967.

97. A. L. Gonor, A. I. Shvets. Study of the distribution of pressure on some star-shaped bodies during Mach number $M \approx 4$, PMTF, No 6, 122-125, 1965.

98. A. A. Nikol'skiy. Some exact solutions of the equations of the three-dimensional/space flows of gas. Coll. of theoretical works on aerodynamics. Oborongiz, M., 27-33, 1957.

99. J. H. Giese. Compressible flows with degenerate hodographs, Quart. Appl. Math., v 9, 237-246 (1951).

100. S. V. Ballander. Developable wings, the herald of L.G.U., No 19, the series of math., mech., astron., iss. 4, 113-120, 1959.

101. D.F. Parker. Bodies with adjoin regions of simple wave supersonic flow, Quart. J Mech. and Appl. Math., v 18 (3), 299-323 (1965).

102. W.G.Vincenti and N.H.Fischer Jr. Calculation of the supersonic pressure distribution on single - curved tape-red wing in regions not influenced by the root or tip, NACA T.N. No 3499 (1955).

103. J.H.Giese, H.Cohn. Two non-linearized conical flows, Quart. Appl. Math., v 11 (1), 101-108 (1953). [Is a Russian translation/conversion in sb. "mechanic", 1954, 5, 107-115].

104. B. M. Bulakh. To the conical flow theory, PMM, Vol. XXI, of iss. 1, 143-144 (1957).

105. M. A. Holt. Vortical singularity in conical Flow, Quart. J Mech. Appl. Math., v 7 (4), 438-445 (1954). [Is a Russian translation/conversion in sb. "mechanic", 1955, 5, 57-63].

106. B. M. Bulakh. Some questions of the conical flow theory, PMM, Vol. XXV, of iss. 2, 229-241 (1961).

107. P. I. Chushkin, V. V. Shchennikov. Calculation of some conical flows without axial symmetry, eng. physical journal, Vol. 3 (7), 88-94 (1960).

108. V. S. Avduyevskiy, K. I. Medvedev. Study of the breakaway of laminar boundary layer on cone at an angle of attack, Izv. of the AS USSR, mechanic of the liquid of gas, No 3, 117-119, 1966.

109. A. P. Bazzhin, I. F. Chelysheva. Application/use of a method of straight lines on the calculation of the flow about the conical bodies at high angles of attack, Proceedings of the AS USSR, mechanic of liquid and gas, No 3, 119-123, 1967.

Page 336.

110. J.W.Britton. Pressure Measurements at Supersonic Speeds on Three Uncambered Conical Wings of Unit Aspect Ratio, A.R.C.C.P, No 641 (1963).

111. R.Legendre. Ecoulement au voisinage de la pointe avant d'une aile à forte fleche aux incidences moyennes, Rech. Aeron, No 30, 3-8, 1952; No 31, 3-6, 1953.

112. R.Legendre. Neppes en Cornet aux bords d'attaque d'un aile en delta. Rech. Aéron. No 70, 3-10, 1959.

113. A.Stanbrook and L.C.Squire. Possible Types of Flow at Swept Leading Edges, Aeronaut. Quart., v XV (1), 72-82 (1964).

114. P.M.Stocker, F.E.Mauger. Supersonic flow past cones of general cross-section, J.Fluid Mech., v 13 (3), 383-399 (1962).

115. D.W.Eastman, M.E.Omar. Flow fields about highly yawed cones by the inverse method, AIAA, J, v 3 (9), 1782-1784 (1965). [Is the Russian translation/conversion: rocket engineering and astronautics, 1965, No 9, 271-273].

116. A.H.Stone. On Supersonic Flow Past a Slightly Yawing Cone, J Math. Phys., I, 27 (1), 67-81 (1948); II, v 30 (4), 220-213 (1952).

117. A.H.Stone. Corections to the Paper "On Supersonic Flow Past a Slightly Yawing Cone", J Math. Phys., v of 31 (4), 300 (1953).

118. B. M. Bulakh. Supersonic flow about the inclined round cone, PMM, Vol. XXVI, iss. 2, 300-307 (1962).

119. K.W.Mangler and J.H.B.Smith. A theory of the flow past slender delta wing with leading edge separation, Proc. Roy. Soc. A v 251-(1265), 200-217 (1959).

120. A.Ferri, N.Ness, T.T.Kaplita. Supersonic flow over conical bodies without axial symmetry, JAS, v 20 (8), 563-571 (1953). [Is a Russian translation/conversion in sb. "mechanic", 1954, No 3, 3-19].

121. G.N.Ward. Linearized theory of steady high-speed flow, Cambridge, 1955.

122. B. M. Bulakh. On some properties of the supersonic conical flows of gas, PMM, Vol. XXV, No 3, 478-484 (1961).

123. F K Moore. Second Approximation to Conical Flow, JAS, v 17 (6), 328-334 (1950).

124. M.D.Van Dyke. The Slender Elliptic Cone as a Model for Nonlinear Supersonic Flow Theory, J Fluid Mech., v 1 (1), 1-15 (1956).

125. M.D.Van Dyke. Subsonic edges in thin-wing and slenderbody theory, NACA TN, No 3343 (1954).

126. E.Y.G.Sun. Nicht-angestellte Delatflügel mit Unterschall und Schallvorderkanten, J Méchanique, v 3 (2), 142-163 (1964).

Page 337.

127. L.E.Fraenkel and K.Watson. The formulation of a uniform approximation for thin conical wings with sonic leading edges, "Sympos. Transsonicum, Aachen, 1962", Berlin-Göttingen-Heidelberg, 249-263, 1964.

128. K. V. Frick. experimental aerodynamics of wings with transonic and supersonic velocities, chapter VII in book of "aerodynamicist of aircraft components at high velocities", IL, M., 1959.

129. M.Sugo. The Second Order Theory of the Supersonic Three-Dinsional Thin-Wing Expansion, Trans. Japan Soc. Aeron. Space Sci. v 2 (2), 1-10 (1959).

130. H.S.Tan. Second Approximation to Conical Flows, Cornell. Univ. Garad. School Aero-Eng., Ithaca, 1950.

131. M.J.Lighthill. Higher Approximation, Sect. E in "General theory of high speed Aerodynamics", Oxford Univ. Press., 345-490, 1955.

132. M.Fenain et P.German. Sur la résoltuion de l'équation regissant en seconde approximation, les écoulements d'un fluide autour d'obstacles tridimensionnels, C.R.Acad. Sci. Paris, t of 241 (3), 276-278 (1955).

133. J.B.Broderick. Supersonic flow past a semi-infinite cone, Quart. J Mech. Appl. Math., v 2 (1), 121-128 (1949).

134. K.Sauer. Supersonic flow about projectile heads of arbitrary shape at small incidence, Luftfahrtforschung, 19 (4), 148-152 (1942).

135. M.J.Lighthill. Supersonic flow past slender pointed bodies of

135. M. J. Lighthill. Supersonic flow past slender pointed bodies of revolution at yaw, J Mech. Appl. Math. 1, 76 (1948).

136. A.Ferri. The linearized characteristics method and its application to practical nonlinear supersonic problem, NACA, TN, No 2515 (1951).

137. R.Vaglio-Laurin, M.D. Van Dyke. A discussion about a slender elliptic cone, J.Fluid Mech., v 3 (6), 638-644 (1958)

138. A.Martellucci. An extension of the linearized characteristic method for calculating the supersonic flow around elliptic cones, JAS, v 27 (9), 667-674 (1960).

139. G.B.W. Young and C.P. Siska. Supersonic flow around cones at large yaw, JAS, v 19 (2), 111-119, 142 (1952).

140. R.C.Roberts and J.D.Riley. A guide to the use of the M.I.T cone tables, JAS, v of 21 (5), 336-342 (1954).

141. F.W.Barry. On the limiting yaw angle for Ston's yawing cone theory, JAS, v 29 (5), 613-614 (1962).

142.
M.Holt and J.J.Blackie. Experiments on circular cones at yaw in supersonic flow, JAS, v 23 (10), 931-936, (1956). [Is a Russian

translation/conversion in the collection of "mechanic", 1957, No 4, 54-62].

143. A.Ferri. Supersonic flow around circular cones. NACA TN, No 2236 (1950).

144. J.E.Willett. Supersonic Flow at the Surface of a Circular Cone at Angle of Attack, JAS, v 27 (12), 907-912, 920 (1960).

145. J.E.Willett. A note on the inviscid streamlines at the surface of a yawed cone in Supersonic stream, JAS, v 29 (3), 375-376 (1962).

Page 338.

146. B.A.Woods. The Flow Close to the Surface of a Yawed Cone in Supersonic Stream, Applied Mechanics (Proc. 10 Intern. Congr. Appl. Mech., Stresa, 1960), Elsever Publ. Comp., Amsterdam - New York, 209-210, 1962.

147. H.K.Cheng. On the Structure of Vortical Layers in Supersonic and Hypersonic Flows, JAS, v 27(2), 155-156 (1960).

148. H.K.Cheng. Hypersonic flow past a yawed circular cone and other poited bodies, J.Fluid Mech., v 12 (2), 169-191 (1962).

149. B.A.Woods. The flow close to the surface of a circular cone at incidence to a supersonic stream, Aeronaut. Quart., v 13 (2), 115-128 (1962).

150. B.A.Woods. Comments on the paper: "Supersonic flow at the surface of a circular cone at angle of attack", by J.E.Willett, JAS, v 29 (4), 481-482 (1962).

151. A.G.Munson. The vortical layer on inclined cone, J.Fluid Mech., v 20 (4), 625-643 (1964).

152. B.A.Woods. The supersonic flow past a curcular cone at incidence, ARC RM, No 3413(1965).

153. Ya. G. Sapunkov. Vorticity layers during the flow about the conical bodies super and by hypersonic flows of gas, dissertation, Saratov university, 1964.

154. V. Zakkay. An Investigation of the Pressure Distribution on Conical Bodies in Hypersonic Flows, JAS, v 26 (7), 457 (1959).

155. W.P.Jacobs. A simplified approximate method for the calculation

of the pressure around conical bodies of arbitrary shape in supersonic and hypersonic flow, JAS, v 28 (12), 987-988 (1961).

156. W.H.Michael. Flow Studies on Flat-Plate Delta Wings at Supersonic Speeds, NACA TN, No 3472 (1955).

157. Mac C.Adams. Leading is Edge Separation from Delta Wings at Supersonic Speeds, JAS, v 20 (6), 430 (1953).

158. R.H.Edwards. Leading-Edge Separation from Delta Wings. JAS, v of 21 (2), 134-135 (1954).

159. C.E.Brown and W.H.Michael Jr. Effect of Leading-Edge Seaparation on the Lift of a Delta Wing, JAS, v of 21 (10), 690-694, 706 (1954).

160. L.C.Squire. The Estimation of the Non-Linear Lift of Delta Wings at Supersonic Speeds. J. Roy. Aeron. Soc., v 67 (632), 476-480 (1963).

161. D.Küchemann. Report of the I.U.T.A.M. symposium on concentrated vortex motions in fluid, J.Fluid Mech., v 21 (1) 1-20 (1965). [Is a Russian translation/conversion in the collection of "mechanic", 1966, 2, 66-84].

162. Zh. Guiraud. Basic questions of the theory of hypersonic flows, "world", M., 1965.

163. I. I. Nochevkin. Flow of supersonic flow about the conical bodies of ideal fluid when angle of attack is present,. Zh. Vychisl. math. and math. physicists, Vol- 1, No 1, 144-150 (1961); t 2, No 3, 511 (1962).

Page 339.

164. V. F. Syagaev. Method of the numerical solution of the task of the flow of supersonic flow about the conical bodies of gas. I. comp. math. physicists, Vol. 3, No 4, 742-754 (1963).

165. I. A. Makhin, V. F. Syagayev. To the numerical solution of the task of the supersonic flow about the conical bodies at an angle of attack. Izv. of the AS USSR, mechanic of liquid and gas, No 1, 140-142, 1966.

166. G.Radhakrishnan. The exact flow behind a yawed sonical shock, Coll. of Aero., Rep. No 116 (1958).

167. B.P.Briggs. Calculation on supersonic flow past bodies shaped like elliptic cones, NASA Rep., no D-24.

168. A. L. Gonor. Flow about the cone at an angle of attack with high supersonic velocity, Izv. of the AS USSR, OTN [Department of Technical Sciences], No 7, 102-105, 1958.

169. A. Ferri. Conial Flow, Section H, Chapter 3, 721-747, in General theory of high speed Aerodynamics, Oxford University Press, 1955.

170. A. A. Dorodnitsin. On one method of the numerical solution of some nonlinear tasks of aerohydrodynamics. Transactions of 3rd All-Union math. congress/descent, Vol. 2, 78, M, 1956; Actes congres international de mechanique applique 9, t I, 485, Bruxelles, 1957.

171. V. P. Kolodochkin. Calculation of the supersonic flow about the cone at the angles of attack, Izv. of the AS USSR, OTN, mechanic and machine-building, No 5, 38-43, 1961.

172. S. M. Gilinskiy, G. F. Tele-Nina, G. P. Tinyakov. Method of the calculation of the supersonic flow about the blunted bodies with detached shock wave, Izv. of the AS USSR, mechanic and machine-building, No 4, 9-28, 1964.

173. A. S. Avdonin. Conical wing in the supersonic flow of gas, the herald of L.G.U., No 7, the series of math., mech., astron., iss. 2, 62-74, 1961.

174. N. V. Smirnov. On supersonic conical flows with the complicated thermodynamics, the herald of L.G.U., No 7, the series of math., mech., astron., 113-118, 1962.

175. S. V. Vallander. Numerical determining of the aerodynamic characteristics of some finite-span wings, herald of L.G.U., No 19, the Series of math., mech., astron., iss. 4, 106-112, 1959.

176. A. P. Bazzhin. Calculation of flow about the lower surface of delta wings at high angles of attack, engineering i., Vol. IV, iss. 2, 242-246 (1964).

177. A. P. Bazzhin. On the calculation of the flow about the flat/plane delta wings at the high angles of attack, Izv. of the AS USSR, mech. liq. and gas, No 5, 104-105, 1966.

178. A. Ferri. Recent Theoretical Work in Supersonic Aerodynamics at the Polytechnic Institute of Brooklyn. Proc. Conf. on High-Speed Aeronautics, 341-362, PIB, 1955.

179. B. M. Bulakh. Observation to L. R. Fauell's article "precise and approximate solution for supersonic delta-like covered", PMM, Vol. XXII, No 3, 404-407 (1958).

Page 340.

180. B. M. Bulakh. Observation to A. Ferri's report "new theoretical work on supersonic aerodynamics in the Brooklyn polytechnic institute" PMM, Vol. XXIII, iss. 3, 576-580 (1959).

181. B. M. Bulakh. Observation to D. U. Reyn's article "Differential-geometric examinations of the conversion of hodograph for conical flow", PMM, Vol. XXVI, No 4, 793-797 (1962).

182. B. M. Bulakh. Shock waves in conical flows, PMM, Vol. 29, No 5, 969-972 (1965).

183. B. M. Bulakh, D. U. Reyn. Comments on the problems of a delta wing. PMM vol. 31, issue 1, 190-192 (1967).

184. G. G. Chernyy. Wings in hypersonic flow, PMM, Vol. 29, iss. 4, 616-634, (1965).

185. D. A. Babaev. Numerical solution of the task of the flow suction side of wing of supersonic flow about, Zh. Vychisl. math. and math. physics, Vol. 2, No 2, 278-289 (1962).

186. St. Maslen. Supersonic Conical Flow, NACA TN 2651, March 1952.

187. L.K.Powell. Exact and Approximate Solutions for the Supersonic Delta Wing, JAS, v 23 (8), 709-720 (1956).

188. W.J.Bannink, C.Nebbeling and J.W.Reyn. Investigation on the Flow Field on the Expansion Side of a Delta Wing with Supersonic Liding Edges, Techn. Univ. Delft, Report VTN-128, August, 1965.

189. D.Treadgold. Experimental Study of the Flow over a Particular Afterbody Shape having a Near-Sonic Ridge Line, ARC CP, No 546, 1961.

190. D.Pierce and D.Treadgold. Some examples of the use of a conical shadowgraph thechnique, ARC CP, No 763, 1964.

191. M.H.Bertram, A.Henderson Jr. Recent hypersonic studies of wings and bodies, ARS J, v of 31 (8), 1129-1139 (1961). [Is the Russian translation/conversion: rocket engineering and astronautics, 1961, No 8, 101-114].

192. D. A. Babaev. Numerical solution of the task of the flow the lower surface of delta wing of the supersonic flow about of gas, Zh. Vychisl. math. and math. physicists, Vol. 2, No 6, 1086-1101 (1962).

193. J.H.Clarke and J.Walles. Uniform second-order solution for supersonic flow over delta wing using reverseflow integral method. J Fluid Mech., v 18 (2), 225-238 (1964).

194. S.P. Sutera. Second-Order Forces and Pressure Distributions on Three Dimensional Bodies by Reverse-Flow Integral Method, AIAA, J, v 3 (3), 441-450 (1965). [Is the Russian translation/conversion: "rocket engineering and astronautics", 3, 75-88, 1965].

195. S. M. Grind-Ter-Minas4nq. Task of the supersonic flow about the lower surface of delta wing, Izv. of the AS USSR, mechanic of liquid and gas, No 6, 147-152, 1966.

196. D. A. Melnikov. Flow of the supersonic flow about flat/plane triangular plate, Izv. of the AS USSR, OTN, mechanic and machine-building, No 6, 33-39, 1962.

Page 341.

197. R. Chapkis. Hypersonic flow over an elliptic cone: Theory and experiment, JAS. v 28, 844-854 (1961).

198. S. Molder. Internal, axisymmetric, conical flow, AIAAJ., v 5, No 7, 1252-1255 (1967). [Is the Russian translation/conversion: rocket engineering and astronautics, 1967, NO 7, 47-51].

199. M.D.High and E.F.Blick. Cone Pressure Distribution at Large and Small Angles of Attack AIAA J, v 2(11), 2054-2055 (1964). [Is the Russian translation/conversion: rocket engineering and astronautics, 1964, No 11, 230-231].

200. Z.Kopal. Tables of supersonic flow around jawing cones, M.I.T., Techn. Rep., No 3, 1947.

201. Z.Kopal. Tables of supersonic flow around cones of large jaw, MIT, Techn. Rep., No 5, 1949.

202. J.P.Guiraund. Newtonian Flow over a Surface-Theory and Applications, Hypersonic Flow, Ed. by A.R.Collar, J.Tinkler, London, Butterworths Sci. Publications, 253-296, 1960.

203. Ya. G. Sapunkov. Hypersonic flow about the round cone. PMM, Vol. XXVII, iss. 1, 190-192 (1963).

204. Ya. G. Sapunkov. Round cone at an angle of attack in hypersonic flow of gas, PMM, Vol. XXVII, iss. 5, 930-939 (1963).

205. Ya. G. Sapunkob. Hypersonic flow around a round core at an angle of attack by a balanced gas flow. Izv. AN SSSR, Mechanics and Machine building, No. 4. 4-8, 1964

206. R.E.Melnik and R.A.Scheuing. Shock layer structure and entropy layers in hypersonic conical flows, Hypersonic Flow Research, Academic Press, New York - London, 379-420, 1962. [Is the Russian

translation/conversion: the "study of hypersonic flows", "world", M., 268-295, 1964].

207. B. M. Bulakh. On the asymmetric hypersonic flow about the round cone. PMM, Vol. XXVI, No 5, 973-976 (1962).

208. P. Laval. Ecoulement supersonique au voisinage d'une surface conique de forme arbitraire, Publ. ONERA, No 97 (1963).

209. Ya. G. Sapunkov. Hypersonic flow about the conical bodies. Izv. of the AS USSR, mechanic of liquid and gas, No 1, 137-139, 1966.

210. R.E.Melnik. Newtonian entropy layer in the vicinity of a conical summetry plane, AIAA J, v 3 (3), 520-522 (1965). (Is the Russian translation/conversion: rocket engineering and astronautics, 1965, No 3, 186-188).

211. A. L. Gonor. On the location of leading wave during the asymmetric flow of flow about the pointed body of gas at high supersonic velocity. Izv. of the AS USSR, OTN, mechanic and machine-building, No 5, 117-118, 1959.

212. I. Ginzel. Two remarks on cones at angle attack in high supersonic flow, J Aero/Space Sci., v 29 (4), 497-498 (1962).

213-214. A. L. Gonor. Flow of flow about the conical bodies of gas at high supersonic velocity, dissertation, MGU [Moscow State University], 1958.

Page 342.

215. A. L. Gonor. The asymmetric conical flows of gas after powerful shock wave, the coll. of the scientific of works all-Union it is external. Machinebuilding institute, iss. 3, 47-60, 1961.

216. A. I. Shvets. Study of the flow about elliptical cones, Izv. of the AS USSR, mechanic of liquid and gas, No 1, 130-137, 1966.

217. A.F.Messiter. Lift of sleuder delta wings according to Newtonian theory. AIAA, J, v 1 (4), 794-802 (1963). [Is the Russian translation/conversion: rocket engineering and astronautics, 1963, 4, 31-41].

218. J.D.Cole and J.J.Brainerd. Slender wings at high angles of attack in hypersonic flows, Hypersonic Flow Research (F.R.Riddel, ed.), Academic Press, New York - London, 321-343, 1962./ [is the Russian translation/conversion: the study of hypersonic flows, "world", M. 233-247, 1964].

219. V. A. Bashkin. Experimental studies of the flow about the flat/plane delta wings with M = 5 in the range of angles of attack from 0 to 70°. Izv. of the AS USSR, mechanic of liquid and gas No 3, 102-107, 1967.

220. K. Hida. Thickness effects on the force of slender delta wings in hypersonic flow, AIAA, J, v 3 (3), 427-433 (1965). [Is Russian translation: Rocket engineering and astronautics, 1965, No 3, 57-64].

221. A. Polak. Note on the Force of Slender Delta Wings, AIAA J, v 4 (3), 556-558 (1966). [Is the Russian translation/conversion: rocket engineering and astronautics, 1966, No 3, 227-229].

222. P. Mandl. A Theoretical Study of the Inviscid Hypersonic Flow about a Conical Flat-Top Wing-Body Combination, AIAA J, v 2 (11), 1956-1964 (1964). [Is the Russian translation/conversion: rocket engineering and astronautics, 1964, No 11, 84-94].

223. L.C.Squire. Applications of linearized characteristics to Sharp-edged conical bodies, Aeron. Quart., v 16 (1), 1-14 (1965).

224. V. V. Sichev. Three-dimensional/space hypersonic flows of gas

about slender bodies at the high angles of attack, PMM, Vol. XXIV, 2, 205-212 (1960).

225. V. N. Gusev, T. V. Klimov, A. S. Korolev, S. G. Kryukova, V. S. Nikolayev. Flow of viscous hypersonic flow about the fine/thin pointed cones of gas, engineering i., Vol. 5, No 3, 416-424 (1965).

226. D. A. Babaev. Flow about the delta wing with large mach numbers. J. of computational mathematics and mathematical physics, vol. 3, NO 2, 397-400 (1963).

227. P. Laval. Hypersonic flow over conical surfaces, ONERA Publ. 106 (1962).

228. N. D. Malmuth. Three-Dimensional Perturbations on Hypersonic Wedge Flow, AIAA J, v 2 (8), 1383-1389 (1964), [is the Russian translation/conversion: "rocket engineering and astronautics", 1964, No 8, 30-37].

Page 343.

229. N.D.Malmuth. Hypersonic Flow over a Delta Wing of Moderate Aspect Ration, AIAA, J, v 4 (3), 555-556 (1966). [Is the Russian translation/conversion: "rocket engineering and astronautics", 1966, No 3, 225-227).

230. V. N. Zhigulev. Conical transonic flows, PMM, Vol. XXI, No 3, 375-380 (1957).

231. G. Birkhgof. Hydrodynamics, IL, M., 1963.

232. J. A. Lukasiewicz. A. R. C. Technical Report, R. a. M. No 2669, 1-10 (1952). [Is a Russian translation/conversion in sb. "mechanic", 1954, No 4, 58-66].

233. R. E. Melnik. Vortical Singularities in Conical Flow, AIAA J, v 5 (4), 631-637 (1967). [Is the Russian translation/conversion: rocket engineering and astronautics, 1967, No 4, 28-36].

234. K. P. Stanyukovich. Unsteady motions of continuous medium, the State Technical Press, M., 1955.

AD-A066 205 FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO F/G 20/4

NONLINEAR CONICAL FLOWS OF GAS,(U)

MAR 78 B M BULAKH

UNCLASSIFIED FTD-ID(RS)T-0266-78 NL

7 OF 7

AD
A066205

END

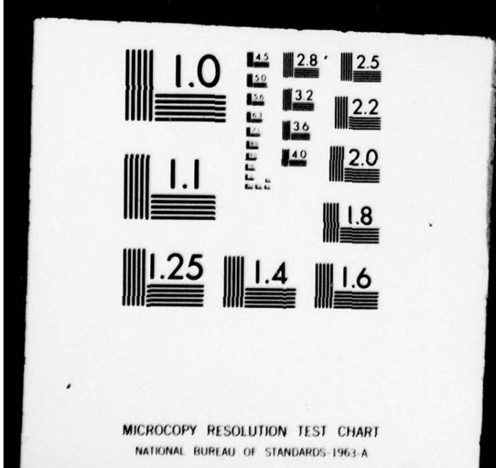
DATE
FILMED

5-79

DDC

7 OF 7

0
066205



ORGANIZATIO

A205 DMATC
A210 DMAAC
B344 DIA/R
C043 USAMI
C509 BALLI
C510 AIR M
LAB
C513 PICAT
C535 AVIAT

C591 FSTC
C619 MIA R
D008 NISC
H300 USAIC
P005 ERDA
P055 CIA/C
NAVORDSTA (

NASA/KSI

AFIT/LD

FTD-ID(RS

DISTRIBUTION LIST

DISTRIBUTION DIRECT TO RECIPIENT

ORGANIZATION		MICROFICHE
A205	DMATC	1
A210	DMAAC	2
B344	DIA/RDS-3C	8
C043	USAMIIA	1
C509	BALLISTIC RES LABS	1
C510	AIR MOBILITY R&D LAB/FIO	1
C513	PICATINNY ARSENAL	1
C535	AVIATION SYS COMD	1
C591	FSTC	5
C619	MIA REDSTONE	1
D008	NISC	1
H300	USAICE (USAREUR)	1
P005	ERDA	1
P055	CIA/CRS/ADD/SD	1
NAVORDSTA (50L)		1
NASA/KSI		1
AFIT/LD		1

ORGANIZATION		MICROFICHE
E053	AF/INAKA	1
E017	AF/ RDXTR-W	1
E404	AEDC	1
E408	AFWL	1
E410	ADTC	1
E413	ESD	2
	FTD	
	CCN	1
	TQIS	3
	NIA/PHS	1
	NICD	2